Asset Safety versus Asset Liquidity

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ABSTRACT

Many economists assume that safer assets are more liquid, and some have practically used "safe" and "liquid" as synonyms. But these terms are not synonyms, and mixing them up can lead to confusion and wrong policy recommendations. We build a multi-asset model where an asset's safety and liquidity are well-defined and distinct, and examine their relationship in general equilibrium. We show that the common belief that "safety implies liquidity" is generally justified, but also identify conditions under which this relationship can be reversed. We use our model to rationalize, qualitatively and quantitatively, a prominent safety-liquidity reversal observed in the data.

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1 Introduction

Recently, there has been a lot of attention on the role of safe assets and liquid assets in the macroeconomy. Many economists, both academics and practitioners, seem to believe that safer assets are also more liquid, and some go a step further by practically using the two terms as synonyms or by merging them into the single term "safe and liquid assets".[1] However, the terms are not synonyms: Safety refers to the probability that the (issuer of the) asset will pay the promised cash flow, at maturity, and liquidity refers to the ease with which an investor can sell the asset if needed, before maturity.[2] Mixing up an asset's safety and liquidity is not just semantics; it can lead to false conclusions and misguided policy recommendations.

For instance, when a credit rating agency characterizes a certain bond as AAA, should investors think of this as an assessment (only) of its safety or also of its liquidity? And, if the answer is affirmative, how can one explain the fact that (the virtually default free) AAA corporate bonds are considered less liquid than their riskier AA counterparts? Moreover, a recent literature in empirical macro-finance measures the so-called safety premium as the spreads between AAA and BAA bonds, assuming that these types of bonds are equally (il)liquid. But if certain assets carry different liquidity premia *because* they have different safety characteristics (as indicated by the conventional wisdom and confirmed by our theory), bonds of "equal liquidity" may be tricky to identify. Finally, policy makers and financial regulators are often concerned about liquidity in certain assets markets. If safety implies liquidity, could we just improve safety and let liquidity follow?

These questions reveal that it is essential to carefully study the relationship between asset safety and asset liquidity, rather than just assume that one implies the other. To do so, we build a multi-asset model in which an asset's safety and liquidity are well-defined and *distinct* from one another. Treating safety as a primitive, we examine the relationship

[1] The examples are numerous, so for the sake of brevity we highlight just two. From the IMF's 2012 Global Financial Stability Report: "Safe assets are a desirable part of a portfolio from an investor's perspective, as they [...] are highly liquid, permitting investors to liquidate positions easily." And at the 2017 American Economic Association meeting, one session was titled: "How safe and liquid assets impact monetary and financial policy".

[2] Although there are economists who adopt slightly different definitions for both of these terms. For instance, Gorton and Ordonez (2021) emphasize that an important aspect of *safe* assets is that they are "information insensitive". Also, a large number of papers in the New Monetarist literature, assume that an asset's *liquidity* refers to the ease with which that asset can be used to purchase consumption, e.g., by serving as a means of payment; see Lagos, Rocheteau, and Wright (2017). For a careful comparison of the various approaches, see the Literature Review (Section 1.1).

between an asset's safety and its liquidity. We show that the commonly held belief that "safer assets will be more liquid" is generally justified, but with important exceptions. We then describe the conditions under which a riskier asset can be more liquid than its safe(r) counterparts, and use our model to rationalize a prominent safety-liquidity reversal observed in the data. Finally, we highlight a surprising implication of our model about the effect of an increase in the supply of safe assets on welfare.

To answer the research question at hand we build a dynamic general-equilibrium model with two assets, A and B. The concept of asset safety is straightforward in our framework: asset A is "safe" in the sense that it always pays the promised cash flow, whereas asset B may default with a certain probability, known to everyone. The concept of liquidity is more involved; specifically, we define an asset's liquidity as the ease with which an agent can sell it for cash (if needed). To capture this idea, we employ the monetary model of Lagos and Wright (2005), extended to incorporate asset trade in over-the-counter (OTC) secondary asset markets subject to search and bargaining frictions, á la Duffie, Gârleanu, and Pedersen (2005). Another important ingredient we introduce is an entry decision made by the agents: each asset trades in a distinct OTC market, and agents choose to visit the market where they expect to find the best terms. Thus, in our model, an asset's liquidity depends on the *endogenous* choice of agents to visit the secondary market where that asset is traded, not the exogenous characteristics of that market.

More precisely, after agents make their portfolio decisions, two shocks are realized. The first is an idiosyncratic shock that determines whether an agent will have a consumption opportunity in that period, and the second is an aggregate shock that determines whether asset B will default in that period. Since purchasing the consumption good necessitates the use of a medium of exchange (i.e., money) and carrying money is costly, in equilibrium, agents who receive a consumption opportunity will visit a secondary market to sell assets and boost their cash holdings. Hence, assets have *indirect liquidity* properties (they can be sold for cash, although they do not serve directly as means of payment), and their equilibrium price in the primary market will typically contain a *liquidity premium*, i.e., it will exceed the fundamental value of holding the asset to maturity.

Our first result is that, other things equal, the safer asset carries a higher liquidity premium, and that premium is increasing in the default probability of asset B.[3] The intuition is as follows. An agent who turns out to be an asset seller can only visit one OTC market at a time; since, typically, assets are costly to own due to the liquidity premium, agents choose to 'specialize' ex-ante in asset A or B. Unlike sellers, who can only sell

[3] In the appendix, we establish a positive relationship between an asset's safety and liquidity under an alternative measure of liquidity, namely, trade volume in the secondary market.

assets they own and are therefore committed to the market corresponding to their chosen asset, asset buyers are free to visit any market they wish, since their money is good to buy any asset. As a result, if asset B defaults, all asset buyers will rush into the market for asset A. A second, more subtle force is that agents who specialize in the (riskier) asset B endogenously choose to carry more money for precautionary reasons. Consequently, asset B sellers need a smaller liquidity boost, which means that the surplus generated from trades in the market for asset B is lower. This force attracts more buyers to the market for asset A *even if asset B does not default*. Taken together, these forces imply that ex post (and regardless of the default shock realization) more buyers are attracted to the market for asset A, which in turn encourages agents to specialize in asset A ex ante, precisely because this asset will be easier to sell down the road. Through this channel a small default probability for asset B can be magnified into a big endogenous liquidity advantage for asset A, even with constant returns to scale (CRS) in the OTC matching technology.

So far we have assumed that all parameters other than asset safety are kept equal. Allowing for varying asset supplies delivers the second important result of the paper.[4] Even with slight increasing returns to scale (IRS) in OTC matching, demand curves can be *upward sloping*, because an asset in large supply is now more likely to be liquid. Consequently, asset B can be more liquid than asset A, despite being less safe, as long as the supply of the former is large enough compared to the latter.

The mechanism is as follows. When the supply of (say) asset B increases, so does the trading volume in its associated OTC market, which in turn makes this market more attractive to enter for both buyers and sellers. With CRS in the matching function, the larger numbers of buyers and sellers balance out so that the matching probabilities of either side are unchanged. And as more agents choose to specialize in asset B, each one of them will hold a smaller amount, whereas the reverse is true for holders of asset A, so that in equilibrium the surplus in any single trade is also balanced out across the two OTC markets. (We dub this the *dilution effect* since the asset in larger supply is 'diluted' among more agents holding it.) Now, with IRS in the matching function, the matching probabilities will *not* remain constant when more buyers and sellers enter market B, but increase for both sides. As a result, following an increase in the supply of asset B, entry into this market will remain attractive until the surplus of a single trade is substantially

[4] There are two more parameters held equal in the background: the efficiency of matching in each OTC market and the bargaining power of buyers versus sellers in each OTC market. Since our goal is to develop a theory that links asset safety and asset liquidity in an unbiased way, we assume that these parameters are always equal in both OTC markets. This guarantees that any difference in liquidity between the two assets is driven exclusively by differences in safety and not by exogenous market characteristics.

smaller than it would be in the market for asset A. In other words, with IRS, increased entry into a market tends to make this market even more attractive on the extensive margin, so entry will continue until the intensive margin becomes sufficiently unattractive via the dilution effect – i.e., agents trading in the thicker market must be trading small amounts, while agents in the smaller market trade larger amounts and get closer to satisfying their liquidity needs. But both of these forces imply that the asset in larger supply (and therefore with the thicker OTC market) will command a higher liquidity premium: first, because of its higher matching probability, and second, because a lower quantity of asset sold increases the marginal value of selling more, which is to say, the marginal value of carrying more of the asset in the first place.

Our model can shed some light on the puzzling empirical observation that, in the U.S., the virtually default-free AAA bonds are less liquid than (the less safe) AA corporate bonds (see Section 4.2.1 for details and empirical evidence). In recent years, regulations introduced to improve the stability and transparency of the financial system (most prominently, the Dodd-Frank Act) have made it especially hard for corporations to attain the AAA score. As a result, the supply of such bonds has fallen dramatically. During the same time, the yield on AA corporate bonds has been *lower* than that on AAA bonds, even without controlling for the risk premium associated with the riskier AA bonds. While it is plausible to attribute this differential to a higher liquidity premium enjoyed by AA corporate bonds – and this is precisely what practitioners have claimed – existing models of asset liquidity cannot capture this stylized fact (for details, see Section 1.1). Our approach of combining the 'indirect liquidity' mechanism with endogenous market entry can capture this stylized fact, and not only qualitatively: in Section 5, we calibrate our model and show it can reproduce the AA-AAA yield reversal quantitatively as well.[5]

The model also delivers a surprising result regarding welfare. A large body of recent literature highlights that the supply of safe assets has been scarce, and that increasing this supply would be beneficial for welfare (see for example Caballero, Farhi, and Gourinchas, 2017). In our model this result is not necessarily true: there exists a region of parameter values for which welfare is decreasing in the supply of the safe asset. The intuition is as follows. In our model agents have the opportunity to acquire additional cash by selling assets in the secondary market. When the safe asset becomes more plentiful, agents expect that it will be easier to acquire extra cash *ex-post* and, thus, choose to hold less money

[5] There are other cases where the common belief "safety and liquidity go together" is violated. Christensen and Mirkov (2019) document yet another class of bonds – Swiss Confederation Bonds – that are extremely safe, but not particularly liquid. And, vice versa, Beber, Brandt, and Kavajecz (2008) report that Italian government bonds are among the most liquid, but also the most risky of Euro-area sovereign bonds.

ex-ante. This channel depresses money demand, which, in turn, decreases the values of money and of the trade that the existing money supply can support.

1.1 Literature Review

Our paper is related to the "New Monetarist" literature (see Lagos et al., 2017) that highlights the importance of liquidity for the determination of asset prices; see Geromichalos, Licari, and Suárez-Lledó (2007), Lagos (2011), Nosal and Rocheteau (2013), Andolfatto, Berentsen, and Waller (2014), and Hu and Rocheteau (2015). Unlike these paper, where assets serve *directly* as means of payment, here liquidity is *indirect*, since agents sell assets for money in secondary asset markets. This approach is empirically relevant and integrates the concepts of liquidity adopted by monetary economics and finance (Geromichalos and Herrenbrueck, 2016). The indirect liquidity approach is employed in several recent papers, including Berentsen, Huber, and Marchesiani (2014, 2016), Han (2015), Mattesini and Nosal (2016), Herrenbrueck and Geromichalos (2017), Herrenbrueck (2019a), and Madison (2019). Our work is also related to the literature that studies how frictions in OTC markets affect asset prices and trade, such as Weill (2007, 2008), Lagos and Rocheteau (2009), Chang and Zhang (2015), and Üslü (2019). Vayanos and Weill (2008) is conceptually related to our paper, since the authors use a model with trade in OTC markets to rationalize a puzzling empirical observation known as the "on-the-run" phenomenon.

Our paper is also related to the growing literature that studies the role of safe assets in the macroeconomy. Examples include Gorton, Lewellen, and Metrick (2012), Piazzesi and Schneider (2016), Caballero et al. (2017), Gorton (2017), Caballero and Farhi (2018), and He, Krishnamurthy, and Milbradt (2019). None of these papers study the relationship between asset safety and asset liquidity. Two recent exceptions are Infante and Ordoñez (2020) and Ozdenoren, Yuan, and Zhang (2021). The former paper shows that increased volatility and illiquidity in the economy will raise the price of safe assets due to their ability to hedge idiosyncratic risks. The latter shows that safe assets earn a liquidity premium due to their ability to mitigate the adverse selection problem in asset markets.

Andolfatto and Martin (2013) consider a model where an asset, whose expected return is subject to a news shock, serves as medium of exchange. They show that the non-disclosure of news can enhance the asset's property as an exchange medium. As already mentioned, the concept of (indirect) liquidity adopted here is different, and so is the concept of safety.[6] Here, an asset's safety is simply the (ex-ante) probability with which the assets will pay the promised cash flow, which, in turn, is a function of the issuer's credit

[6] The authors never use the term "safety". However, the idea that some assets are more "information sensitive" is close to the definition of safety adopted by Gorton and Ordonez (2021); see footnote 2.

worthiness. This probability is public knowledge and can be thought of (or approximated by) a credit rating agency's score. Rocheteau (2011) studies a model where bonds serve as media of exchange alongside with money. The author shows that if the bond holders (and goods buyers) have private information about the bond's return, then money will endogenously arise as more liquid asset (i.e., a better medium of exchange). Our paper studies the link between asset safety and liquidity, assuming that the various assets' safety characteristics are public knowledge, i.e., we do not have a story of private information.

Our paper is also related to Lagos (2010), who considers a model where bonds, whose return is deterministic, and stocks, whose return is stochastic, compete as media of exchange. The author quantitatively demonstrates that the equity premium puzzle can be explained through a liquidity differential between the safe and the risky asset. Jacquet (2021) employs a similar model, but includes a larger variety of asset classes and ex-ante heterogeneous agents. The author shows that the equilibrium displays a "class structure" in the sense that agents with different liquidity needs will only be willing to hold assets of a certain risk structure. Our paper differs from the aforementioned papers, not only because it employs a different model of liquidity, but also because it predicts that an asset in large(r) supply may carry a higher liquidity premium. This result cannot be obtained in Lagos (2010), or other related papers, as in these papers the asset demand curve is decreasing. Thus, our model of indirect liquidity and endogenous market entry has the unique ability to rationalize why assets in limited supply can be highly illiquid, even when they enjoy a high credit rating (e.g., AAA corporate bonds in the U.S.).

He and Milbradt (2014) study a one-asset model where defaultable corporate bonds are traded in an OTC secondary market, and show that the inverse bid-ask spread, which is their proxy for bond liquidity, is positively related with credit ratings. However, in their model the probability of trade between agents is exogenous. We define liquidity as the ease with which an investor can sell her assets, if needed. We build a two-asset (easily extended to an N-asset) model, where the probability of selling an asset depends on the *endogenous* decision of agents to visit the various asset markets, which, in turn, is a function of each asset's safety characteristics. Also, He and Milbradt (2014) employ the model of Duffie et al. (2005) where assets are indivisible, i.e., agents can hold either 0 or 1 units of the asset. Our model also incorporates OTC secondary asset trade *à la* Duffie et al. (2005), but does so within the monetary model of Lagos and Wright (2005), which allows us to study perfectly divisible asset holdings, and opens up a number of new insights. Such insights include the possibility of upward-sloping demand curves, the possibility that a riskier asset can be more liquid in general equilibrium, and the fact that welfare can be decreasing in the supply of safe assets.

In related empirical work, Krishnamurthy and Vissing-Jorgensen (2012) distinguish between asset safety and liquidity, and extract safety and liquidity premia from the data to explain why Treasury yields have been decreasing. Their model identifies the safety premium through the spreads between AAA and BAA bonds, assuming that both of these types of bonds are equally illiquid. The present paper demonstrates that certain assets may carry different liquidity premia precisely because they are characterized by different default risks. This, in turn, highlights that we need more theory that studies the relationship between asset safety and liquidity, and we view the present paper as a part of this important agenda.

The rest of this paper is organized as follows. In Section 2, we describe the baseline model, and we solve it in Section 3. We present our main results in Section 4, and calibrate the model in Section 5. Finally, in Section 6, we discuss and provide microfoundations for key assumptions of our model. The Appendix contains further details and proofs.

2 The model

Our model is a hybrid of Lagos and Wright (2005) (henceforth, LW) and Duffie et al. (2005). Time is discrete and continues forever. Each period is divided into three subperiods, characterized by different types of trade (for an illustration, see Figures 1-2 below). In the first subperiod, agents trade in OTC secondary asset markets. In the second subperiod, they trade in a decentralized goods market (DM). Finally, in the third subperiod, agents trade in a centralized market (CM). The CM is the typical settlement market of LW, where agents settle their old portfolios and choose new ones. The DM is a decentralized market characterized by anonymity and imperfect commitment, where agents meet bilaterally and trade a special good. These frictions make a medium of exchange necessary, and we assume that only money can serve this role. The OTC markets allow agents with different liquidity needs to rebalance their portfolio by selling assets for money.

Agents live forever and discount future between periods, but not subperiods, at rate $\beta \in (0, 1)$. There are two types of agents, consumers and producers, distinguished by their roles in the DM. The measure of each type is normalized to the unit. Consumers consume in the DM and the CM and supply labor in the CM; producers produce in the DM and consume and supply labor in the CM. All agents have access to a technology that transforms one unit of labor in the CM into one unit of the CM good, which is also the numeraire. The preferences of consumers and producers within a period are given by $\mathcal{U}(X, H, q) = X - H + u(q)$ and $\mathcal{V}(X, H, q) = X - H - q$, respectively, where X denotes consumption of CM goods, H is labor supply in the CM, and q stands for DM goods produced and consumed. We assume that u is twice continuously differentiable, with $u' > 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $u'' < 0$. The term q^* denotes the first-best level of trade in the DM, i.e., it satisfies $u'(q^*) = 1$. All goods are perishable between periods.

Notice that in this model the agents dubbed "producers" will never choose to hold any assets, as long as these assets are priced at a premium for their liquidity. (Why pay this premium when they know they will never have a liquidity need in the DM?) As a result, all the interesting portfolio choices are made by the "consumers". Thus, henceforth, we will refer to the "consumers" simply as "agents". When we use the terms "buyer" and "seller", it will be exclusively to characterize the role of these agents in the secondary asset market.

There is a perfectly divisible object called fiat money that can be purchased in the CM at the price φ in terms of CM goods. The supply of money is controlled by a monetary authority, and follows the rule $M_{t+1} = (1+\mu)M_t$, with $\mu > \beta - 1$. New money is introduced if $\mu > 0$, or withdrawn if $\mu < 0$, via lump-sum transfers in the CM. Money has no intrinsic value, but it possesses all the properties that make it an acceptable medium of exchange in the DM, e.g., it is portable, storable, and recognizable. Using the Fisher equation, we summarize the money growth rate by $i = (1 + \mu - \beta)/\beta$; this rate will be a useful benchmark as the yield on a completely illiquid asset. (Thus, i should not be thought of as representing the yield on T-bills; see Geromichalos and Herrenbrueck, 2021.)

There are also two types of assets, asset A and asset B. These are one-period, nominal bonds with a face value of one dollar; their supply is exogenous and denoted by S_A, S_B. Asset $j = \{A, B\}$ can be purchased at price p_j in the CM, which we think of as the primary market. After leaving the CM agents receive an idiosyncratic consumption shock (discussed below) and may trade these assets before maturity in a secondary OTC market. Each asset j trades in a distinct secondary market, which we dub OTC$_j$, $j = \{A, B\}$. To make things tractable, we assume that agents can only hold asset A or asset B, and can visit only one OTC market per period. (In Section 6 we discuss this assumption in detail, and we provide microfoundations for it.) Thus, we say they "specialize" in asset A or B. However, agents are free to choose any quantity of money and the asset of their choice.

The economy is characterized by two shocks, both of which are revealed after the CM closes and before the OTC round of trade opens. The first is an aggregate shock that determines whether asset B will default or not in that period.[7] More precisely, we define

[7] The timing adopted here, i.e., assuming that the aggregate shock is revealed before agents go to the OTC, captures the idea that when an asset defaults, trade in its secondary market will also be impeded, which will affect its liquidity. Exploring this link between default and asset liquidity is one of the main

the aggregate state $s = \{n, d\}$ (n for "normal" and d for "default"); with probability π, $s = n$ and each unit of asset B pays the promised dollar, but with probability $1 - \pi$, $s = d$ where asset B defaults and pays nothing. Throughout the paper we assume that asset A is a perfectly safe and default-free asset.[8] This aggregate default shock is *iid* across time.

The second shock is an idiosyncratic consumption shock determining whether an agent will have an opportunity/desire to consume in the DM. Only a fraction $\ell < 1$ of agents obtains this opportunity. Thus, a measure ℓ of agents will be of type C ("Consuming"), and a measure $1 - \ell$ of agents will be of type N ("Not consuming"). This shock is *iid* across agents and time. Since the various types are realized after agents have made their portfolio choices in the CM, N-types will typically hold some cash that they do not need in the current period, and C-types may find themselves short of cash, since carrying money is costly. Placing the OTC round of trade after the CM but before the DM allows agents to reallocate money into the hands of the agents who need it most, i.e., the C-types.[9]

As we have discussed, agents can only trade in one OTC market per period, and they will choose to trade in the market where they expect to find the best terms. Suppose that a measure C_j of C-types and a measure N_j of N-types have chosen to trade in the market for asset $j = \{A, B\}$. Then, the matching technology:

$$f(C_j, N_j) = \left(\frac{C_j N_j}{C_j + N_j} \right)^{1-\eta} (C_j N_j)^{\eta}, \quad \eta \in [0, 1], \tag{1}$$

determines the measure of successful matches in OTC$_j$. The suggested matching function satisfies $f(C, N) \leq \min\{C, N\}$, and is useful because it admits both constant and increasing returns to scale (CRS and IRS, respectively) as subcases: when $\eta = 0$, the matching technology features CRS, while $\eta > 0$ implies IRS. Within each successful match the buyer and seller split the available surplus based on proportional bargaining (Kalai, 1977), with $\theta \in (0, 1)$ denoting the seller's (C-type's) bargaining power.[10] Notice that the matching

goals of our paper. Thus, assuming that agents find themselves in the secondary round of trade after learning that asset B has defaulted better corresponds to our research question.

[8] Our results are robust to different model specifications. For instance, modeling asset A as a default-free asset is done for simplicity and because many real-world assets characterized as AAA are virtually default free. However, all one needs is that asset A defaults with a lower probability than asset B. And to begin with, we assume that when asset B defaults, it defaults completely; however, we explore the case of partial default (fraction $\rho < 1$ of asset B's face value can be recovered) in Appendix C.1.

[9] The first paper to incorporate this idea into the LW framework is Berentsen, Camera, and Waller (2007), but there the reallocation of money takes place through a competitive banking system.

[10] The proportional bargaining solution is more tractable than the Nash solution. Moreover, in recent work, Rocheteau, Hu, Lebeau, and In (2021) solve a sophisticated model of bargaining with strategic foundations, and find that, under fairly general conditions, their solution converges to the proportional one.

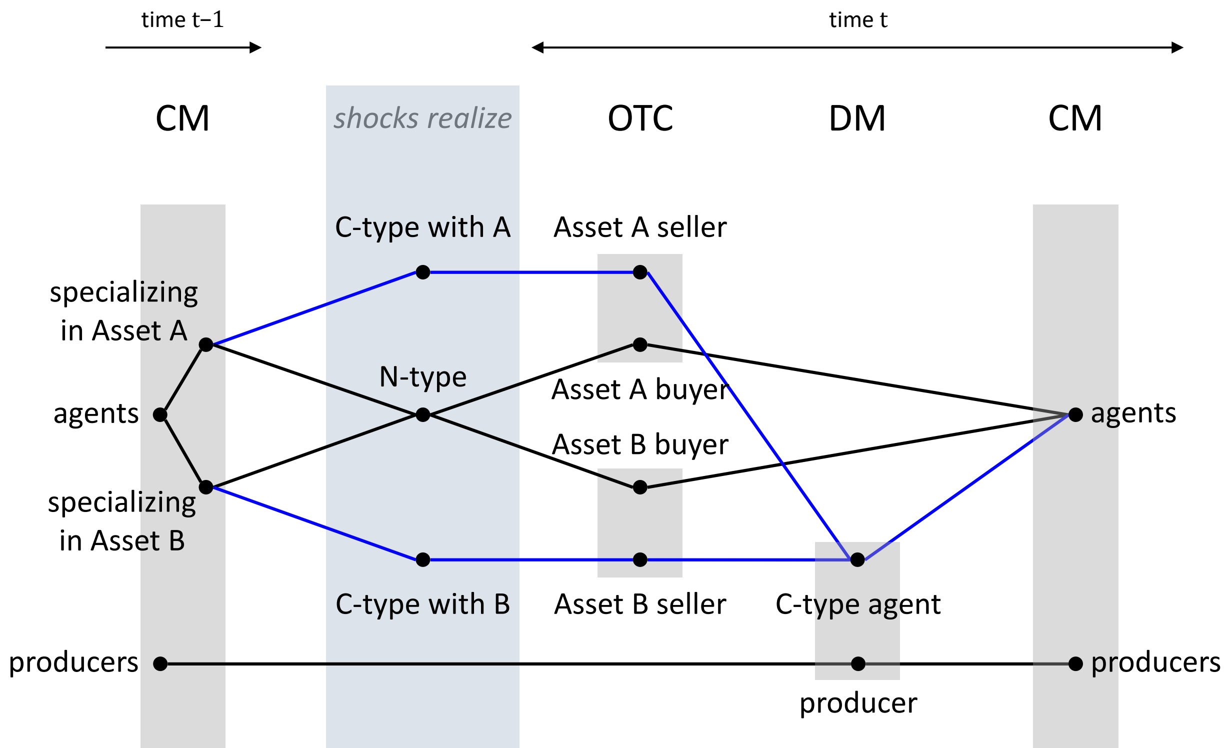


Figure 1. Timeline when both assets pay out

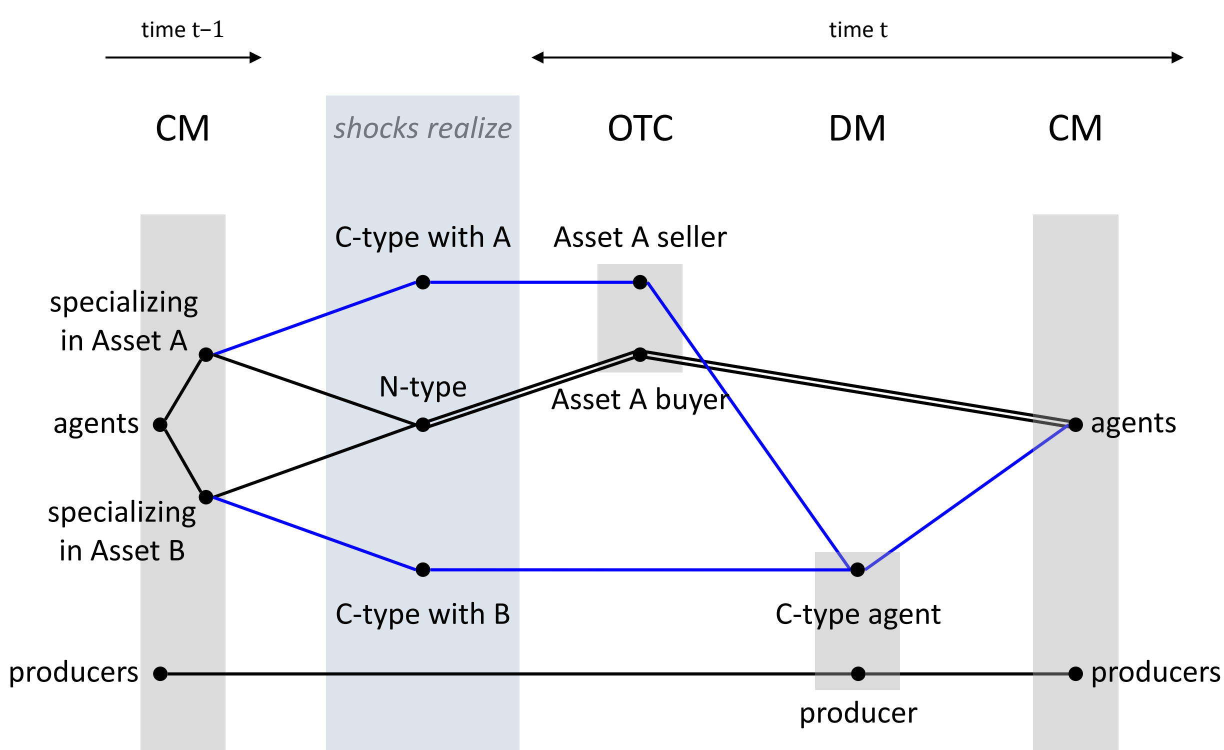


Figure 2. Timeline when asset B defaults

technology and the bargaining protocol are identical in both OTC markets. This guarantees that any differences in liquidity between assets A and B will be driven by differences in safety, and not by exogenous market characteristics (see footnote 4).

Since all the action of the model takes place in the CM and, more importantly, the OTC markets, we wish to keep the DM as simple as possible. To that end, we assume that all C-type consumers match with a producer, and they make a take-it-or-leave-it offer.

Figures 1 and 2 summarize the timing of events and the important economic actions of the model. A few details are worth emphasizing. First notice that agents who turn out to be C-types are *committed* to visit the OTC market of the asset they chose to specialize in. (One cannot sell asset B in OTC$_A$.) However, this is not true for N-types: an agent who turns out to be an N-type can visit either OTC market, because *her money is good* to buy any type of assets. This has an important consequence. In the default state (see Figure 2), OTC$_B$ will shut down so *all* N-types will rush into OTC$_A$. And what about the agents who specialized in asset B and turned out to be C-types? Unfortunately, they must proceed to the DM only with the money that they carried from the CM. But it is important to remember that agents are aware of this possibility and may choose to hold asset B anyway. Part of what makes this choice optimal is that they may pay a low(er) price for asset B and choose to carry more money as a precaution.

3 Analysis of the model

3.1 Value functions, bargaining, and matching probabilities

In order to streamline the analysis, we relegate the derivations of the value functions and the solutions of the various bargaining problems to Appendix A.1-A.2. Here is a summary of this analysis. As is standard in models that build on LW, all agents have linear value functions in the CM, a result that follows from the (quasi) linear preferences. This makes the bargaining solution in the DM easy to characterize. Consider a DM meeting between a producer and a C-type agent who carries m units of money, and define $m^* \equiv q^*/\varphi$ as the amount of money that (given the price φ) allows the agent to purchase the first-best quantity, q^*. Then, either $m \geq m^*$, and the buyer can purchase q^*, or $m < m^*$, and she spends all of her money to purchase the amount $q = \varphi m < q^*$.

Next, consider a meeting in OTC$_j$, $j = \{A, B\}$, where the N-type brings a quantity $\tilde{m}$ of money, and the C-type brings a portfolio (m, d_j) of money and asset j. Since money is costly to carry, in equilibrium we will have $m < m^*$, and the C-type will want to acquire the amount of money that she is missing in order to reach m^*, namely, $m^* - m$. Whether she will be able to acquire that amount of money depends on her asset holdings. If her asset holdings are enough (of course, how much is "enough" depends on the bargaining power θ), then she will acquire exactly $m^* - m$ units of money. If not, then she will give up all her assets to obtain an amount of money $\xi(m, d_j) < m^* - m$, which is increasing in d_j (the more assets she has, the more money she can acquire) and decreasing in m (the more money she carries, the less she needs to acquire through OTC trade). This last case, where assets are scarce, is especially interesting, because it is precisely then that having a few more assets would have allowed the agent to alleviate the binding cash constraint, which is why an asset price will carry a liquidity premium.

This discussion assumes that $m+\tilde{m} \geq m^*$, i.e., that the money holdings of the C-type and the N-type pooled together are enough to allow the C-type to purchase the first best quantity q^*. We restrict attention to this case for two reasons. First, our calibration in Section 5 reveals that this case is the relevant one (see footnote 25). Second, the equilibrium associated with the complementary case, where $m + \tilde{m} < m^*$, would imply that none of the assets carry a positive liquidity premium, a result which is clearly unrealistic. The reason is simple: when $m + \tilde{m} < m^*$, the money of the two agents pooled together is so scarce that asset holdings become relatively plentiful. Given these two points, we choose to restrict attention to $m+\tilde{m} \geq m^*$. This simplifies the analysis since it implies that we do not need to keep track of N-types' money holdings.

One of the innovations of our paper is that the OTC matching probabilities are a function of agents' entry choices. Let $e_C \in [0, 1]$ be the fraction of C-type agents who specialize in asset A and are thus committed to trading in OTC$_A$, no matter the eventual aggregate state. And let $e_N^s \in [0, 1]$ be the fraction of N-type agents who enter OTC$_A$ in aggregate state $s = \{n, d\}$. (These terms will be carefully described in Section 3.4.) Then, in state s, $e_C\ell$ is the measure of C-types and $e_N^s(1-\ell)$ is the measure of N-types who enter OTC$_A$. Similarly, $(1-e_C)\ell$ is the measure of C-types and $(1-e_N^s)(1-\ell)$ is the measure of N-types who enter OTC$_B$. As a result, we need to track eight different matching probabilities α_{ij}^s, of an i-type who enters OTC$_j$ in state s (though $\alpha_{CB}^d = \alpha_{NB}^d = 0$ will be trivially true in the full-default version of our model).

The matching probability of a C-type who enters OTC$_A$ in a non-default state provides a representative example:

$$\alpha_{CA}^n \quad \equiv \quad \frac{f[e_C\ell, e_N^n(1-\ell)]}{e_C\ell} \quad = \quad \left[\frac{e_N^n(1-\ell)}{e_C\ell + e_N^n(1-\ell)}\right] \cdot [e_C\ell + e_N^n(1-\ell)]^\eta.$$

It is the product of two terms: a standard *market tightness* term whereby an asset seller benefits from asset buyers entering on the other side of the market but is congested by asset sellers entering on the same side, and a *market size* term whereby any market participant benefits from any other market participant's presence. This market size term is governed by the elasticity η, and in particular it is shut down when $\eta = 0$.

3.2 Optimal portfolio choice

As is standard in models that build on LW, all agents choose their optimal portfolio in the CM independently of their trading histories in previous markets. In our model, in addition to choosing an optimal portfolio of money and assets, $(\hat{m}, \hat{d}_A, \hat{d}_B)$, agents also choose which OTC market they will enter in order to sell or buy assets, once the shocks have been realized. The agent's choice can be analyzed with an objective function, $J(\hat{m}, \hat{d}_A, \hat{d}_B)$, which we derive in Appendix A.3 and reproduce here for convenience:

$$\begin{aligned} J(\hat{m}, \hat{d}_A, \hat{d}_B) \equiv & -\varphi(\hat{m} + p_A \hat{d}_A + p_B \hat{d}_B) + \beta \hat{\varphi}(\hat{m} + \hat{d}_A + \pi \hat{d}_B) \\ & + \beta \ell \Big(u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi \max\{\alpha^n_{CA} \mathcal{S}_{CA}, \alpha^n_{CB} \mathcal{S}_{CB}\} + (1-\pi)\alpha^d_{CA} \mathcal{S}_{CA} \Big), \end{aligned}$$

where $\mathcal{S}_{Cj}$ is the surplus of an agent who turns out to be a C-type and trades in OTC$_j$:

$$\mathcal{S}_{Cj} = u(\hat{\varphi}(\hat{m} + \xi_j(\hat{m}, \hat{d}_j))) - u(\hat{\varphi}\hat{m}) - \hat{\varphi}\chi_j(\hat{m}, \hat{d}_j).$$

In the above expression, ξ_j stands for the amount of money that the agent can acquire by selling assets, and χ_j stands for the amount of assets sold in OTC$_j$, $j = \{A, B\}$.

The interpretation of J is straightforward. The first term is the cost of choosing the portfolio $(\hat{m}, \hat{d}_A, \hat{d}_B)$. This portfolio yields the expected payout $\hat{\varphi}(\hat{m} + \hat{d}_A + \pi \hat{d}_B)$ in next period's CM (the second term of J). The portfolio also offers certain liquidity benefits, but these will only be relevant if the agent turns out to be a C-type; thus, the term in the second line of J is multiplied by ℓ. The C-type can enjoy at least $u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m}$ just with the money that she brought from the CM. Furthermore, she can enjoy an additional benefit by selling assets for cash in the secondary market. How large this benefit is depends on the market choice of the agent (the term inside the $\max$ operator) and on the realization of the aggregate shock: if asset B defaults, an event that happens with probability $1 - \pi$, a C-type who specialized in that asset has no benefit. A default of asset B is not the only reason why the C-type may not trade in the OTC markets; it may just be that she did not match with a trading partner. This is why the various surplus terms $\mathcal{S}_{Cj}$ are multiplied by the α-terms, i.e., the matching probabilities discussed in Section 3.1.[11]

[11] There are two reasons why the objective function does not contain any term that represents the event in which the agent is an N-type. First, and most obviously, N-types are defined as the agents who do not get to consume in the DM. Second, the OTC terms of trade, χ and ξ, depend only on the portfolio of the C-type. An intuitive explanation was presented in Section 3.1; for the details, see Appendix A.2.

3.3 Equilibrium

We focus on steady-state equilibria. Before we move on to characterizing possible equilibria, we first need to understand their structure. We have twelve endogenous variables to be determined in equilibrium (not including the terms of trade in the OTCs):

- prices: φ, p_A, p_B
- real balances: z_A, z_B
- entry choices: $e_C(\equiv e_C^n = e_C^d)$, e_N^n, e_N^d
- DM production: $q_{0A}(\equiv q_{0A}^n = q_{0A}^d)$, $q_{1A}(\equiv q_{1A}^n = q_{1A}^d)$, $q_{0B}(\equiv q_{0B}^n = q_{0B}^d)$, $q_{1B}(\equiv q_{1B}^n)$

In this list of equilibrium variables, the asset prices are obvious, and z_j, $j = \{A, B\}$, is simply the real balances held by an agent who chooses to specialize in asset j. The remaining terms deserve some discussion. First, notice that the fraction of C-types who enter OTC$_A$, e_C, does not depend on the aggregate state $s = \{n, d\}$. This is because C-types are committed to visiting the OTC market of the asset they chose to specialize in (and this choice is effectively made before the realization of the shock).

Regarding the DM production quantities q_{kj}, $k = \{0, 1\}$ indicates whether the C-type did ($k = 1$) or did not ($k = 0$) trade in the preceding OTC market, and $j = \{A, B\}$ indicates the asset in which she specializes. For example, q_{0A} is the amount of DM good purchased by an agent who specialized in asset A and did not match in OTC$_A$, and so on. These quantities do not depend on the aggregate state $s = \{n, d\}$. To see why, notice that q_{0A} depends only on the amount of real balances that the agent carried from the CM (this agent did not trade in the OTC), and that choice was made before s was realized. The same reasoning applies to q_{0B}. How about the term q_{1A}? This term depends on the real balances that the agent carried from the CM (which, we just argued, is independent of the shock realization), and on the amount of assets this agent carries from the CM (see Section 3.1). How many assets does this agent carry? The answer is S_A/e_C: the exogenous asset supply, S_A, divided by the measure of agents who specialize in asset A. Since S_A is a parameter, and e_C is independent of the state s, the same will be true for the term q_{1A}.

To simplify the exposition of the equilibrium analysis, we focus on the block of variables $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e_N^n\}$, which we refer to as the "core" variables. In Appendix A.6, we establish that the remaining variables, $\{z_A, z_B, \varphi, p_A, p_B\}$, follow immediately once the core variables have been determined. Of course, e_N^d is always equal to 1. To determine our six core variables, we proceed as follows.

First, we have two money demand equations for agents who specialize in asset A and B: Equations (A.14)-(A.15) in Appendix A.6, derived in Appendix A.5. What is important to remember here is that agents who choose to specialize in different assets will typically carry different amounts of money; not surprisingly, agents who choose to carry the less safe asset B self-insure against the probability of default (and the shutting down of OTC$_B$) by carrying more money.

Next, we use asset market clearing to solve for the outcome of the OTC bargaining protocol in a form that involves only core equilibrium variables: Equations (A.16)-(A.17) in Appendix A.6. Intuitively, they state that if the agent's asset holdings are large, then $q_{1j} = q^*$, because the agent will acquire (through selling assets) the money necessary to purchase the first-best quantity, and no more. In contrast, if the agent's asset holdings are scarce, she will give up all her assets and purchase an amount of DM good equal to q_{0j} (the amount she could have purchased without any OTC trade) plus the additional amount she can now afford by selling assets for extra cash.

Our last two equilibrium conditions come from the optimal OTC market entry decisions of agents. An important remark is that the OTC surplus of N-types does not depend on their portfolios (see Section 3.1 or A.2), whereas the OTC surplus of C-types does depend on their portfolios. Hence, in making their entry decisions, C-types consider not only the expected surplus of entering in either market, as is the case for N-types, but also the cost associated with each entry decision. Another way of stating this is to say that e_C is determined ex ante and represents the decision to specialize in asset A, while e_N^n is determined ex post and represents the fraction of N-types who enter OTC$_A$ in the normal state. Therefore, the optimal entry of C-types is characterized by:

$$e_C = \begin{cases} 1, & \tilde{\mathcal{S}}_{CA} > \tilde{\mathcal{S}}_{CB} \\ 0, & \tilde{\mathcal{S}}_{CA} < \tilde{\mathcal{S}}_{CB} \\ \in [0,1], & \tilde{\mathcal{S}}_{CA} = \tilde{\mathcal{S}}_{CB} \ , \end{cases} \tag{2}$$

where the *ex-ante* surplus terms $\tilde{\mathcal{S}}_{CA}$ and $\tilde{\mathcal{S}}_{CB}$ are defined in Equations (A.18)-(A.19). The optimal entry of N-types is characterized by:

$$e_N^n = \begin{cases} 1, & \alpha_{NA}^n \mathcal{S}_{NA} > \alpha_{NB}^n \mathcal{S}_{NB} \\ 0, & \alpha_{NA}^n \mathcal{S}_{NA} < \alpha_{NB}^n \mathcal{S}_{NB} \\ \in [0,1], & \alpha_{NA}^n \mathcal{S}_{NA} = \alpha_{NB}^n \mathcal{S}_{NB} \ , \end{cases} \tag{3}$$

in the normal state, where the *ex-post* surplus terms $\mathcal{S}_{NA}$ and $\mathcal{S}_{NB}$ are defined in Equations (A.20)-(A.21), and by:

$$e_N^d = \begin{cases} 1, & e_C > 0 \\ \in [0,1], & e_C = 0 \ , \end{cases} \tag{4}$$

in the default state.

We can now define the steady-state equilibrium of the model:

Definition 1. For given asset supplies $\{A, B\}$, the steady-state equilibrium for the core variables of the model consists of the equilibrium quantities and entry choices, $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e_N^n\}$, such that (A.14), (A.15), (A.16), (A.17), (2) and (3) hold. The equilibrium real balances, $\{z_A, z_B\}$, satisfy (A.10), the equilibrium price of money, φ, solves (A.11), and the equilibrium asset prices, $\{p_A, p_B\}$, solve (A.12) and (A.13).

3.4 Equilibrium market entry

In this section we analyze the optimal entry decision of agents, which is at the heart of our model. To begin with, we fix a value of e_C (the fraction of agents who specialize in asset A and, if they should turn out to be C-types, are thus committed to sell in OTC$_A$), compute the rest of the core equilibrium variables optimally (including the entry decision of N-types in the normal state, $e_N^n(e_C)$), and then evaluate the best response of a representative agent to our initial guess of e_C. This task becomes easier by recognizing that there are three opposing forces at work. We dub them the congestion effect, the coordination effect, and the dilution effect; they are described in detail in Appendix A.7, so we focus on an intuitive summary here.

First, a high initial e_C will *discourage* the representative agent from holding asset A because it implies that there will be many sellers in OTC$_A$; holding the measure of buyers constant, it becomes harder to match for each of the sellers. This is the "congestion effect". However, a high e_C also attracts buyers to market A, i.e., it implies a high e_N^n, which in turn improves sellers' matching probability and *encourages* the representative agent to hold asset A; this is the "coordination effect". With CRS the two effects tend to balance out, while with IRS the coordination effect becomes stronger. A more subtle force is the "dilution effect". When e_C is high, many agents specialize in asset A, and each one of them carries a small fraction of the (fixed) asset supply. As a result, the surplus generated per meeting in OTC$_A$ will be small, another force that *discourages* the representative agent from holding asset A when many others already do so.

Moving to the formal analysis, we construct equilibria as fixed points of e_C. To be specific: first, we fix a level of e_C; then we solve for the optimal portfolio choices through equations (A.14)-(A.17) and (3); and finally, we define the C-types' *best response function*:

$$G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB},$$

where the surplus terms have the optimal choices substituted. This function measures the relative benefit to an *individual* C-type from specializing in asset A over asset B, assuming a proportion e_C of all *other* C-type agents specialize in A, and all other decisions (portfolios and entry of N-types) are conditionally optimal. We say that a value of e_C is part of an "interior" equilibrium if $e_C \in (0,1)$ and $G(e_C) = 0$, or a "corner" equilibrium if $e_C = 0$ and $G(0) \le 0$ (B-corner) or $e_C = 1$ and $G(1) \ge 0$ (A-corner).

Proposition 1. *The following types of equilibria exist, and have these properties:*

(a) There exists a corner equilibrium where $e_C = 0$, $e_N^n = e_N^d = 0$.

(b) There exists a corner equilibrium where $e_C = 1$, $e_N^n = e_N^d = 1$.

(c) Assume $\eta = 0$ *(CRS) and asset supplies are low enough so that assets are scarce in OTC trade. Then,* $\lim_{e_C \to 0+} G(e_C) > 0 > G(0)$*; the equilibrium at the* B*-corner is not robust to small trembles.*

(d) Assume $\eta = 0$ *(CRS) and asset supplies are low enough so that assets are scarce in OTC trade. Then,* $\lim_{e_C \to 1} G(e_C) < G(1)$*. If* $\pi \to 1$*, then the limit is negative, and the equilibrium at the* A*-corner is not robust, either.*

(e) Assume $\eta = 0$ *(CRS),* $\pi \to 1$*, and asset supplies are low enough so that assets are scarce in OTC trade. Then, there exists at least one interior equilibrium which is robust.*

(f) Given $\eta > 0$ *(IRS),* $\lim_{e_C \to 0+} G(e_C) \neq G(0)$.

(g) Given $\eta > 0$ *(IRS),* $\lim_{e_C \to 1-} G(e_C) = G(1) > 0$*; the equilibrium at the* A*-corner is robust.*

Proof. See Appendix B.1. □

Figures 3 and 4 illustrate these results for the CRS and the IRS case, respectively. The left panel of each figure depicts the individual C-type's best response function, $G(e_C)$. Since this function depends not only on the behavior of fellow C-types, but also on that of N-types, on the right panel of each figure we show the optimal entry choice of N-types, e_N^n, as a function of e_C. The figures also illustrate how equilibrium entry is affected by changes in the supply of asset A, keeping the supply of asset B constant.

As indicated in the right panel of each figure, we have $e_N^n(0) = 0$ and $e_N^n(1) = 1$: when all C-types are concentrated in one market, the N-types will follow. Generally, the higher e_C is, the more N-types would like to go to market A: this is just the coordination effect and it tends to make $e_N^n(e_C)$ increasing. Whether it will be strictly increasing or not, ultimately depends on the strength of the dilution effect relative to the coordination effect. This is why in both figures, $e_N^n(e_C)$ is increasing when A is large: it is a large asset supply that weakens the dilution effect.[12]

[12] There is also a difference between the two figures. In Figure 3 (CRS case), $e_N^n(e_C)$ is strictly increasing in its entire domain. However, in Figure 4 (IRS case), and for the case of large S_A, $e_N^n(e_C)$ reaches 1 for a

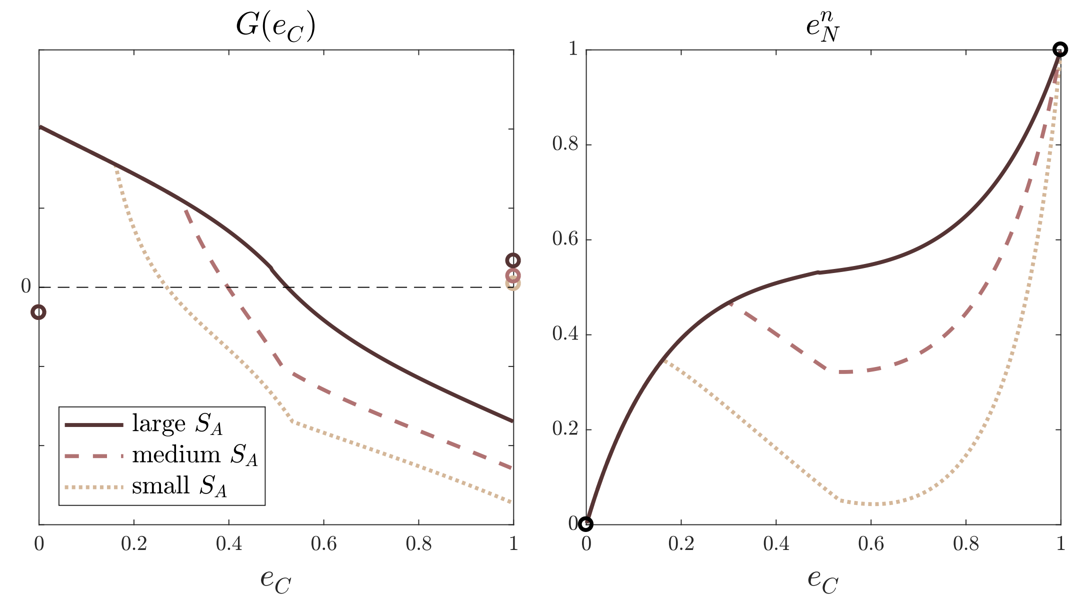


Figure 3. C-types' incentive to deviate and N-types' optimal entry choice, given e_C, for the case of CRS

Notes: The figure depicts the function $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB}$ (left panel) and the optimal response of N-types, e_N^n (right panel), as functions of aggregate e_C, assuming CRS in matching ($\eta = 0$). Equilibrium entry is illustrated for three levels of asset supply S_A, keeping the supply of asset B constant. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $\pi = 0.95$, $S_A = \{0.05, 0.09, 0.14\}$, $S_B = 0.14$, with log utility. We refer to the corner with $e_C = 0$ (all C-types are specializing in asset B) as the B-corner, and the corner with $e_C = 1$ (all C-types are specializing in asset A) as the A-corner.

Next, we have $G(0) < 0$ and $G(1) > 0$, while $e_N^n(0) = 0$ and $e_N^n(1) = 1$. This illustrates parts (a) and (b) of Proposition 1; the corners are always equilibria (marked with circles on the left panel of the figures). However, with CRS these equilibria are not robust because G is discontinuous at the corners; this illustrates parts (c) and (d) of the proposition.[13] Also, with CRS the congestion effect is so dominant that the G-function is globally decreasing in the interior, as shown in part (e) of the proposition and illustrated in Figure 3. Therefore, there exists a robust interior equilibrium where the representative C-type is indifferent between entering market A or B; i.e., $G(e_C) = 0$. As the supply of asset A increases, so

rather small value of e_C and becomes flat afterwards. This is because with IRS, the desire of N-types to go to the market with many C-types, i.e., the coordination effect, is supercharged.

[13] More precisely, they are not "trembling hand perfect" Nash equilibria. Consider for example the equilibrium with $e_N^n = e_C = 1$ (a similar argument applies to the one with $e_N^n = e_C = 0$). Since all N-types visit market A, the representative C-type also wishes to visit that market. (Why try to trade in a ghost town, which OTC$_B$ is in this case?) However, if an arbitrarily small measure ε of N-types visited market B by error, the representative C-type would have an incentive to deviate to market B, where her chance of matching is now extremely high (since $e_C = 1$, she would be the only C-type in that market).

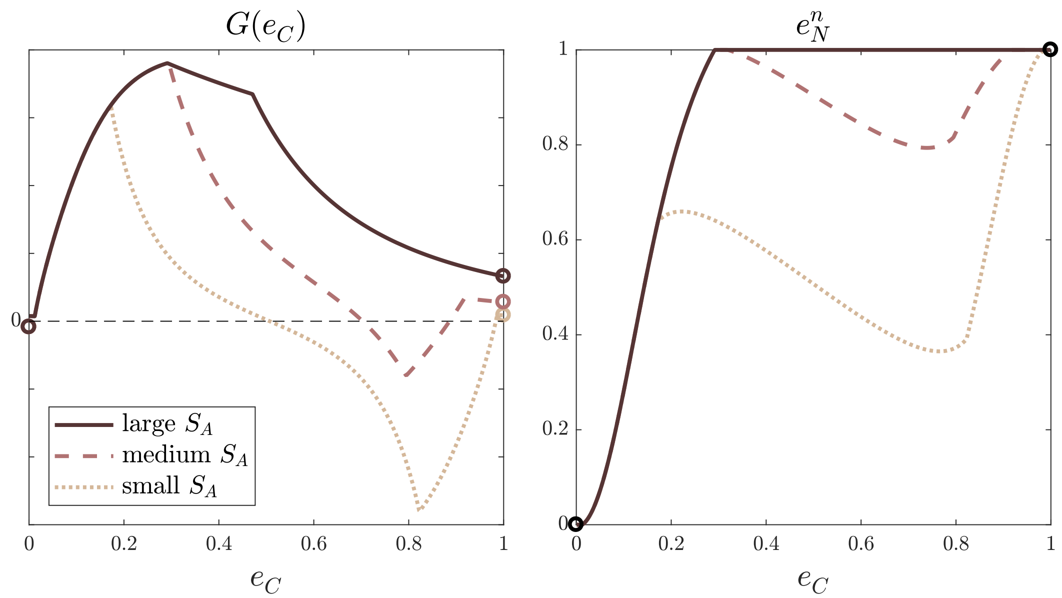


Figure 4. C-types' incentive to deviate and N-types' optimal entry choice, given e_C, for the case of IRS

Notes: The figure depicts the function $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB}$ (left panel) and the optimal response of N-types, e_N^n (right panel), as functions of aggregate e_C, assuming IRS in matching ($\eta = 0.3$). Equilibrium entry is illustrated for three levels of asset supply S_A, keeping the supply of asset B constant. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $\pi = 0.95$, $S_A = \{0.05, 0.09, 0.14\}$, $S_B = 0.05$, with log utility. We refer to the corner with $e_C = 0$ (all C-types are specializing in asset B) as the B-corner, and the corner with $e_C = 1$ (all C-types are specializing in asset A) as the A-corner.

does the equilibrium value of e_C, because a larger asset supply weakens the dilution effect and increases the incentives of agents to concentrate on market A.

Moving on to the IRS case, the two corner solutions are still equilibria, and since IRS strengthen the coordination effect, the equilibrium where all agents go to OTC$_A$ ($e_N^n = e_C = 1$) is now robust (part (g) of the proposition). This may or may not be true for the B-corner ($e_N^n = e_C = 0$), depending on the values of π and η.[14] Figure 4 demonstrates the case of non-robustness; as shown in part (f) of the proposition, the best response

[14] Consider first the equilibrium with $e_N^n = e_C = 1$. With IRS the desire to go to OTC$_A$ (where all agents are concentrated) is so strong that, even if some N-types visit OTC$_B$ by error, the representative C-type no longer has an incentive to deviate to that market (unlike the CRS case; see footnote 13). But the channel described so far is relevant for both corners. So why is the equilibrium where all agents go to OTC$_B$ not always robust as well? Because OTC$_B$ is the market of the asset that may default. When that happens (ex-post), all N-types will rush to market A, i.e., $e_N^d = 1$, and this creates an additional incentive for the representative C-type to deviate to market A (a decision made ex-ante). This additional incentive will be relatively large, when π is low (high default probability) and η is low (weak coordination effect). Therefore, the equilibrium with $e_N^n = e_C = 0$ is likely to be non-robust for relatively low values of π and η.

function is discontinuous at the B-corner, though it is continuous at the A-corner. With the coordination effect amplified, multiple interior equilibria are typical (as in the case of "small S_A" and "medium S_A"). However, the only robust interior equilibrium is the one where G has a negative slope. A rise in S_A will lead to an increase in the (interior and robust) equilibrium value of e_C. But with IRS, another interesting possibility arises: if S_A is large enough, the desire of agents to coordinate on OTC$_A$ is so strong that interior equilibria cease to exist. This is depicted in the "large S_A" case, where one can see that the A-corner (with $e_N^n = e_C = 1$) is the unique robust equilibrium entry outcome.

3.5 Liquidity premia

Most of our main results will be about the *liquidity premia* assets A and B may carry. In equilibrium, asset prices consist of a "fundamental value" multiplied by a premium that reflects the possibility of selling the asset in the OTC market. We define the fundamental value of an asset as the equilibrium price that would emerge if this possibility was eliminated. In that case, agents would value the assets only for their payouts at maturity, and the equilibrium prices would be given by $1/(1+i)$, for asset A, and $\pi/(1+i)$, for asset B.

The liquidity premium of asset j, denoted by L_j, is therefore defined as the percentage difference between an asset's price and its fundamental value:

$$p_A = \frac{1}{1+i}(1+L_A), \qquad\qquad p_B = \frac{\pi}{1+i}(1+L_B), \tag{5}$$

where p_A and p_B are described by the asset pricing equations (A.12) and (A.13), and:

$$\begin{aligned} L_A &= \ell \cdot \left(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\right) \cdot \frac{\theta}{\omega_\theta(q_{1A})} \cdot (u'(q_{1A}) - 1), \\ L_B &= \ell \cdot \alpha_{CB}^n \cdot \frac{\theta}{\omega_\theta(q_{1B})} \cdot (u'(q_{1B}) - 1). \end{aligned} \tag{6}$$

Each liquidity premium is the product of four terms. First, the probability that an agent turns out to be a C-type and thus needs liquidity at all (ℓ). Second, given that the agent is a C-type, the expected probability of matching in the respective OTC market, conditional on entering that market. Third, the share of the marginal surplus captured by the C-type (θ/ω_θ), which is endogenous but constrained to the interval $(0, \theta]$. And fourth, the marginal surplus of the match: the utility gained by a consumer who brings one more unit of real balances into the DM, net of the production cost ($u'(q_{1j}) - 1$).

Thus, there are two ways a liquidity premium can be zero: either the relevant OTC market is closed ($\alpha_{Cj} = 0$), or assets are so plentiful that selling an extra asset in the OTC does not create additional surplus in the DM ($q_{1j} = q^*$, thus $u'(q_{1j}) = 1$). In the latter case, the asset is still "liquid", but its liquidity is *inframarginal* so it does not affect the price.

4 Main Results

4.1 Result 1: Safe and liquid

The first result of the paper is that, other things equal, the safer asset (A) tends to be more liquid. We demonstrate this result adopting the liquidity premium as the measure of liquidity. However, in Appendix B.4, we confirm that this result remains valid if one adopts trade volume in the OTC market as the measure of asset liquidity (this approach is common in the finance literature). Throughout Section 4.1, we assume that the supplies of the two assets are equal ($S_A = S_B$), in order to focus on liquidity differences purely due to the assets' safety differential. Because of the complexity of our model, we break our analysis into two stages.[15] First, we take a local approximation of our model around $\pi \to 1$, assuming CRS ($\eta = 0$). The perturbation of this specification with small changes in π can be solved in closed form (see Appendix B.2). Second, in order to obtain global results away from $\pi \to 1$, and to conduct comparative statics with respect to η, we solve the model numerically. The following proposition summarizes our analytical results:

Proposition 2. *Assume that asset supplies S_A and S_B are equal and are low enough so that assets are scarce in OTC trade. Then:*

(a) At $\pi = 1$, there exists a symmetric equilibrium where $e_C = e_N = 0.5$, $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $L_A = L_B$.

(b) Assume $\eta = 0$ (CRS) and $(1 - \ell)\theta$ is sufficiently large. Then, locally, $\pi < 1$ implies $L_A > L_B$: the safer asset is more liquid.

Proof. See Appendix B.2. □

Naturally, when $\pi = 1$, the two assets are perfect substitutes and their equilibrium prices (and liquidity premia) will be equal. However, as π falls below 1, the liquidity premium of asset A generally exceeds that of asset B. Near the symmetric equilibrium, the derivative of the difference between the liquidity premia with respect to π is:

$$\left.\frac{d(L_A - L_B)}{d\pi}\right|_{\pi \to 1} = \ell\theta \frac{u'(q_1) - 1}{w_\theta(q_1)}(\alpha^n_{CA} - \alpha^d_{CA}) \ \ldots$$

[15] Our model has six 'core' equilibrium variables, most of which show up in multiple equations; these equations are non-linear and include kinks, due to the various branches of the bargaining solutions and the agents' market entry decisions. Simply put, every time a parameter value changes, all six endogenous variables are affected by simultaneous and, typically, opposing forces. For more detail, one can inspect matrix equation (B.8) in the Appendix, which describes the effect of changes in π on the core variables in general equilibrium, keeping in mind that this matrix is evaluated at the limit as $\pi \to 1$.

$$+\ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}\times\frac{d(q_{1A}-q_{1B})}{d\pi}+\ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}\times\frac{d(\alpha^n_{CA}-\alpha^n_{CB})}{d\pi}$$

The first term on the right-hand side represents a negative *direct effect*: the probability of meeting a buyer for asset A is always lower in the normal state than in the state where B defaults ($\alpha^n_{CA} < \alpha^d_{CA}$), therefore the liquidity advantage of asset A increases as B becomes less safe ($\pi\downarrow$, i.e., $d\pi < 0$). But this liquidity advantage is magnified by the endogenous responses of agents to *perceived* default risk, which affect what happens even in the normal state. Consider the second term in the equation. An agent who specializes in asset B despite the default risk will self-insure by carrying more money, which translates (after OTC trade) to a higher q_{1B}, resulting in a lower marginal utility of selling asset B (indicated by multiplication with $u''(q_1) < 0$) and thus a lower liquidity premium for asset B; thus, this *intensive margin effect* is also negative and always reinforces the direct effect.

Finally, there is the the third term in the equation, representing an *extensive margin effect*: generally, when $\pi < 1$, N-types respond more strongly to the lower trading surplus in the B-market, thus the matching probability for C-types is higher in OTC$_A$. If so, then all three effects point in the same direction and thus the overall sign of the equation is negative, as per part (b) of Proposition 2. Analytically, we can show that this is indeed the case when $(1-\ell)\theta$ is sufficiently large; numerically, we can find counterexamples, but the overall negative sign is still the predominant result.[16]

Figure 5 illustrates our result for a range of π, and for both CRS and an intermediate degree of IRS. In each of these cases, the difference between L_A and L_B is positive and strictly decreasing in π. It is important to remind the reader that this differential is purely due to liquidity; it is not a risk premium. Indeed, decreasing π makes agents less willing to hold asset B because that asset is now at higher risk of default, but that effect is already included in the fundamental value of the assets (see equations 5). The new result here is that as asset B becomes less safe it also enjoys a smaller liquidity premium on top of the smaller fundamental value.

The intuition behind Result 1 is as follows. Unlike C-types, who are committed to visit the market of the asset in which they chose to specialize, N-types are free to visit any market they wish, since their money is good to buy any asset. Consequently, *if asset*

[16] To be precise, we checked the sign for all combinations of θ and ℓ in $\{0.1, 0.5, 0.9\}$, and asset supplies of $S_A = S_B \in \{.02, .05, .10, .15\}$, with $\eta = 0$, $i = .1$, and $M = 1$ maintained. Out of these 36 parameter combinations, in four of them the assets are so plentiful that both liquidity premia are zero for any π; in three of them, all with maximal ℓ and minimal asset supplies, the sign is reversed so that $L_A < L_B$ when $\pi < 1$, i.e., the safer asset is less liquid; in the remaining 29 cases, we have the 'normal' result where the safer asset is more liquid. For more details, see Appendix B.2.

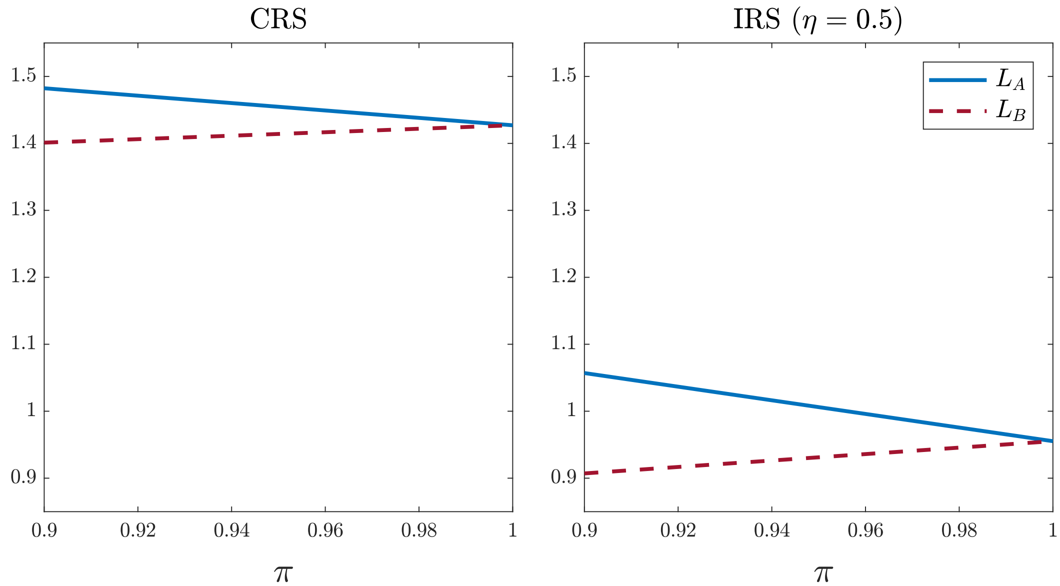


Figure 5. Liquidity premia as functions of π

Notes: The figure depicts the liquidity premia of assets A, B as functions of π, assuming symmetric asset supplies. The left panel illustrates the case of a CRS matching technology, and the right panel represents the the case of IRS ($\eta = 0.5$). The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $S_A = S_B = 0.05$, with log utility.

B *defaults*, all the N-types (even those who had chosen to specialize in asset B) will rush into OTC$_A$. But there is also a more subtle force at work: precisely because asset B may default, agents who specialize in that asset B endogenously choose to carry more money as insurance. Consequently, asset B sellers need a smaller liquidity boost, which means that the surplus generated from asset trades in OTC$_B$ is lower. This is a force that attracts more N-types to OTC$_A$ *even if asset* B *does not default*. Taken together, these two forces imply that *ex-post* (and regardless of the default shock realization) more buyers are attracted to OTC$_A$. This, in turn, incentivizes agents to specialize in asset A *ex-ante*, as they realize that in this market they will have a high trade probability, if they turn out to be sellers.

The discussion following equations (6) reveals why this is important for liquidity: an agent who buys an asset today (in the primary market) is willing to pay a higher price if she expects that it will be easy to sell that asset 'down the road', and, importantly, it is the C-types who sell assets down the road. Through this channel, any positive default probability for asset B translates into a matching advantage for C-types in OTC$_A$. This, in turn, translates into a higher liquidity premium for asset A, because that premium depends on the (anticipated) ease with which the agent can sell the asset if she turns out to be a C-type. Naturally, this channel, and the liquidity differential between the two assets, will be magnified if matching is characterized by IRS. This last point can be seen more

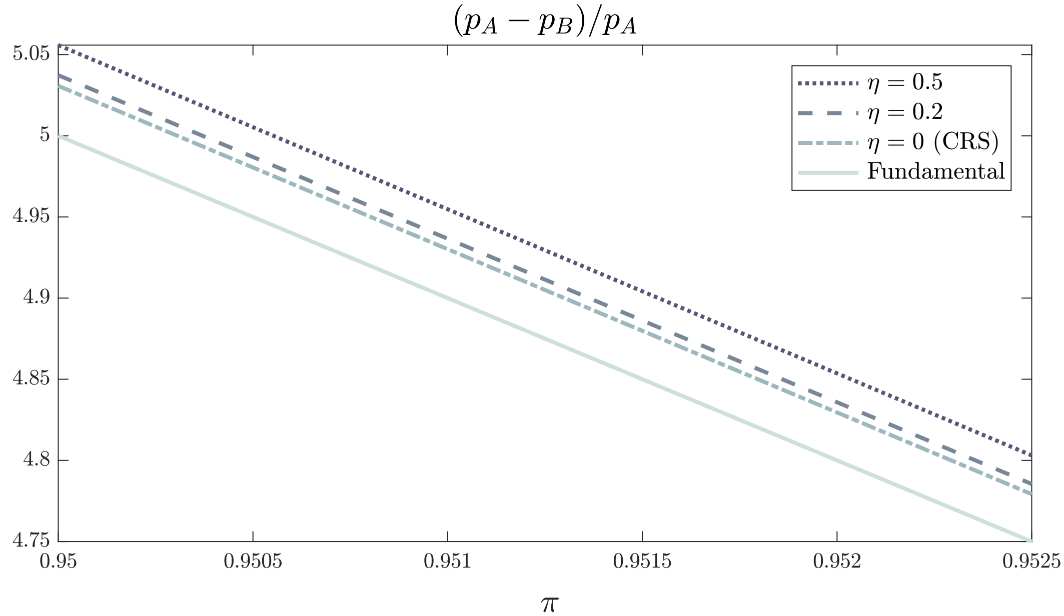


Figure 6. Price differentials as functions of π

Notes: The figure depicts the price differential $(p_A - p_B)/p_A$ as a function of π, for various values of η, assuming symmetric asset supplies. The curve dubbed "Fundamental" represents the percentage difference between the price of assets A and B, if the liquidity channel was shut down, namely, the term $1 - \pi$. The difference between the "Fundamental" curve and the curves corresponding to the various η's represent a pure liquidity difference between the two assets. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $S_A = S_B = 0.05$, with log utility.

clearly in Figure 6. Instead of liquidity premia, we plot the percentage difference between the two asset prices, $(p_A - p_B)/p_A$, for various values of η, and we contrast them to the difference between the fundamental values. Thus, any difference between the curve labeled "fundamental" and the curves representing the various η's is a *pure* liquidity difference.

4.2 Result 2: Safer yet less liquid

The previous section established that a safer asset will also be more liquid – other things being equal. Of course, other things are not always equal, and of those we are particularly interested in asset supplies. Allowing for differences in asset supplies delivers the second important result of the paper: even with slight IRS in OTC matching, the coordination channel becomes so strong that asset demand curves can be upward sloping. Consequently, asset B can carry a higher liquidity premium than the safe asset A, as long as the supply of the former is sufficiently larger than that of the latter.

Figure 7 depicts the liquidity premia for assets A and B as functions of the supply

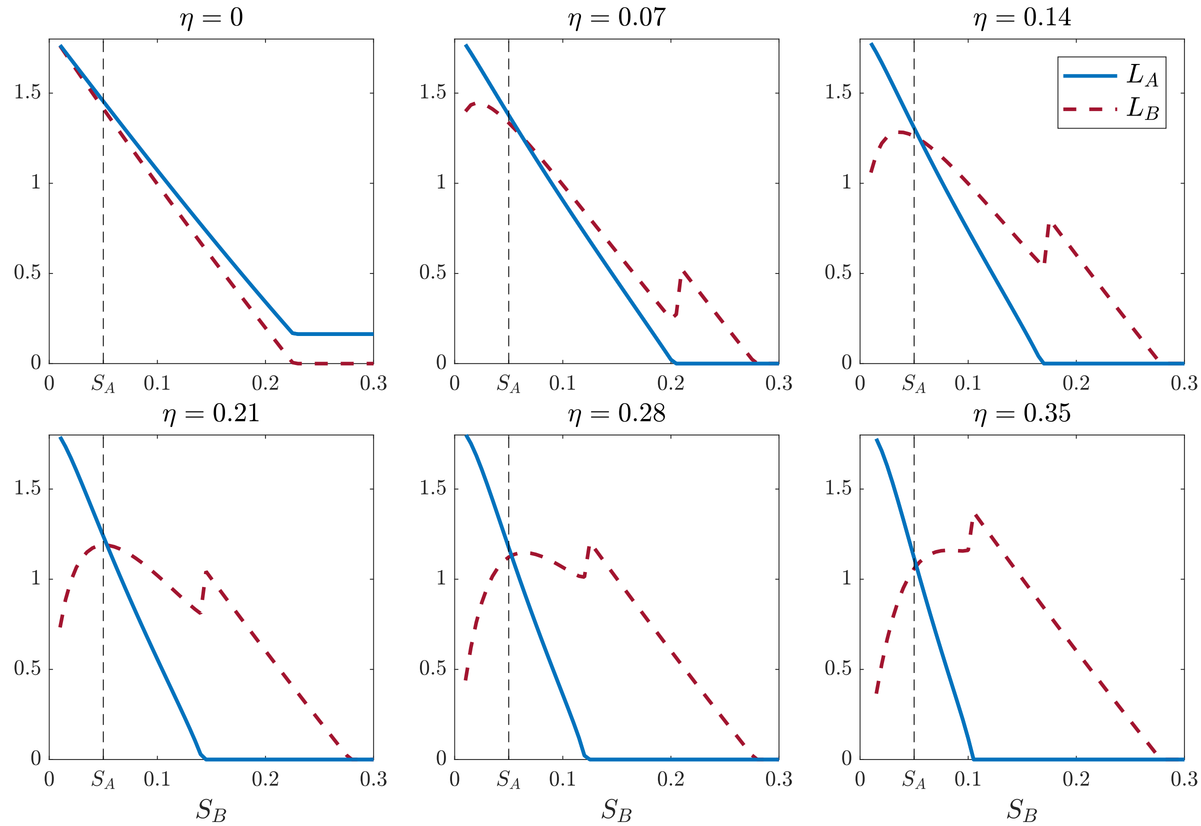


Figure 7. Liquidity premia with varying degrees of IRS

Notes: The figure depicts the liquidity premia of assets A and B as functions of S_B, for a constant S_A, and for varying degrees of IRS. The dashed vertical line indicates the (fixed) supply of asset A. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $\pi = 0.95$, $S_A = 0.05$, with log utility.

S_B, keeping S_A fixed, and for various degrees of IRS in matching. First, notice that the liquidity premium on asset A is always decreasing in S_B. With CRS (top-left panel), this is also true for the liquidity premium on asset B, as is standard in existing models of asset liquidity. However, with even a small degree of IRS, the asset demand curves have upward-sloping segments. And if S_B exceeds S_A by a large enough amount, we observe $L_B > L_A$, i.e., the less safe asset emerges as more liquid.

The mechanism of this result is as follows. As we have seen, our model has a channel whereby a safer asset also enjoys an endogenous liquidity advantage. However, whether this advantage will materialize depends on the relative strength of other channels. Consider a change in an asset's supply (say, $S_B\uparrow$): in equilibrium, this will be accommodated by a shift in the proportion of agents holding this asset ($e_C \downarrow$) so that each individual agent, whether holding A or B, can obtain a similar surplus when selling that asset in the corresponding OTC. This is the dilution effect described in the previous section, and

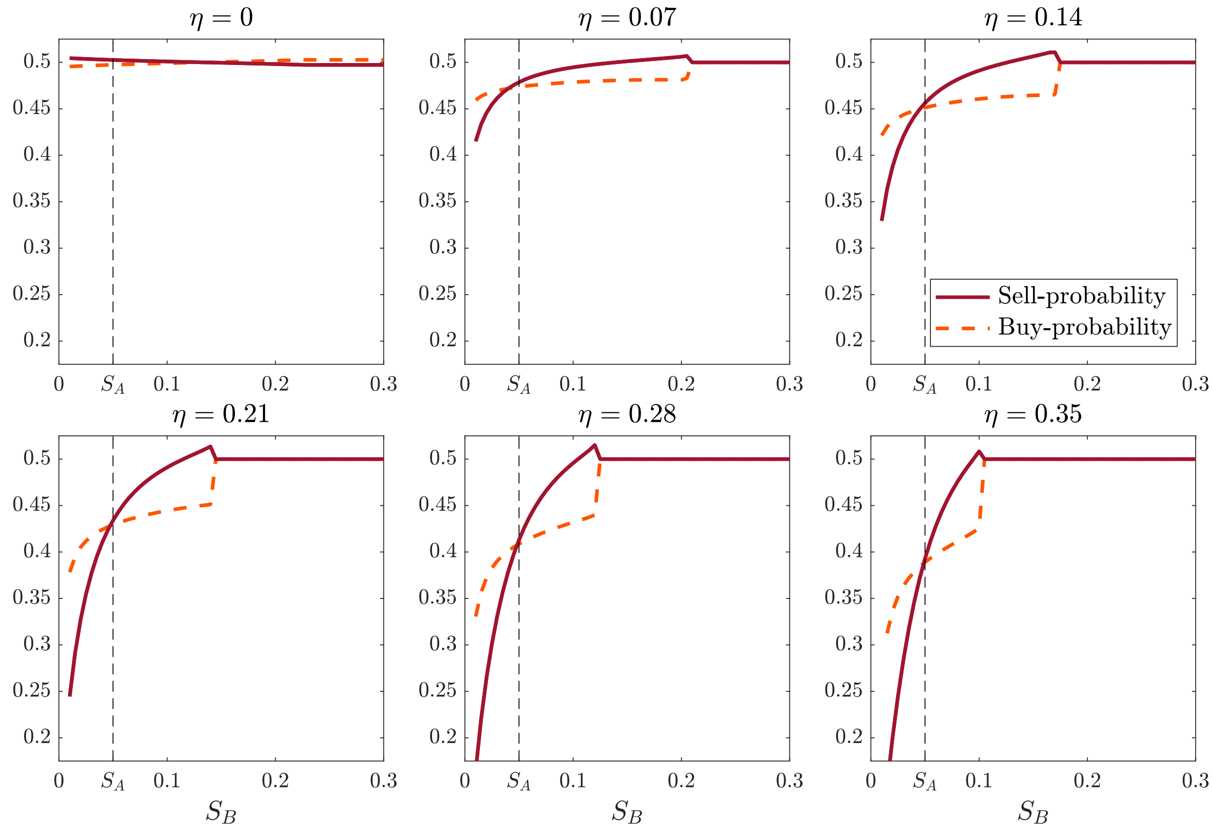


Figure 8. Sell- and buy-probabilities in OTC$_B$

Notes: The figure depicts the sell-probability, α^n_{CB}, and the buy-probability, α^n_{NB}, in the secondary market for asset B, in the normal state, as a function of S_B (and for varying degrees of IRS). The dashed vertical line indicates the (fixed) supply of asset A. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $\pi = 0.95$, $S_A = 0.05$, with log utility.

as long as we have constant returns in matching the other two effects (congestion and coordination) offset, so this is the end of the story.

With IRS, however, the coordination effect (whereby entry of sellers in an OTC market begets the entry of buyers, and vice versa) is strengthened and dominates the congestion effect (see Appendix A.7). What does this mean for the effects of an increase in S_B? Following the logic described above, e_C decreases (more agents hold asset B and thus enter OTC$_B$ as sellers), and following the logic described in Section 3.4, e^n_N decreases too (assuming asset B does not default, more buyers enter OTC$_B$). But with a strong coordination effect, this means that the matching probability increases for *both* sides in OTC$_B$, and decreases for both sides in OTC$_A$, which begets a further shift in entry rates. We illustrate this channel in Figure 8.[17] (In principle, the coordination effect could be so

[17] Close inspection of the figure reveals another interesting effect: the sell-probability always responds

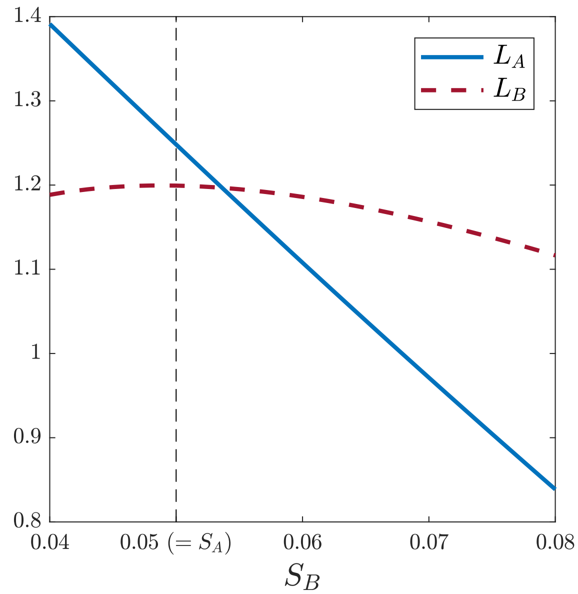


Figure 9. Liquidity premia

Notes: The figure depicts the liquidity premia of assets A and B as functions of S_B, for a constant S_A, and for $\eta = 0.2$. The dashed vertical line indicates the (fixed) supply of asset A. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $\pi = 0.95$, $S_A = 0.05$, with log utility.

strong as to dominate both the congestion and dilution effects combined, in which case the asset in larger supply would be the only liquid one, irrespective of its safety; however, this only happens for values of η far higher than our empirical study in Section 5 allows.) Thus, with IRS in matching, there is a "thick market" channel which is so strong that the premium an agent is willing to pay for an asset can be increasing in that asset's supply.

Figure 9 summarizes Results 1 and 2. It depicts the liquidity premia of assets A and B as functions of S_B, keeping S_A fixed, with a slight degree of IRS, $\eta = 0.2$. When the supplies of the two assets are equal ($S_B = S_A$), asset A carries a higher liquidity premium (Result 1). However, as S_B increases further, we enter the region where the demand for asset B becomes upward sloping, until eventually L_B surpasses L_A (Result 2).

more strongly to any impulse than the buy-probability. This reflects the fact that N-types always respond more elastically to any change in market attractiveness, which happens because their market entry choice is made *ex post* and any cost of holding money or assets is already sunk; the C-types, on the other hand, must take the cost of holding assets into account. This fact serves to further *amplify* our result of upward-sloping demand curves, but the result would still hold if we forced $e_C = e_N^n$ in the background.

4.2.1 Application: Rationalizing the illiquidity of AAA corporate bonds

An interesting fact that has recently drawn the attention of practitioners (but not so much that of academic researchers yet) is that, in the U.S., the virtually default-free AAA bonds are less liquid than (the riskier) AA corporate bonds. Here, we highlight our model's ability to explain this puzzle qualitatively, while in Section 5 we use the example of AA and AAA bonds for a quantitative test of our model.

Figure 10 plots the time-series yields of AAA versus AA corporate bonds (as well as their difference) on the top panel, and, as a reference point, it does the same for the AAA versus AA municipal bond yields in the bottom panel.[18] The bottom panel is consistent with what one would expect to see: the riskier AA municipal bonds command a higher yield than the one on AAA municipal bonds, because investors who choose to hold the former want to be compensated for their higher default probability.

Interestingly, this logical pattern is reversed in the case of corporate bonds. On the top panel of the figure, we see that in the last 5 years of our sample, the yield on AA corporate bonds has been consistently lower than that on AAA bonds. Why do investors command a higher yield to hold (the virtually default-free) AAA corporate bonds? Many practitioners have claimed that this is so because the secondary market for AAA corporate bonds is extremely illiquid. This narrative is consistent with the observations depicted in Figure 10, and it is supported by further evidence. For instance, He and Milbradt (2014) document that the bid-ask spread in the market for AAA corporate bonds is higher than the one in the market for AA corporate bonds. Additionally, in recent years Bloomberg has ceased constructing its price index for AAA-rated corporate bonds, due to the dearth of outstanding bonds and the lack of secondary market trading. Of course, a high bid-ask spread and a low trade volume are both strong indicators of an illiquid market.

Our model could shed some light on this empirical observation, if it was the case that AAA corporate bonds have a scarce supply relative to AA corporate bonds. This turns out to be overwhelmingly true. In the years following the financial crisis, regulations introduced to improve the stability and transparency of the financial system (such as the Dodd-Frank Act) have made it especially hard for corporations to attain the AAA score. This resulted in a large decrease in the outstanding supply of this class of bonds.[19]

[18] The data on municipal bonds comes from Standard & Poor's, and the data on corporate bonds comes from Federal Reserve Economic Data (FRED). The original data is on a daily base, but, to make the graphs more legible, it is converted to a monthly base. The graphs show the historical yields for the past 10 years.

[19] The number of AAA-rated corporations in the U.S., never high, decreased to four – Automatic Data Processing, Exxon Mobil, Johnson & Johnson, and Microsoft – in 2011. Automatic Data Processing got downgraded in 2014, and Exxon Mobil in 2016. Today, there are only two AAA-rated companies.

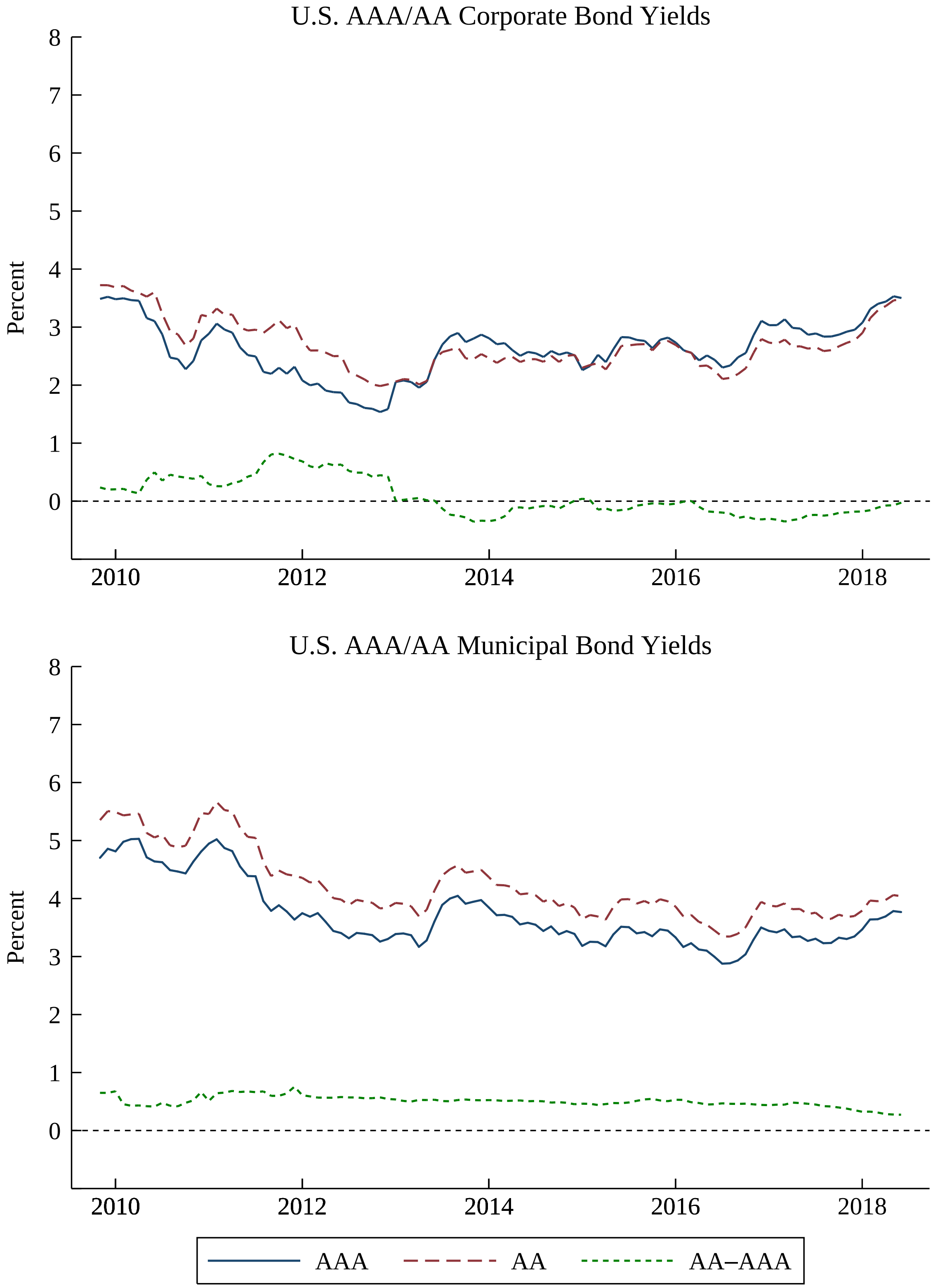


Figure 10. Historical yields of AAA/AA corporate/municipal bonds

Sources: FRED; Standard & Poor's.

As a benchmark of comparison, in June 2018, the outstanding supply of AAA over AA corporate bonds was 1/10, while the same statistic for municipal bonds was 1/3.

While it is plausible to attribute the irregularity observed on the top panel of Figure 10 to 'some liquidity story', existing models of liquidity cannot help us understand this puzzling observation (see a review of the literature in Section 1.1). In these papers, the asset demand curves are decreasing, hence, an asset in large (small) supply will tend to have a low (high) liquidity premium. Our model formalizes the idea that an asset in very scarce supply will be illiquid, even if it maintains an excellent credit rating. And our 'indirect liquidity' approach, coupled with endogenous market entry, is key for delivering this empirically relevant result.

It should be pointed out that the case of AAA versus AA US corporate bonds is not the only one where the commonly held belief that "safety and liquidity go together" is violated. Christensen and Mirkov (2019) highlight yet another class of bonds – Swiss Confederation Bonds – that are considered extremely safe, yet not particularly liquid. Furthermore, Beber et al. (2008) report that Italian government bonds are among the most liquid, despite also being among the riskiest Euro-area sovereign bonds. The authors justify this observation by pointing to the large supply of Italian debt, which is consistent with our model's prediction.

4.3 Result 3: Safe asset supply and welfare

In our final result, we highlight an important implication of our model about the effect of an increase in the supply of safe assets on welfare. A large body of recent literature highlights that the supply of safe assets has been scarce, and that increasing this supply would be beneficial for welfare (see, for example, Caballero et al., 2017). In our model this result is not necessarily true. In particular, welfare may not be monotonic in S_A.

First, let us define the welfare function of this economy, which is the C-type agent's surplus in the DM, averaged between agents who had the opportunity to rebalance their portfolios in the OTC round of trade, and those who did not.[20] Clearly, one also needs to remember that here we have agents who chose to specialize in different assets, and two possible aggregate states (default and no-default). In the normal state, welfare is:

$$\begin{aligned}\mathcal{W}^n = &\big(e_C\ell - f(e_C\ell, e_N^n(1-\ell))\big)\cdot\big(u(q_{0A}) - q_{0A}\big) + f(e_C\ell, e_N^n(1-\ell))\cdot\big(u(q_{1A}) - q_{1A}\big)\\ &+ \big((1-e_C)\ell - f((1-e_C)\ell, (1-e_N^n)(1-\ell))\big)\cdot\big(u(q_{0B}) - q_{0B}\big)\end{aligned}$$

[20] In models that build on LW, steady-state welfare depends only on the volume of DM trade. Hence, a sufficient statistic for welfare is how close the average DM production is to the first-best quantity, q^*.

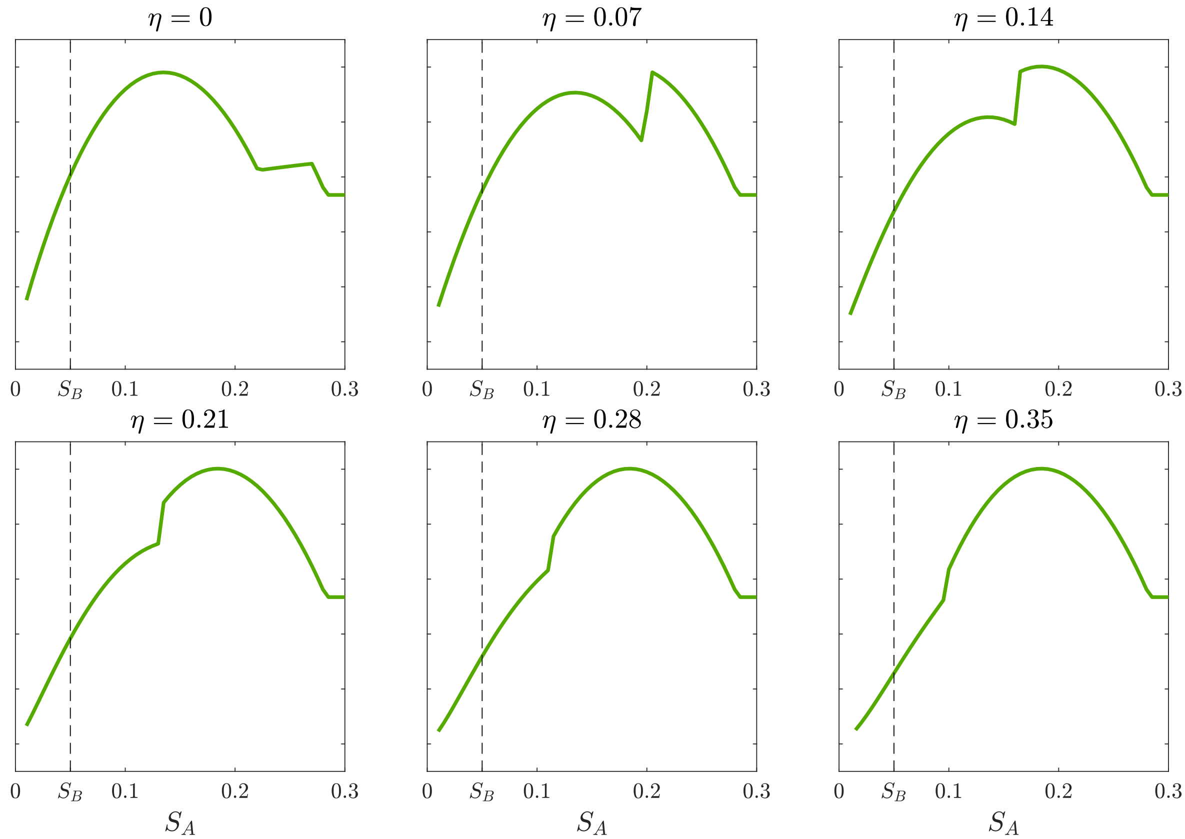


Figure 11. Safe asset supply and welfare

Notes: The figure depicts equilibrium welfare as a function of S_A, for various values of η, including $\eta = 0$ (CRS). The dashed vertical line indicates the (fixed) supply of asset B. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $\pi = 0.95$, $S_A = 0.05$, with log utility.

$$
\begin{aligned}
&+ f((1-e_C)\ell, (1-e_N^n)(1-\ell)) \cdot \big(u(q_{1B}) - q_{1B}\big) \\
&= e_C\ell \cdot \big[(1-\alpha_{CA}^n)\big(u(q_{0A}) - q_{0A}\big) + \alpha_{CA}^n\big(u(q_{1A}) - q_{1A}\big)\big] \\
&\quad + (1-e_C)\ell \cdot \big[(1-\alpha_{CB}^n)\big(u(q_{0B}) - q_{0B}\big) + \alpha_{CB}^n\big(u(q_{1B}) - q_{1B}\big)\big],
\end{aligned}
$$

and in the default state, it is:

$$
\begin{aligned}
\mathcal{W}^d &= \big(e_C\ell - f(e_C\ell, 1-\ell)\big) \cdot \big(u(q_{0A}) - q_{0A}\big) \\
&\quad + f(e_C\ell, 1-\ell) \cdot \big(u(q_{1A}) - q_{1A}\big) + (1-e_C)\ell \cdot \big(u(q_{0B}) - q_{0B}\big) \\
&= e_C\ell \cdot \big[(1-\alpha_{CA}^d)\big(u(q_{0A}) - q_{0A}\big) + \alpha_{CA}^d\big(u(q_{1A}) - q_{1A}\big)\big] + (1-e_C)\ell \cdot \big(u(q_{0B}) - q_{0B}\big).
\end{aligned}
$$

We define aggregate welfare as:

$$
\mathcal{W} = \pi\mathcal{W}^n + (1-\pi)\mathcal{W}^d. \tag{7}
$$

Figure 11 plots equilibrium welfare as a function of the supply of the safe asset, and highlights the case in which welfare is non-monotonic in S_A. This result may seem surprising at first. A higher supply of asset A enhances the liquidity role of that asset (or, equivalently, allows for more secondary market asset trade), which, in turn, should allow agents to purchase more goods in the DM. While not wrong, this intuition is incomplete. What is missing is that when the safe asset becomes more plentiful, agents expect that it will be easier to acquire extra cash ex-post and, thus, they choose to hold less of it ex-ante. In other words, our model is characterized by an externality: agents prefer to carry assets rather than money, and they wish to acquire money in the secondary market(s) only after they have learned that they really need it (i.e., only if they have turned out to be a C-type). But someone has to bring the money, and that someone will not be adequately compensated. This channel depresses the demand for money, which, in turn, decreases the value of money and the volume of trade that the existing money supply can support.

An interesting detail seen in Figure 11 is that welfare always decreases when S_A is large enough. This feature of equilibrium can be explained a follows. As S_A increases, the amount of DM goods purchased by an agent who traded in OTC$_A$, q_{1A}, also increases, because that agent was able to sell more assets and boost her money holdings. On the other hand, as S_A increases, the amount of DM goods purchased by an agent who did not trade in OTC$_A$, q_{0A}, decreases, because the higher asset supply induced that agent to carry fewer money balances *ex-ante* (see previous paragraph). Hence, an increase in S_A generates two opposing effects on welfare: the surplus term $u(q_{1A})-q_{1A}$ (involving agents who traded in OTC$_A$) increases, but the surplus term $u(q_{0A}) - q_{0A}$ (involving agents who did not trade in OTC$_A$) decreases.[21] While it is hard to know which effect prevails for any value of S_A, what is certain is that if S_A keeps rising, there will come a point where the *marginal* liquidity benefit of more A-assets will be zero (because $q_{1A} \to q^*$ implies $u'(q_{1A}) \to 1$). Near that point, an increase in S_A still hurts welfare by depressing $u(q_{0A}) - q_{0A}$ (because $u'(q_{0A}) \gg 1$), but now it generates no countervailing benefit.

5 Quantitative Analysis

Motivated by the dramatic decrease in the number of AAA-rated corporations in the early 2010s, discussed in Section 4.2.1, and the resulting fall in the supply of AAA bonds, we want to test whether the model can quantitatively match the observed changes in the spread between AAA and AA yields. To do so, we calibrate the model to data on the U.S.

[21] Of course, this is a general equilibrium model where any change in S_A affects not only the terms q_{0A}, q_{1A}, but also the terms q_{0B}, q_{1B}. However, the latter is a secondary effect which turns out to be quantitatively not too important.

	Description	Value
S_A	supply of AAA corporate bonds in the "before" period	0.0109
	supply of AAA corporate bonds in the "after" period	0.0049
S_B	supply of AA corporate bonds in the "before" period	0.0477
	supply of AA corporate bonds in the "after" period	0.0445
M	money supply	1
i	nominal interest rate on an illiquid bond (quarterly)	1.75%
ℓ	fraction of C-type agents	0.5
σ	elasticity of marginal utility	0.34
ρ	recovery rate in case of default	0.4
π	$1-$ default probability	0.996
θ	relative bargaining power of C-types	0.8
η	elasticity of the OTC matching function	0.1

Table 1: Key Parameter Values

economy and bond markets, split into a "before" (2004–2011) and "after" (2012–2018) period. We feed the model data on bond supplies before and after the split, and show that it is capable of matching the spreads observed in those periods.[22]

For the utility function, we define $u(q) = q^{1-\sigma}/(1-\sigma)$. Thus, we need to calibrate ten parameters: the supplies of bonds and money (S_A, S_B, and M), the elasticity of marginal utility (σ), the nominal interest rate on an illiquid bond ($i = (1+\mu-\beta)/\beta$, which subsumes time preference and expected inflation), the fraction of C-type agents (ℓ), the relative bargaining power of C-types (θ), the degree of returns to scale in the OTC matching function (η), and the default probability ($1-\pi$). Finally, for the calibration we use the version of our model that allows for partial default (presented in Appendix C.1), thus, we also need to calibrate the recovery rate in case of default (ρ).

Some of these parameters have straightforward empirical targets, while others do not. For the latter, we start by showing that for some (arguably) reasonable choices, our model delivers a safer-yet-less-liquid yield reversal similar to the one we observe in the data. Additionally, we perform the inverse experiment: what combinations of parameters are required in order to match the yield spreads *exactly*, before and after the change in relative supply?

First, for the bond supplies S_A and S_B, we use the market valuation of the ICE BofA

[22] 2011 is the year the number of AAA-rated corporations in the U.S. decreased to four (see Footnote 19). We also repeated our analysis with the sample split a year earlier or later, and obtained similar results.

AAA US Corporate Index (C0A1) and ICE BofA AA US Corporate Index (C0A2), and for M, we use the MZM monetary aggregate (from FRED). We divide the bond supplies by the money stock for each year, and then average over 2004–2011 and 2012–2018 to obtain our "before" and "after" values of AAA (0.0109 vs 0.0049) and AA (0.0477 vs 0.0445) bond supplies, relative to a normalized $M = 1$. For i, we cannot use any observed interest rate since no traded asset is perfectly illiquid; instead, we use an estimate of 7%/year based on time preference, expected real growth, and expected inflation (Herrenbrueck, 2019b). We set $\ell = 0.5$ for symmetry (equal numbers of potential buyers and sellers in the secondary asset markets). Next, we follow the procedure of Rocheteau, Wright, and Zhang (2018) (where to be consistent with the rest of our calibration, we use MZM as the monetary aggregate) and set $\sigma = 0.34$ to match the slope of the empirical U.S. money demand function. For the recovery rate ρ, we use 0.4 (Moody's Investors Service, 2003); for the probability that asset B pays out, we set $\pi = 0.996$.[23]

This leaves us with the bargaining power θ and the scale elasticity η, which have no direct counterparts in the data. For θ, since it is the bargaining power of asset sellers in the OTC markets, it scales liquidity premia on both bonds approximately proportionally (see Equation 6), without favoring one or the other. Since it is precisely these liquidity premia that make our model work, we want θ to be on the high side (while still less than 1 so that asset buyers have a meaningful market entry decision); thus, we set $\theta = 0.8$. For η, we already know that some amount of IRS is necessary for our Result 2, thus it must be positive. We also know from our numerical examples (e.g., Figure 8) that η does not have to be particularly high, and indeed if η is too high then one of the assets will attract all secondary market trade, and the other will be completely illiquid. Thus, we set $\eta = 0.1$.[24]

Table 2 summarizes our findings. As we can see, the supply of AA corporate bonds did not change much across the two periods, but the supply of AAA bonds decreased

[23] The 1982–2003 issuer-weighted mean recovery rate of all bonds was 40.2%; this is the most recent data we could find. For AA bonds specifically, the rate is 41.1% if default happens five or more years later – at which point the bond might not be rated AA anymore – but higher if default happens closer in time to the latest AA rating. Furthermore, eventual recovery rates do not necessarily cover all costs (legal, etc.) associated with recovering assets after a default. Historical default probabilities are lower than our value for π implies, but similarly hard to define for the reason discussed above: defaults are usually preceded by a downgrade. Of course, from the point of view of an investor, it does not matter if a bond defaults outright or if it is downgraded before eventually defaulting.

[24] There is also the question of how we deal with the multiplicity of equilibrium (see Proposition 1). Numerically, we use $e_C^0 \equiv S_A/(S_A + S_B)$ as a starting point, then iterate the function $G(e_C)$ in the direction of its sign, until $G(e_C) \to 0$ or until reaching a corner. If the corners are not robust, this procedure will always find a robust interior equilibrium.

	2004–2011	2012–2018	Change
AAA supply / M, observed	0.0109	0.0049	−55%
AA supply / M, observed	0.0477	0.0445	−7%
Spread AA–AAA (%), observed	0.2245	−0.0409	−0.2654
Spread AA–AAA (%), predicted	0.1961	−0.0648	−0.2609

Table 2: Prediction of the model with Table 1 parameters

by more than half. The spread between AA and AAA yields was positive in the earlier period, as every textbook would have predicted, then became negative in the later period. The final row of the table shows the outcome of our calibrated model, which captures both the positive spread in the earlier period when AAA supplies were relatively high and the negative spread in the later period when AAA supplies had shrunk. More than just getting the sign right, the table shows that our calibrated model can also capture the magnitude of the spreads, and their reversal.[25]

Having shown that our model can quantitatively capture the change in AA–AAA spreads following the big reduction in the supply of AAA corporate bonds, for reasonable choices of parameters that do not have clear empirical targets, we now perform the inverse exercise: what would those parameters have to be in order to match the spreads exactly? We have three parameters (θ, π, and η) and two targets (the AA-AAA spread in the earlier and later periods). Since we have an extra degree of freedom, we fix a value of the bargaining power θ and let our model (and the data) tell us what the implied default probability and scale elasticity should be.

The results of this exercise are shown in Table 3. For $\theta < 0.6$, no exact match is possible, reflecting the fact that low values of θ result in low liquidity premia (and thus low power for our model). For $\theta > 0.6$, our model can match the data perfectly, up to $\theta \to 1$ in the limit. The levels of π and η adjust as we vary θ: π in order to match the level of the spreads (since lower π, everything else equal, increases the spread via the default premium), and η in order to match the size of the change (since η governs how sensitive the spread is to the difference in bond supplies).

[25] Our calibration implies that $q_{0A}/q^* = 0.8886$ and $q_{0B}/q^* = 0.8922$ for the 2004–2011 period and $q_{0A}/q^* = 0.8839$ and $q_{0B}/q^* = 0.8869$ for the 2012–2018 period. These numbers are significantly higher than 0.5, which is the threshold below which the combined money holdings of the C-type and the N-type are not sufficient to purchase q^*. (Given the take-it-or-leave-it assumption in the DM, q_{0j} coincides with the real balances of an agent specializing in asset j; see Equation (A.10).) This verifies our claim, made on page 12, that the range for which $m + \tilde{m} < m^*$ is not relevant for the analysis.

θ	π	η
0.6	0.9949	0.212
0.7	0.9957	0.135
0.8	0.9958	0.103
0.9	0.9958	0.083
0.99	0.9956	0.069

Table 3: Combinations of θ, π, and η that result in an exact match of our model to the observed AA-AAA spreads in the "before" and "after" periods.

The resulting parameter values are all plausible; the most novel and interesting one is the scale elasticity of the matching function (which to our knowledge has never been estimated for financial markets). Our calibrated value of η ranges from 0.069 to 0.212, which corresponds to an elasticity of the matching function with respect to market size of $[1.069, 1.212]$. This can be interpreted as saying that if the measure of traders in an OTC market (buyers *and* sellers) increases by 1%, overall matches increase by 1.069% to 1.212% – or, equivalently, the matching probabilities for individual traders increase by 0.069% to 0.212%. Quantitatively, the estimates in this range are closer to CRS than to the congestion-free matching functions employed in most of the finance search literature (which our formulation nests for $\eta = 1$; e.g., Duffie et al., 2005 and Vayanos and Wang, 2007). But they are still robustly different from CRS, enough so that an asset class in dramatically smaller supply than a competitor – for example, AAA bonds versus AA bonds, or Swiss bonds compared to German or Italian ones – can end up being substantially less liquid in equilibrium.

6 Segmented Markets: Extensions and Microfoundations

A key assumption in our analysis is that secondary markets are segmented and agents can hold only one asset per period. The first part of this assumption is certainly realistic: different classes of assets trade in distinct secondary markets, and most fixed income dealers do not intermediate multiple kinds of securities, since there is a cost to becoming an expert in a specific security. (For instance, in investment banks different "desks" are in charge of trading different types of securities, so they are committed ex-ante to a specific class of assets.) The second part of the assumption, according to which agents must hold only one asset, is stronger but, clearly, it is not meant to be taken literally. It is just a stark way to capture the idea that even if investors can hold multiple assets and visit multiple secondary markets, they will visit more frequently the market where they expect to find better trading conditions.[26]

There is an extensive empirical finance literature reporting that asset markets are segmented from one another. For a detailed review of this literature, see Allen and Gale (1994) and Edmond and Weill (2012). Another important argument in favor of (some) market segmentation is that in its absence every asset would be equally easy to trade (equally "liquid"). But the empirical finance literature abounds with examples of assets that have (almost) identical characteristics, yet they sell at significantly different prices because they are traded in secondary markets with different degrees of liquidity. For example, Andreasen, Christensen, and Riddell (2021) show that, after controlling for expected inflation, TIPS bonds sell at a significant discount compared to common Treasuries, and suggesting that this differential is due to the fact that the TIPS market is less liquid.

Our model aims to explain the puzzling observation that the virtually default-free AAA corporate bonds are less liquid than the less safe AA corporate bonds. As we discuss in Section 6.1, a necessary condition for this "disconnect" between safety and liquidity is that the assets in question trade in segmented secondary markets. Interestingly, in a report on the corporate bond market structure (BlackRock, 2014), the authors characterize the corporate bond market as "fragmented" and make a number of proposals to increase the "deteriorating liquidity" of corporate bonds. Most notably they suggest that regulators should work towards consolidating the secondary markets for corporate bonds. This view is supported by the empirical findings of Oehmke and Zawadowski (2016), who find that "the *fragmented* nature of the corporate bond market impedes its liquidity" (emphasis added).

The discussion thus far suggests that adopting a model with some market segmentation is not only theoretically interesting, but also empirically relevant. However, one may still wonder whether we need this strong version of market segmentation, whereby each agent can only hold one asset at a time. The answer is that this version of the market segmentation assumption is necessary only for tractability, i.e., for obtaining analytical solutions (e.g., Propositions 1 and 2), but it is not necessary for the main results. The goal of this section is to prove this claim, and we proceed in several steps. First, in Section 6.1 we discard the segmentation assumption altogether, and assume that agents can trade both assets in a consolidated secondary market. This helps the reader understand exactly

[26] This assumption is analogous to a discrete choice model; such models are popular in economics (especially in Industrial Organization) because they offer a tractable way of modeling individual consumer choice over various goods while permitting a realistic description of aggregate market shares.

which results the segmentation assumption is responsible for. We show that market segmentation is necessary for Result 2 (the "disconnect" between asset safety and liquidity), but it is not necessary for Result 1 (safety implies liquidity). Then, in Sections 6.2 and 6.3 we consider versions of the model with weaker segmentation assumptions. In both cases, we show that the strong market segmentation adopted in the baseline model *as an assumption* can arise as an *endogenous equilibrium outcome* in a more general model where agents are given the option to avoid such segmentation.

6.1 Consolidated secondary asset market

In this section, we fully abandon the segmentation assumption and study a model where agents can trade both assets in a single *consolidated* OTC market. The purpose of this section is twofold. First, it helps the reader understand exactly which results the segmentation assumption is responsible for. Second, it serves as an input for Section 6.3, where we study a model where agents can endogenously choose whether to trade in an environment characterized by market segmentation as in the baseline model or not.

The main difference between this section and the baseline model is that now there exists a unique consolidated secondary market where agents can liquidate assets. With a unique OTC market, agents have a trivial entry decision, i.e., they all visit the consolidated market. Within that market, the matching technology and the bargaining protocol remain unaltered. The details of the model, including characterization of the equilibrium, are presented in Appendix C.2. Here we discuss the main results.

Proposition 3. *We restrict attention to equilibria with positive liquidity premia. In the consolidated market model, regardless of the individual asset supplies, S_A, S_B, it is always true that $L_A > L_B$. Thus, Result 1 holds in the model with a unique consolidated market, but Result 2 fails.*

Proof. See Appendix B.3. □

Proposition 3 implies that market segmentation is not a necessary condition for Result 1 (other things equal, safety implies liquidity), but it is a necessary condition for Result 2 (a less safe asset could be more liquid). The reason why Result 1 holds here is different, and simpler, than the justification provided in Section 4.1: a liquidity premium reflects downward sloping demand for liquidity, and, in the default state, when there is less liquidity, agents value the remaining liquidity (the one provided by the safe asset A) more. However, since $L_A > L_B$ for *any* asset supplies, the riskier asset B can never be more liquid than asset A. We conclude that some form of segmentation is necessary for the safer asset to be less liquid.

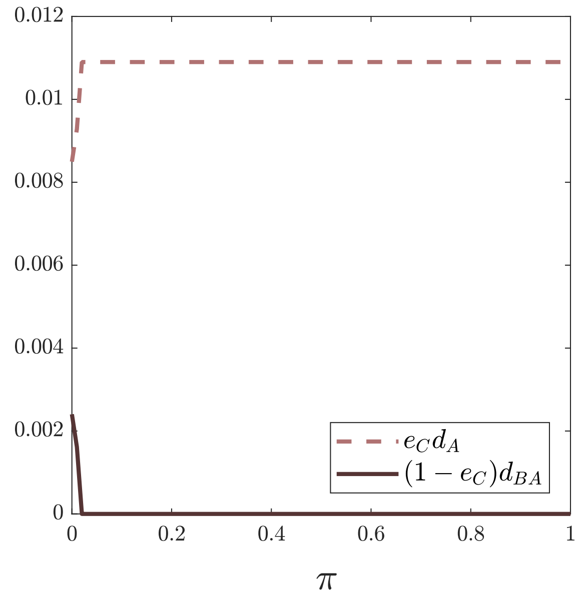


Figure 12. Optimal holdings of asset A by the various agents as functions of π

Notes: The figure depicts the amounts of asset A held by agents who specialize in asset A ($e_C d_A$) and agents who specialize in asset B ($(1 - e_C)d_{BA}$), which sum up to the entire supply of asset A. The figure is based on the calibrated parameters presented in Table 1, except that here we focus on full default, i.e., $\rho = 0$. The figure uses corporate bond supply data for the period 2004–2011.

6.2 Endogenous asset specialization

In this section, we return to a model with market segmentation, but we consider a weaker version of it. More precisely, we relax the assumption that each agent must hold only one asset, and replace it with the assumption that they can hold both assets but visit only one OTC market per period. This allows agents to "specialize" in asset B (and visit OTC_B in the normal state), while carrying some asset A for precautionary motives, i.e., with an intention to sell it in OTC_A in the case that asset B defaults. We find that despite giving agents the option to carry asset A as a precaution, they choose to not exercise it. Thus, instead of having asset specialization imposed on them, agents will endogenously choose to specialize in one asset, hence the heading "Endogenous asset specialization".

The details of the analysis have been relegated to Appendix C.3. Here we provide a discussion of the main results. We find that the structure of equilibrium looks similar to the baseline model, in the sense that we still have some agents specializing in asset A and some others specializing in asset B. Obviously, agents who hold asset A do not have an incentive to carry any asset B: OTC_A is always open, and if asset B carries even the slightest liquidity premium (a case that we consider to be the interesting one), it is not optimal for the agent to pay a liquidity premium for two types of assets, while she only gets to benefit from the liquidity services of one of them. However, that is not necessarily true for agents who specialize on asset B. These agents may well choose to carry some asset A as a precaution, and liquidate it in OTC_A in the event that asset B defaults. Thus, the current version of the model has a new key equilibrium variable, denoted by d_{BA}, that captures the optimal amount of asset A chosen by an agent who otherwise specializes in asset B and plans to visit OTC_B in the normal state.

Figure 12 illustrates the equilibrium value of the crucial variable d_{BA}, which epitomizes the difference between this version of the model and the baseline one. The figure is based on the calibrated parameters discussed in Section 5. For comparison, the figure also presents the optimal asset A holdings of agents who choose to only hold that asset. The main finding is that even though agents who specialize in asset B have the option to bring some asset A for safety, they typically do not exercise this option. More precisely, we observe that for any (roughly) $\pi > 0.02$, $d_{BA} = 0$, i.e., agents who specialize in asset B will find it optimal to carry a positive amount of asset A, only if the probability of default is in the order of 98% per period. But, as we explained in Section 5, the probability of default for which our model matches the data well is in the order of 0.4–0.5%.

The main reason why agents do not carry asset A for precautionary motives is that asset A does not provide a very cheap insurance; it carries a hefty liquidity premium (see Result 1). Moreover, agents in our baseline model already had a way of insuring themselves against the default shock; namely, they could bring money. Generally, agents are more likely to exercise this new option and carry some asset A for safety when asset A is cheaper and inflation is higher. It should be noted that this analysis has assumed full default, i.e., $\rho = 0$. But notice that for any value of $\rho > 0$ agents would become even less willing to insure themselves against the default shock by carrying asset A, thus strengthening our result.

In conclusion, we considered a version of the model with weaker segmentation assumptions and found that agents choose to endogenously specialize exclusively in one asset. Thus, full segmentation and specialization in one asset, which was imposed as an assumption in the baseline model, here arises endogenously as an equilibrium result.

6.3 Model with two marketplaces

The goal of this section, similar to that of Section 6.2, is to emphasize that the market segmentation imposed in the baseline model by assumption can arise endogenously in a model where agents have the option to avoid such segmentation. To give them this option we develop a model with two distinct *marketplaces*. Agents can pay an ex-ante real

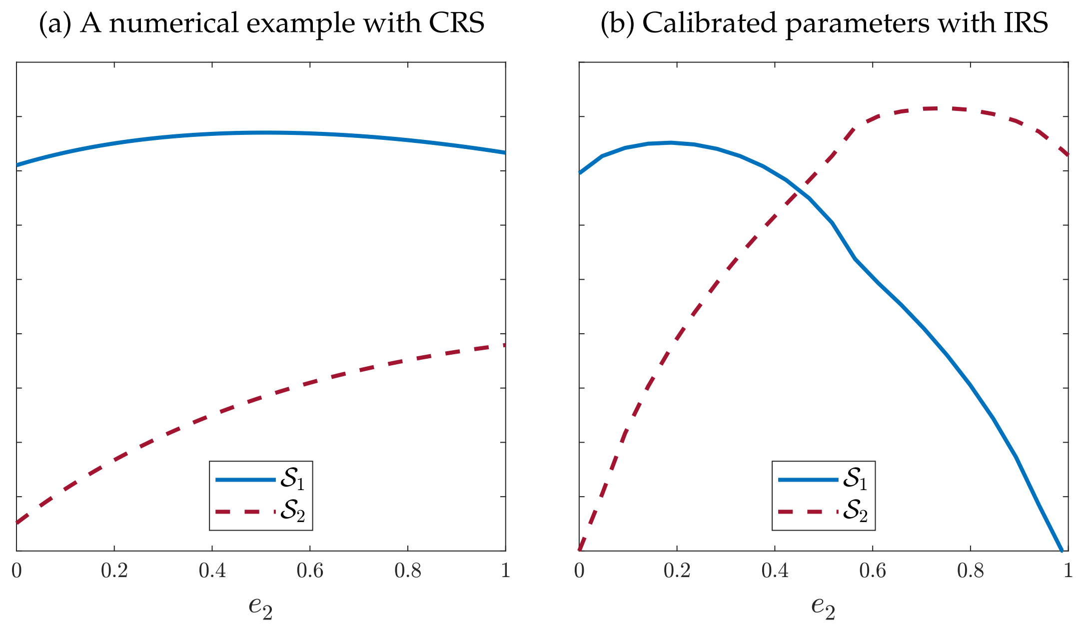


Figure 13. The agent's surplus from visiting the segmented and the consolidated marketplaces as a function of e_2, the fraction of agents visiting the consolidated marketplace.

Notes: $\mathcal{S}_1$ and $\mathcal{S}_2$ are the surpluses of the typical agent from visiting the segmented and the consolidated marketplaces, respectively. These terms are defined in (C.40) and (C.41) in the appendix. Panel (a) (CRS) illustrates the case where $S_A = S_B = 0.05$ and $\pi = 0.95$. Panel (b) is based on the calibrated parameters presented in Table 1. The figure uses corporate bond supply data for the period 2004–2011.

cost, κ_1, and visit the "segmented marketplace", which consists of two submarkets, OTC$_A$ and OTC$_B$. Once in this marketplace, agents must choose to visit either OTC$_A$ or OTC$_B$, exactly as in our baseline model. Alternatively, agents can pay an ex-ante real cost, κ_2, and visit the "consolidated marketplace", where they can trade both assets A and B in a single OTC submarket. This is the market described in Section 6.1. Within any submarket, matching and bargaining remain the same as in the baseline model.

The details of the model are presented in Appendix C.4. Here we discuss the main results. Figure 13 (a) illustrates the surplus of the typical agent from visiting the segmented and the consolidated marketplaces, in the case of CRS, as a function of e_2, which denotes the fraction of agents visiting the consolidated marketplace. Since the consolidated marketplace offers agents the option to trade both assets, one may expect that all agents would be attracted to that market. Our results show this is not the case because our model also contains a force that makes the segmented marketplace desirable. The intuition is as follows. The agent's entry decision weighs her benefit from visiting the two marketplaces. As a C-type the agent may (or may not) prefer the consolidated marketplace because that marketplace offers a form of insurance in case asset B defaults. However, as an N-type the agent *always* prefers the segmented marketplace because that environment offers her the *ex-post* flexibility of visiting either submarket, and, crucially, avoiding OTC$_B$ in the default state. In contrast, in the consolidated marketplace, the whole market, including the N-types, is subject to the default of asset B, and there is no way to avoid it.

Our numerical simulations show that the latter force (making the segmented marketplace desirable for N-types) typically prevails, so that in the model with CRS *having all agents visit the segmented marketplace is the only robust equilibrium*.[27] (When we use the term "robust", we mean robust to small trembles, as defined in Proposition 1.) To be more precise, in our numerical simulations, the representative agent's surplus from visiting the segmented marketplace is higher than the surplus from visiting the consolidated marketplace. It should be emphasized that this result is obtained without any additional assumptions regarding the size of κ_1, κ_2. (The surpluses illustrated in Figure 13 do not include the entry cost terms κ.) In other words, full concentration in the segmented marketplace arises as the only robust equilibrium not because we give that marketplace an exogenous advantage, say, by making κ_2 larger than κ_1, but because our model entails a hardwired channel that attracts agents to the segmented marketplace.

However, with several forces pointing in different directions, providing an analytical proof of this result is hard, and one cannot exclude the possibility that for some parameter values the consolidated marketplace is operating. Therefore, we now study the equilibrium numerically using the calibrated parameters discussed in Section 5. The results of this exercise are illustrated in Figure 13 (b). An important difference compared to Figure 13 (a) is that now the IRS are amplifying the coordination effect, so that the representative agent has a strong desire to visit the consolidated marketplace if and only if she expects many other agents to be there (e_2 is large). Our analysis implies that there exist two robust corner equilibria; one where all agents concentrate in the consolidated marketplace, and one where all agents concentrate in the segmented marketplace. Of course, the latter coincides with the equilibrium described in our baseline model, where segmentation was imposed as an assumption.

Finally, note that the analysis so far has abstracted from the entry fees κ_1, κ_2. In other words, we have established that even if $\kappa_1 = \kappa_2$, having all agents concentrate in

[27] Another way of viewing this result is that the C-type's motive to visit the consolidated marketplace for insurance reasons is not strong enough to exceed the motive of the N-type to visit the segmented marketplace. This result should not be surprising, since we have already seen a manifestation of it in Section 6.2. There, agents were offered the option to insure themselves against the default probability of asset B, by carrying some asset A, but they optimally chose to turn it down. The reason was that agents already had a good way of insuring themselves against the default shock (as they do here): carry more money.

the segmented marketplace is a robust equilibrium of the model. However, one may expect that the cost associated with visiting the consolidated marketplace (where agents can trade all assets) will be greater than the cost associated with visiting the segmented marketplace. If κ_2 was significantly greater than κ_1, then concentrating in the segmented marketplace would become the unique robust equilibrium.

7 Conclusion

We argue that understanding the link between an asset's safety and its liquidity is crucial. To this end, we present a general equilibrium model where asset safety and asset liquidity are well-defined and *distinct* from one another. Treating safety as a primitive, we examine the relationship between an asset's safety and liquidity. We show that the commonly held belief that "safety implies liquidity" is generally justified, but there may be exceptions. In particular, we highlight that a safe asset in scarce supply may be less liquid than a less-safe asset in large supply. Next, we calibrate our model to rationalize the puzzling observation that AAA corporate bonds in the U.S. are less liquid than (the riskier) AA corporate bonds, and show that our model can explain this reversal as resulting from a recent decrease in the relative supply of AAA bonds. Finally, and contrary to a recent literature on the role of safe assets, we show that in our model increasing the supply of the safe asset is not always beneficial for welfare.

Appendix

A Baseline Model: Details

A.1 Value functions

First, we analyze the value functions in the CM. Consider an agent who enters the CM with m units of money and d_j units of asset j, $j = \{A, B\}$ in state $s = \{n, d\}$. The value function of the agent is given by

$$W_s(m, d_A, d_B) = \max_{\substack{X,H,\\ \hat{m},\hat{d}_A,\hat{d}_B}} \left\{ X - H + \beta\, \mathbb{E}_{s,i}\Big[\max\Big\{\Omega^s_{iA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^s_{iB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\}\Big] \right\}$$

$$\begin{aligned}
\text{s.t. } & X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A + d_B), && \text{if } s = n \quad \text{(normal)},\\
& X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A), && \text{if } s = d \quad \text{(default)},
\end{aligned}$$

where variables with hats denote portfolio choices for the next period, and $\mathbb{E}$ is the expectation operator over states and types of consumers. Ω^s_{ij} denotes a value function of an i-type agent, $i = \{C, N\}$, who enters the OTC market for asset j in state s, and it is described below. Replacing $X - H$ from the budget constraint yields

$$\begin{aligned}
W_s(m, d_A, d_B) = {} & \varphi(m + \mu M + d_A + d_B\cdot\mathbb{I}\{s = n\}) \\
& + \max_{\hat{m},\hat{d}_A,\hat{d}_B} \Big\{ -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) \\
& \qquad + \beta\pi\ell \max\Big\{\Omega^n_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^n_{CB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta\pi(1-\ell) \max\Big\{\Omega^n_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^n_{NB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta(1-\pi)\ell \max\Big\{\Omega^d_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^d_{CB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta(1-\pi)(1-\ell) \max\Big\{\Omega^d_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^d_{NB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \Big\},
\end{aligned} \tag{A.1}$$

where $\mathbb{I}$ is the indicator function, and we have used the fact that asset B defaults with probability $1 - \pi$ and that an agent becomes C-type with probability ℓ. This can be simply written as follows:

$$\begin{aligned}
W_n(m, d_A, d_B) &= \varphi(m + d_A + d_B) + \Lambda,\\
W_d(m, d_A, d_B) &= \varphi(m + d_A) + \Lambda,
\end{aligned} \tag{A.2}$$

where Λ collects the remaining terms that do not depend on the current states.

The value function for a producer is much simpler. Note that producers will not want to leave the CM with a positive amount of money or assets, as long as the assets are priced at a liquidity premium. The reason is that a producer's identity is permanent; so, there is no reason for her to bring money or buy assets with paying a liquidity premium when she knows that she will never have a liquidity need in the DM. Therefore, when entering the CM, a producer will only hold money that she received as payment in the preceding DM. Thus, the value function for a producer is given by

$$W^P(m) = \max_{X,H} \left\{ X - H + \beta\, \mathbb{E}_s\big[V_s^P\big] \right\}$$
$$\text{s.t. } \; X = H + \varphi m,$$

where V_s^P denotes a value function of a producer in the DM in state s that will be described later. Notice that the value function does not depend on states of the economy. Using the budget constraint, we can re-write the value function as follows:

$$W^P(m) = \varphi m + \beta \left(\pi V_n^P + (1-\pi) V_d^P\right) \equiv \varphi m + \Lambda^P.$$

Note that all agents have linear value functions in the CM. This is standard in models that build on LW, a result that follows from the (quasi) linear preferences, and it makes the bargaining solution in the DM easy to characterize.

We now move to the value functions in the OTC markets. C-type agents are selling assets, and N-type agents are buying assets. Let $\Omega_{ij}^s(m, d_A, d_B)$ denote a value function of an agent of type i who decides to enter OTC$_j$ in state s. ξ_j is the amount of money that gets transferred to a C-type, and χ_j the amount of asset j that gets transferred to an N-type in a typical match in OTC$_j$. These terms of trade are described in the next section. The value functions are given by

$$\begin{aligned}
\Omega_{CA}^n(m, d_A, d_B) &= \alpha_{CA}^n V_n(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha_{CA}^n) V_n(m, d_A, d_B),\\
\Omega_{CA}^d(m, d_A, d_B) &= \alpha_{CA}^d V_d(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha_{CA}^d) V_d(m, d_A, d_B),\\
\Omega_{CB}^n(m, d_A, d_B) &= \alpha_{CB}^n V_n(m + \xi_B, d_A, d_B - \chi_B) + (1 - \alpha_{CB}^n) V_n(m, d_A, d_B),\\
\Omega_{CB}^d(m, d_A, d_B) &= V_d(m, d_A, d_B),\\
\Omega_{NA}^n(m, d_A, d_B) &= \alpha_{NA}^n W_n(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha_{NA}^n) W_n(m, d_A, d_B),\\
\Omega_{NA}^d(m, d_A, d_B) &= \alpha_{NA}^d W_d(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha_{NA}^d) W_d(m, d_A, d_B),\\
\Omega_{NB}^n(m, d_A, d_B) &= \alpha_{NB}^n W_n(m - \xi_B, d_A, d_B + \chi_B) + (1 - \alpha_{NB}^n) W_n(m, d_A, d_B),\\
\Omega_{NB}^d(m, d_A, d_B) &= W_d(m, d_A, d_B),
\end{aligned} \tag{A.3}$$

where V_s denotes a C-type agent's value function in the DM in state s. Note that OTC$_B$ shuts down when asset B defaults, and thus N-type agents proceed directly to the CM, whereas C-type agents move on to the DM.

Finally, consider the value functions in the DM. C-type agents meet producers. Let q denote the quantity of DM goods traded and τ the total payment in units of money. These terms of trade are described in the next section. The value function of an agent who enters the DM with a portfolio (m, d_A, d_B) in state s is given by

$$V_s(m, d_A, d_B) = u(q) + W_s(m - \tau, d_A, d_B), \tag{A.4}$$

The value function of a producer, who enters with no money or assets, is given by

$$V^P = -q + W^P(\tau),$$

which does not depend on states of the economy.

A.2 Terms of trade

First, we describe the terms of trade in the DM. Consider a meeting between a producer and a C-type agent with a portfolio (m, d_A, d_B). The two parties bargain over a quantity q to be produced by the producer and a cash payment τ to be made by the agent. The agent makes a take-it-or-leave-it offer maximizing her surplus subject to the producer's participation condition and the cash constraint:

$$\max_{\tau,\, q} \Big\{ u(q) + W_s(m - \tau, d_A, d_B) - W_s(m, d_A, d_B) \Big\}$$
$$\text{s.t.} \quad -q + W^P(\tau) - W^P(0) = 0, \quad \tau \leq m.$$

Using the linearity of the CM value functions, the C-type agent's surplus becomes $u(q) - \varphi\tau$ and the producer's surplus $-q + \varphi\tau$. This implies that the bargaining solution must satisfy $q(m) = \varphi\tau(m)$ – that is, the producer will require $\tau(m)$ units of money for producing $q(m)$ of goods. When the agent has enough money to have the optimal level produced, that is, when $\varphi m \geq q^*$, q^* will be produced. Otherwise, φm will be produced. Define $m^* \equiv q^*/\varphi$ as the amount of money that allows an agent to purchase the first-best quantity, q^*. Then, the solution can be expressed in a concise way:

$$\begin{aligned} q(m) &= \min\{q^*, \varphi m\} \big(= \varphi\tau(m)\big), \\ \tau(m) &= \min\{m^*, m\}, \quad m^* \equiv q^*/\varphi. \end{aligned} \tag{A.5}$$

Since an agent will never choose to hold $m > m^*$ due to the cost of carrying money, we will focus on the binding branch of the bargaining solution, $q(m) = \varphi m$ and $\tau(m) = m$.

Next, we turn to the terms of trade in the OTC markets. Consider a meeting in OTC$_j$ between a C-type agent with a portfolio (m, d_A, d_B) who wants to sell assets and an N-type agent with $(\tilde{m}, \tilde{d}_A, \tilde{d}_B)$ who wants to buy assets. Let χ_j be the amount of asset j will be traded for ξ_j amount of money as a result of bargaining. The Kalai bargaining applies with the asset seller's bargaining power denoted by θ. Then, the bargaining surpluses of an i-type consumer from an OTC$_j$ trading in state s, $\mathcal{S}^s_{ij}$, are given by

$$\begin{aligned}
\mathcal{S}^n_{CA} &= V_n(m + \xi_A, d_A - \chi_A, d_B) - V_n(m, d_A, d_B) = u(\varphi(m + \xi_A)) - u(\varphi m) - \varphi\chi_A, \\
\mathcal{S}^n_{NA} &= W_n(\tilde{m} - \xi_A, \tilde{d}_A + \chi_A, \tilde{d}_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_A + \varphi\chi_A, \\
\mathcal{S}^d_{CA} &= V_d(m + \xi_A, d_A - \chi_A, d_B) - V_d(m, d_A, d_B) = u(\varphi(m + \xi_A)) - u(\varphi m) - \varphi\chi_A, \\
\mathcal{S}^d_{NA} &= W_d(\tilde{m} - \xi_A, \tilde{d}_A + \chi_A, \tilde{d}_B) - W_d(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_A + \varphi\chi_A, \\
\mathcal{S}^n_{CB} &= V_n(m + \xi_B, d_A, d_B - \chi_B) - V_n(m, d_A, d_B) = u(\varphi(m + \xi_B)) - u(\varphi m) - \varphi\chi_B, \\
\mathcal{S}^n_{NB} &= W_n(\tilde{m} - \xi_B, \tilde{d}_A, \tilde{d}_B + \chi_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_B + \varphi\chi_B.
\end{aligned}$$

Notice that $\mathcal{S}^n_{CA} = \mathcal{S}^d_{CA}$ and $\mathcal{S}^n_{NA} = \mathcal{S}^d_{NA}$; thus, the solutions will not depend on states of the economy. $\mathcal{S}^d_{CB}$ and $\mathcal{S}^d_{CB}$ are not defined since OTC$_B$ shuts down when asset B defaults. Thus, we will simply write as follows: $\mathcal{S}_{CA}(\equiv \mathcal{S}^n_{CA} = \mathcal{S}^d_{CA})$, $\mathcal{S}_{NA}(\equiv \mathcal{S}^n_{NA} = \mathcal{S}^d_{NA})$, $\mathcal{S}_{CB}(\equiv \mathcal{S}^n_{CB})$, and $\mathcal{S}_{NB}(\equiv \mathcal{S}^n_{NB})$. The expressions for the surpluses can be simplified as follows:

$$\begin{aligned}
\mathcal{S}_{Cj} &= u(\varphi(m + \xi_j)) - u(\varphi m) - \varphi\chi_j, \\
\mathcal{S}_{Nj} &= -\varphi\xi_j + \varphi\chi_j.
\end{aligned}$$

Since money is costly to carry, in equilibrium, C-type agents will bring $m < m^*$ and want to acquire the amount of money that she is missing in order to reach m^*, namely, $m^* - m$. Whether she will be able to acquire that amount of money depends on her asset holdings. If her asset holdings are enough, then she will be able to acquire $m^* - m$ units of money. If not, she will give up all her assets to obtain as much money as possible.

An assumption behind this discussion is that N-type's money holdings never limit the trade. That is, we assume that $m + \tilde{m} \geq m^*$, i.e., that the money holdings of the C-type and the N-type pulled together is enough to allow the C-type to purchase the first best quantity q^*, hence ignoring the constraint $\xi_j \leq \tilde{m}$ in the bargaining problem. This will be true in equilibrium as long as inflation is not too large so that all agents carry at least *half* of the first-best amount of money (see also page 12).

Thus, the bargaining problem is described by

$$\max_{\xi_j, \chi_j} \mathcal{S}_{Cj} \quad \text{s.t.} \quad \mathcal{S}_{Cj} = \frac{\theta}{1-\theta} \mathcal{S}_{Nj}, \quad \chi_j \leq d_j.$$

From the Kalai constraint, we get

$$\varphi \chi_j = z(\xi_j) \equiv (1-\theta)\Big(u(\varphi(m+\xi_j)) - u(\varphi m)\Big) + \theta \varphi \xi_j,$$

which says that the asset seller has to give up $z(\xi_j)/\varphi$ amount of asset j to acquire ξ_j amount of money. Note that $z'(\xi_j) > 0$, and recall that the optimal amount of money that the asset seller wants to achieve is $m^* - m$. When the asset seller has enough assets to compensate $m^* - m$, that is, when $\varphi d_j \geq z(m^* - m)$, $m^* - m$ will be traded. Otherwise, d_j will be traded. The solution can be expressed in a concise way:

$$\begin{aligned} &\chi_j(m, d_j) = \min\{d_j^*, d_j\} \big(= z(\xi_j(m, d_j))/\varphi\big), \quad d_j^* \equiv \frac{z(m^* - m)}{\varphi}, \\ &\xi_j(m, d_j) = \min\{m^* - m, \tilde{\xi}_j(m, d_j)\}, \quad \varphi d_j = z(\tilde{\xi}_j). \end{aligned} \tag{A.6}$$

With the discussion above in mind, note that the solution does not depend on the N-type consumer's portfolio, but only on the C-type's. Also, note that $\xi_j(m, d_j)$ is increasing in d_j (the more assets a C-type has, the more money she can acquire) and decreasing in m (the more money a C-type carries, the less she needs to acquire through OTC trade).

A.3 Objective function

As is standard in models that build on LW, all agents choose their optimal portfolio in the CM independently of their trading histories in previous markets. In our model, in addition to choosing an optimal portfolio of money and assets, $(\hat{m}, \hat{d}_A, \hat{d}_B)$, agents also choose which OTC market they will enter in order to sell or buy assets, once the shocks have been realized. To analyze the agent's choice, we substitute the agent's value functions in the OTC markets and the DM (equations (A.3) and (A.4)) into the maximization operator of the CM value function (A.1) and use the linearity of the CM value functions (equation (A.2)), dropping the terms that do not depend on the choice variables, to obtain

$$\begin{aligned} -\varphi(\hat{m} + p_A \hat{d}_A + p_B \hat{d}_B) &+ \beta\pi\ell\Big(\hat{\varphi}(\hat{d}_A + \hat{d}_B) + u(\hat{\varphi}\hat{m}) + \max\{\alpha_{CA}^n \mathcal{S}_{CA}, \alpha_{CB}^n \mathcal{S}_{CB}\}\Big) \\ &+ \beta\pi(1-\ell)\hat{\varphi}(\hat{m} + \hat{d}_A + \hat{d}_B) \\ &+ \beta(1-\pi)\ell\Big(\hat{\varphi}\hat{d}_A + u(\hat{\varphi}\hat{m}) + \alpha_{CA}^d \mathcal{S}_{CA}\Big) \\ &+ \beta(1-\pi)(1-\ell)\hat{\varphi}(\hat{m} + \hat{d}_A), \end{aligned}$$

from which we finally get the objective function:

$$
\begin{aligned}
J(\hat{m}, \hat{d}_A, \hat{d}_B) \equiv & -\varphi(\hat{m} + p_A \hat{d}_A + p_B \hat{d}_B) + \beta\hat{\varphi}(\hat{m} + \hat{d}_A + \pi \hat{d}_B) \\
& + \beta\ell\Big(u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi \max\{\alpha^n_{CA}\mathcal{S}_{CA}, \alpha^n_{CB}\mathcal{S}_{CB}\} + (1-\pi)\alpha^d_{CA}\mathcal{S}_{CA}\Big) \\
= & -\beta\hat{\varphi} i\hat{m} - \beta\hat{\varphi}(1+i)\left(p_A - \frac{1}{1+i}\right)\hat{d}_A - \beta\hat{\varphi}(1+i)\left(p_B - \frac{\pi}{1+i}\right)\hat{d}_B \\
& + \beta\ell\Big(u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi \max\{\alpha^n_{CA}\mathcal{S}_{CA}, \alpha^n_{CB}\mathcal{S}_{CB}\} + (1-\pi)\alpha^d_{CA}\mathcal{S}_{CA}\Big),
\end{aligned} \tag{A.7}
$$

with $i \equiv (1+\mu)/\beta - 1$, where

$$
\mathcal{S}_{Cj} = \theta\left[u(\hat{\varphi}(\hat{m} + \xi_j(\hat{m}, \hat{d}_j))) - u(\hat{\varphi}\hat{m}) - \hat{\varphi}\xi_j(\hat{m}, \hat{d}_j)\right].
$$

A.4 Asset demand

Asset demand equations are derived from the first-order conditions of the objective function (A.7) with respect to $\hat{d}_A$ and $\hat{d}_B$:

$$
\begin{aligned}
\{\hat{d}_A\} \quad & (1+i)p_A - 1 = \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CA}}{\partial \hat{d}_A}, \\
\{\hat{d}_B\} \quad & (1+i)p_B - \pi = \ell\pi\alpha^n_{CB}\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CB}}{\partial \hat{d}_B},
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial \mathcal{S}_{Cj}}{\partial \hat{d}_j} = \frac{\partial \mathcal{S}_{Cj}}{\partial \tilde{\xi}_j}\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j} &= \theta\left(u'(\hat{\varphi}(\hat{m} + \tilde{\xi}_j)) - 1\right)\hat{\varphi}\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j}, \\
\hat{\varphi} = z'(\tilde{\xi}_j)\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j} &= \hat{\varphi}\left(\theta + (1-\theta)u'(\hat{\varphi}(\hat{m} + \tilde{\xi}_j))\right)\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j},
\end{aligned}
$$

where the second equation is from total differentiation of $\hat{\varphi}\hat{d}_j = z(\tilde{\xi}_j(\hat{m}, \hat{d}_j))$.

From above, we can get the asset demand equations (A.12) and (A.13) by expressing in terms of the equilibrium quantities:

$$
\begin{aligned}
(1+i)p_A - 1 &= \ell\theta\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A}) - 1)\frac{1}{\theta + (1-\theta)u'(q_{1A})}, \\
(1+i)p_B - \pi &= \ell\theta\pi\alpha^n_{CB}(u'(q_{1B}) - 1)\frac{1}{\theta + (1-\theta)u'(q_{1B})}.
\end{aligned} \tag{A.8}
$$

A.5 Money demand

Money demand equations are derived from the first-order conditions of the objective function (A.7) with respect to $\hat{m}$:

$$
\{\hat{m}\} \quad i = \ell\left((u'(\hat{\varphi}\hat{m}) - 1) + \Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CA}}{\partial \hat{m}}\right),
$$

$$i = \ell\left((u'(\hat{\varphi}\hat{m}) - 1) + \pi\alpha_{CB}^n \frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CB}}{\partial \hat{m}}\right),$$

where

$$\begin{aligned}
\frac{\partial \mathcal{S}_{Cj}}{\partial \hat{m}} &= \theta\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j)) - u'(\hat{\varphi}\hat{m})\right) + \theta\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j)) - 1\right)\hat{\varphi}\frac{\partial \tilde{\xi}_j}{\partial \hat{m}}, \\
0 &= (1-\theta)\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j)) - u'(\hat{\varphi}\hat{m})\right) + (1-\theta)u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))\frac{\partial \tilde{\xi}_j}{\partial \hat{m}} + \theta\frac{\partial \tilde{\xi}_j}{\partial \hat{m}},
\end{aligned}$$

where the second equation is from total differentiation of $\hat{\varphi}\hat{d}_j = z(\tilde{\xi}_j(\hat{m}, \hat{d}_j))$.

From above, we can get the money demand equations by expressing in terms of the equilibrium quantities:

$$\begin{aligned}
i &= \ell\Big(1 - \theta\Big(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\Big)\Big)(u'(q_{0A}) - 1) \\
&\quad + \ell\theta\Big(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\Big)(u'(q_{1A}) - 1)\left(1 - \frac{(1-\theta)(u'(q_{1A}) - u'(q_{0A}))}{\theta + (1-\theta)u'(q_{1A})}\right), \qquad \text{(A.9)} \\
i &= \ell(1 - \theta\pi\alpha_{CB}^n)(u'(q_{0B}) - 1) + \ell\theta\pi\alpha_{CB}^n(u'(q_{1B}) - 1)\left(1 - \frac{(1-\theta)(u'(q_{1B}) - u'(q_{0B}))}{\theta + (1-\theta)u'(q_{1B})}\right).
\end{aligned}$$

A.6 Characterization of equilibrium conditions

We start by describing how the variables $\{z_A, z_B, \varphi, p_A, p_B\}$ can follow directly from the "core" variables $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e_N^n\}$. (Recall that e_N^d is always equal to 1.) First, since the C-types have all the bargaining power in the DM, the equilibrium real balances satisfy

$$z_A = q_{0A},\ z_B = q_{0B}. \qquad \text{(A.10)}$$

Second, the equilibrium price of money solves the money market clearing condition:

$$\varphi M = e_C q_{0A} + (1 - e_C) q_{0B}. \qquad \text{(A.11)}$$

Third, the equilibrium asset prices solve the following asset demand equations (reproduced from (A.8) in Appendix A.4):

$$p_A = \frac{1}{1+i}\left(1 + \ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\Big)(u'(q_{1A}) - 1)\right), \qquad \text{(A.12)}$$

$$p_B = \frac{1}{1+i}\left(\pi + \ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha_{CB}^n(u'(q_{1B}) - 1)\right), \qquad \text{(A.13)}$$

where:

$$\omega_\theta(q) \ \equiv\ \theta + (1-\theta)u'(q) \ \geq\ 1.$$

Next, we study the determination of $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e_N^n\}$. To determine the six core variables we have six equilibrium conditions. First, we have the two money demand equations for agents who specialize in asset A and B:

$$\begin{aligned} i = \ell\Big(1-\theta\Big(\pi\alpha_{CA}^n+(1-\pi)\alpha_{CA}^d\Big)\Big)(u'(q_{0A})-1) \\ +\ell\theta\frac{\omega_\theta(q_{0A})}{\omega_\theta(q_{1A})}\Big(\pi\alpha_{CA}^n+(1-\pi)\alpha_{CA}^d\Big)(u'(q_{1A})-1), \end{aligned} \tag{A.14}$$

$$i = \ell(1-\theta\pi\alpha_{CB}^n)(u'(q_{0B})-1)+\ell\theta\frac{\omega_\theta(q_{0B})}{\omega_\theta(q_{1B})}\pi\alpha_{CB}^n(u'(q_{1B})-1). \tag{A.15}$$

Next, we have the trading protocol in OTC$_j$, $j=\{A,B\}$, that links q_{0j} and q_{1j}:

$$\begin{aligned} q_{1A} &= \min\Big\{q^*,\, q_{0A}+\varphi\tilde{\xi}_A\Big\}, \quad \varphi d_A = z(\tilde{\xi}_A), \\ q_{1B} &= \min\Big\{q^*,\, q_{0B}+\varphi\tilde{\xi}_B\Big\}, \quad \varphi d_B = z(\tilde{\xi}_B), \end{aligned}$$

where:

$$\begin{aligned} z(\tilde{\xi}) &\equiv (1-\theta)\Big(u(\varphi(m+\tilde{\xi}))-u(\varphi m)\Big)+\theta\varphi\tilde{\xi}, \\ d_A &= \begin{cases} S_A/e_C, & \text{if } e_C>0, \\ 0, & \text{otherwise}, \end{cases} \\ d_B &= \begin{cases} S_B/(1-e_C), & \text{if } e_C<1, \\ 0, & \text{otherwise}. \end{cases} \end{aligned}$$

The equations for d_A, d_B can be interpreted as asset market clearing with 'free disposal': we require agents to choose either asset A or B to specialize in, so it is possible that everyone chooses the same asset. In that case, demand for the other asset is zero. If demand for an asset is positive, the market for that asset clears with equality.

The other equations also have intuitive interpretations. They state that if the agent's asset holdings are large, then $q_{1j}=q^*$, because the agent will acquire (through selling assets) the money necessary to purchase the first-best quantity, and no more. On the other hand, if the agent's asset holdings are scarce, she will give up all her assets and purchase an amount of DM good equal to q_{0j} (the amount she could have purchased without any OTC trade) plus $\varphi\tilde{\xi}_j$ (the additional amount she can now afford by selling assets for extra cash). The terms d_j represent the amount of assets held by the typical agent who specializes in asset j. With some additional work, we can re-write the OTC bargaining protocols in a form that involves only core equilibrium variables (and parameters):

$$q_{1A} = \min\Big\{q^*,\, q_{0A}+\frac{1}{\theta}\,\frac{S_A}{M}\,\frac{e_Cq_{0A}+(1-e_C)q_{0B}}{e_C}-\frac{1-\theta}{\theta}\Big(u(q_{1A})-u(q_{0A})\Big)\Big\}, \tag{A.16}$$

$$q_{1B} = \min\left\{q^*,\, q_{0B} + \frac{1}{\theta}\frac{S_B}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{1-e_C} - \frac{1-\theta}{\theta}\Big(u(q_{1B}) - u(q_{0B})\Big)\right\}. \tag{A.17}$$

Our last two equilibrium conditions come from the optimal OTC market entry decisions. An important remark is that the OTC surplus of N-types does not depend on their portfolios (see Section 3.1 or A.2), whereas the OTC surplus of C-types does depend on their portfolios. Hence, in making their entry decisions, C-types consider not only the expected surplus of entering in either market, as is the case for N-types, but also the cost associated with each entry decision. Another way of stating this is to say that e_C is determined *ex-ante* and represents the decision to specialize in asset A, while e_N^n is determined *ex-post* and represents the fraction of N-types who enter OTC$_A$ in the normal state.

The C-types' surplus terms defining their optimal specialization choice 2 are:

$$\begin{aligned}\tilde{\mathcal{S}}_{CA} =& -iq_{0A} - ((1+i)p_A - 1)\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big)\\ &+ \ell(u(q_{0A}) - q_{0A}) + \ell\Big(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\Big)\mathcal{S}_{CA},\\ \mathcal{S}_{CA} =& \,\theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big),\end{aligned} \tag{A.18}$$

and:

$$\begin{aligned}\tilde{\mathcal{S}}_{CB} =& -iq_{0B} - ((1+i)p_B - \pi)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big)\\ &+ \ell(u(q_{0B}) - q_{0B}) + \ell\pi\alpha_{CB}^n\mathcal{S}_{CB},\\ \mathcal{S}_{CB} =& \,\theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big),\end{aligned} \tag{A.19}$$

where the negative terms represent the 'holding cost' associated with the various assets.

The N-types make their market entry choice *ex post*, hence their surplus terms defining 3 do not include a holding cost:

$$\mathcal{S}_{NA} = (1-\theta)\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big), \tag{A.20}$$

$$\mathcal{S}_{NB} = (1-\theta)\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big), \tag{A.21}$$

A.7 General-equilibrium effects of market entry choices

In Section 3.4, we mention three general-equilibrium effects – congestion, coordination, and dilution – of a presumed exogenous shift in e_C, the share of agents visiting market A as opposed to B. Here, we discuss these effects in more detail than the flow of that section allows.

The "congestion effect" is simply the obvious fact that as more sellers enter a market, matching becomes harder for other sellers in that market. This effect is reduced if η

increases, and altogether disappears if $\eta = 1$, hence we sometimes refer to the matching function with $\eta = 1$ as "congestion-free".

The "coordination effect" consists of two stages: first, the fact that as more buyers enter a market, sellers match more easily, and vice versa; second, the fact that if (exogenously) more sellers were to enter a market, then buyers' match probability will improve, which makes more buyers (endogenously) enter that market, which in turn improves sellers' matching probability and thus rationalizes the presumed movement of sellers.

As is well-known in the search-and-matching literature, with constant returns to scale in the matching function and constant surpluses per meeting, the congestion and coordination effects exactly offset (which is perhaps why, to our knowledge, they have not been given names before). With extremely increasing returns to scale – for instance, our matching function (Equation 1) with $\eta = 1$ – the coordination effect is maximal and the congestion effect is nil. For intermediate levels of increasing returns, represented in our model via $\eta \in (0, 1)$, something in-between is true: the congestion effect exists but the coordination effect is stronger.

Of course, in our model, surpluses per meeting are *not* constant as e_C varies, because e_C represents *both* the share of C-types entering OTC$_A$ and the share of agents holding asset A in the CM, precisely because in our model agents can only sell assets they hold (no short-selling). If a larger share of agents seeks to hold asset A, then each of them will hold a smaller amount of it, which down the line will cause a smaller per-meeting surplus in OTC$_A$. We name this force the "dilution effect", and it is precisely the reason why interior equilibria, where some agents hold asset A and others hold B, tend to be robust at all in our model (when η is not too large).

B Proofs of Propositions

B.1 Proof of Proposition 1: Classification of equilibria

(a) Assume $e_C = 0$. Then, $e_N^n = e_N^d = 0$ is clearly the best response of N-types. We claim that there is no profitable deviation of a C-type, i.e., $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB} < 0$ when $e_C = 0$, $e_N^n = e_N^d = 0$. First, notice that when $e_C = 0$, that is, when nobody is holding asset A, $q_{0A} = q_{1A}$ $(\equiv \bar{q})$ and $\tilde{\mathcal{S}}_{CA} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$. If it were the case that nobody was purchasing asset B either, then $q_{0B} = q_{1B}$ $(= \bar{q})$, $\tilde{\mathcal{S}}_{CB} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$ and $G(e_C) = 0$. Since q_{1B} increases and q_{0B} decreases as agents hold asset B, it remains to show that $d\tilde{\mathcal{S}}_{CB}/dq_{1B} > 0$, keeping in mind that q_{0B} also changes as q_{1B} changes.

When $e_C = 0$ and $e_N^n = e_N^d = 0$, we have $\alpha_{CB}^n = 1 - \ell$:

$$\tilde{\mathcal{S}}_{CB} = -iq_{0B} - \left(\ell \frac{\theta}{\omega_\theta(q_{1B})}\pi(1-\ell)(u'(q_{1B})-1)\right)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)$$
$$+\ell(u(q_{0B})-q_{0B})+\ell\pi(1-\ell)\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big).$$

Then,

$$\frac{d\tilde{\mathcal{S}}_{CB}}{dq_{1B}} = -\frac{dq_{0B}}{dq_{1B}}\left(i - \frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta+(1-\theta)u'(q_{1B})} + \frac{\ell(\theta+(1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}\right)$$
$$-\frac{1}{(\theta+(1-\theta)u'(q_{1B}))^2}(1-\ell)\ell\pi\theta\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)u''(q_{1B}).$$

The coefficient of dq_{0B}/dq_{1B} in the first term is

$$i - \frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta+(1-\theta)u'(q_{1B})} + \frac{\ell(\theta+(1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}$$
$$= i - \left(\frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta+(1-\theta)u'(q_{1B})} - \frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}\right)$$
$$-\left(\frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})} - \frac{\ell(\theta+(1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}\right)$$
$$= i - \ell\left(1 - \frac{\theta}{\theta+(1-\theta)u'(q_{1B})}\pi(1-\ell)\right)(u'(q_{0B})-1) - \ell\frac{\theta}{\theta+(1-\theta)u'(q_{1B})}\pi(1-\ell)(u'(q_{1B})-1),$$

which is 0 since it is equivalent to the money demand equation (A.15). Thus,

$$\frac{d\tilde{\mathcal{S}}_{CB}}{dq_{1B}} = -\frac{1}{(\theta+(1-\theta)u'(q_{1B}))^2}(1-\ell)\ell\pi\theta\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)u''(q_{1B}) > 0. \tag{B.1}$$

Therefore, $G(e_C) < 0$ when $e_C = 0$, $e_N^n = e_N^d = 0$. □

(b) Assume $e_C = 1$. Then, $e_N^n = e_N^d = 1$ is clearly the best response of N-types. We claim that there is no profitable deviation of a C-type, i.e., $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB} > 0$ when $e_C = 1$, $e_N^n = e_N^d = 1$. First, notice that when $e_C = 1$, that is, when nobody is holding asset B, $q_{0B} = q_{1B}$ ($\equiv \bar{q}$) and $\tilde{\mathcal{S}}_{CB} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$. If it were the case that nobody was purchasing asset A either, then $q_{0A} = q_{1A}$ ($= \bar{q}$), $\tilde{\mathcal{S}}_{CA} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$ and $G(e_C) = 0$. Since q_{1A} increases and q_{0A} decreases as agents hold asset A, it remains to show that $d\tilde{\mathcal{S}}_{CA}/dq_{1A} > 0$, keeping in mind that q_{0A} also changes as q_{1A} changes.

When $e_C = 1$ and $e_N^n = e_N^d = 1$, we have $\alpha_{CA}^n = \alpha_{CA}^d = 1 - \ell$:

$$\tilde{\mathcal{S}}_{CA} = -iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}(1-\ell)(u'(q_{1A})-1)\right)\Big((1-\theta)(u(q_{1A})-u(q_{0A}))+\theta(q_{1A}-q_{0A})\Big)$$

$$+\ell(u(q_{0A})-q_{0A})+\ell(1-\ell)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big).$$

Then,

$$\begin{aligned}\frac{d\tilde{\mathcal{S}}_{CA}}{dq_{1A}}=&-\frac{dq_{0A}}{dq_{1A}}\left(i-\frac{\ell(\ell\theta+(1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta+(1-\theta)u'(q_{1A})}+\frac{\ell(\theta+(1-(2-\ell)\theta)u'(q_{1A}))}{\theta+(1-\theta)u'(q_{1A})}\right)\\&-\frac{1}{(\theta+(1-\theta)u'(q_{1A}))^2}(1-\ell)\ell\theta\Big((1-\theta)(u(q_{1A})-u(q_{0A}))+\theta(q_{1A}-q_{0A})\Big)u''(q_{1A}).\end{aligned}$$

The coefficient of dq_{0A}/dq_{1A} in the first term is

$$\begin{aligned}&i-\frac{\ell(\ell\theta+(1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta+(1-\theta)u'(q_{1A})}+\frac{\ell(\theta+(1-(2-\ell)\theta)u'(q_{1A}))}{\theta+(1-\theta)u'(q_{1A})}\\=&\,i-\left(\frac{\ell(\ell\theta+(1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta+(1-\theta)u'(q_{1A})}-\frac{\ell(\ell\theta+(1-\theta)u'(q_{1A}))}{\theta+(1-\theta)u'(q_{1A})}\right)\\&-\left(\frac{\ell(\ell\theta+(1-\theta)u'(q_{1A}))}{\theta+(1-\theta)u'(q_{1A})}-\frac{\ell(\theta+(1-(2-\ell)\theta)u'(q_{1A}))}{\theta+(1-\theta)u'(q_{1A})}\right)\\=&\,i-\ell\left(1-\frac{\theta}{\theta+(1-\theta)u'(q_{1A})}(1-\ell)\right)(u'(q_{0A})-1)-\ell\frac{\theta}{\theta+(1-\theta)u'(q_{1A})}(1-\ell)(u'(q_{1A})-1),\end{aligned}$$

which is 0 since it is equivalent to the money demand equation (A.14). Thus,

$$\frac{d\tilde{\mathcal{S}}_{CA}}{dq_{1A}}=-\frac{1}{(\theta+(1-\theta)u'(q_{1A}))^2}(1-\ell)\ell\theta\Big((1-\theta)(u(q_{1A})-u(q_{0A}))+\theta(q_{1A}-q_{0A})\Big)u''(q_{1A})>0. \tag{B.2}$$

Therefore, $G(e_C)>0$ when $e_C=1$ and $e_N^n=e_N^d=1$. □

(c) First observe the values of α_{CA}^n, α_{CA}^d, α_{CB}^n as $e_C\to 0+$. While $\alpha_{CA}^n=\alpha_{CA}^d=0$ at $e_C=0$, this is not the case when $e_C\to 0+$. From the optimal entry decision by N-types (3),

$$e_N^n=\frac{e_C(1-e_C\ell-\ell\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\ell\mathcal{S}_{NB}/\mathcal{S}_{NA})}{-(1-\ell)(-e_C-\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\mathcal{S}_{NB}/\mathcal{S}_{NA})}.$$

Using this, as $e_C\to 0+$,

$$\begin{aligned}\alpha_{CA}^n&\to\lim_{e_C\to 0+}\frac{(1-\ell)e_N^n}{\ell e_C+(1-\ell)e_N^n}=\lim_{e_C\to 0+}\frac{(1-\ell)\frac{e_C(1-e_C\ell-\ell\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\ell\mathcal{S}_{NB}/\mathcal{S}_{NA})}{-(1-\ell)(-e_C-\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\mathcal{S}_{NB}/\mathcal{S}_{NA})}}{\ell e_C+(1-\ell)\frac{e_C(1-e_C\ell-\ell\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\ell\mathcal{S}_{NB}/\mathcal{S}_{NA})}{-(1-\ell)(-e_C-\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\mathcal{S}_{NB}/\mathcal{S}_{NA})}}\\&=\lim_{e_C\to 0+}1+e_C\ell(-1+\frac{\mathcal{S}_{NB}}{\mathcal{S}_{NA}})-\ell\frac{\mathcal{S}_{NB}}{\mathcal{S}_{NA}}=1-\ell\frac{\mathcal{S}_{NB}}{\mathcal{S}_{NA}}\ (>1-\ell),\\\alpha_{CA}^d&\to\lim_{e_C\to 0+}\frac{1-\ell}{\ell e_C+(1-\ell)}=1.\end{aligned}$$

On the other hand, α^n_{CB} continuously converges to its value, $1-\ell$, at $e_C = 0$ as $e_C \to 0+$. Hence, as $e_C \to 0+$, $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} > \pi\alpha^n_{CB}$. Therefore,

$$\begin{aligned}\tilde{\mathcal{S}}_{CA} =& -iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\right)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big)\\ &+ \ell(u(q_{0A})-q_{0A}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\\ >& -iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}\pi\alpha^n_{CB}(u'(q_{1A})-1)\right)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big)\\ &+ \ell(u(q_{0A})-q_{0A}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\\ \geq& -iq_{0B} - \left(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1)\right)\Big((1-\theta)(u(q_{1B})-u(q_{0B})) + \theta(q_{1B}-q_{0B})\Big)\\ &+ \ell(u(q_{0B})-q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big) = \tilde{\mathcal{S}}_{CB},\end{aligned}$$

where the first inequality comes from

$$\begin{aligned}&\Big[-iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\right)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big)\\ &\quad+ \ell(u(q_{0A})-q_{0A}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\Big]\\ &-\Big[-iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}\pi\alpha^n_{CB}(u'(q_{1A})-1)\right)\Big((1-\theta)(u(q_{1A})-u(q_{0A})) + \theta(q_{1A}-q_{0A})\Big)\\ &\quad+ \ell(u(q_{0A})-q_{0A}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\Big]\\ =&\ \frac{\ell[(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}) - \pi\alpha^n_{CB}]\theta(q_{1A}-q_{0A})}{\theta + (1-\theta)u'(q_{1A})}\left(\frac{u(q_{1A})-u(q_{0A})}{q_{1A}-q_{0A}} - u'(q_{1A})\right) > 0,\end{aligned}$$

in which we used $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} > \pi\alpha^n_{CB}$ and the strict concavity of u; and the second inequality comes from that $\lim_{e_C\to 0+} q_{1B} \leq \lim_{e_C\to 0+} q_{1A} = q^*$ and (B.1). Therefore, $G(e_C) > 0$ as $e_C \to 0+$. □

(d) All we need to show is that one of α^n_{CA}, α^d_{CA} and α^n_{CB} is discontinuous at $e_C = 1$. Here, the discontinuity arises in α^n_{CB}. From the optimal entry decision by N-types (3),

$$e^n_N = \frac{e_C(1-e_C\ell - \ell\mathcal{S}_{NB}/\mathcal{S}_{NA} + e_C\ell\mathcal{S}_{NB}/\mathcal{S}_{NA})}{-(1-\ell)(-e_C - \mathcal{S}_{NB}/\mathcal{S}_{NA} + e_C\mathcal{S}_{NB}/\mathcal{S}_{NA})}.$$

Using this, as $e_C \to 1$,

$$\alpha^n_{CB} \to \lim_{e_C\to 1}\frac{(1-\ell)(1-e^n_N)}{\ell(1-e_C) + (1-\ell)(1-e^n_N)} = \lim_{e_C\to 1}\frac{(1-\ell)\left(1-\frac{e_C(1-e_C\ell-\ell\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\ell\mathcal{S}_{NB}/\mathcal{S}_{NA})}{-(1-\ell)(-e_C-\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\mathcal{S}_{NB}/\mathcal{S}_{NA})}\right)}{\ell(1-e_C) + (1-\ell)\left(1-\frac{e_C(1-e_C\ell-\ell\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\ell\mathcal{S}_{NB}/\mathcal{S}_{NA})}{-(1-\ell)(-e_C-\mathcal{S}_{NB}/\mathcal{S}_{NA}+e_C\mathcal{S}_{NB}/\mathcal{S}_{NA})}\right)}$$

$$= \lim_{e_C \to 1} 1 + \ell\bigg(-1 + e_C - e_C \frac{\mathcal{S}_{NA}}{\mathcal{S}_{NB}}\bigg) = 1 - \ell \frac{\mathcal{S}_{NA}}{\mathcal{S}_{NB}} \, (> 1 - \ell).$$

On the other hand, $\alpha^n_{CB} = 0$ at $e_C = 1$.

Now assume $\pi \to 1$. Unlike α^n_{CB}, α^n_{CA} and α^d_{CA} continuously converge to their values at $e_C = 1$, which are both $1 - \ell$. Hence, as $e_C \to 1$ and $\pi \to 1$, $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} < \pi\alpha^n_{CB}$. Therefore,

$$\begin{aligned}
\tilde{\mathcal{S}}_{CB} =& - iq_{0B} - \bigg(\ell \frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B}) - 1)\bigg)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) \\
&+ \ell(u(q_{0B}) - q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big) \\
>& - iq_{0B} - \bigg(\ell \frac{\theta}{\omega_\theta(q_{1B})}\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1B}) - 1)\bigg)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) \\
&+ \ell(u(q_{0B}) - q_{0B}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big) \\
\geq& - iq_{0A} - \bigg(\ell \frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A}) - 1)\bigg)\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big) \\
&+ \ell(u(q_{0A}) - q_{0A}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big) = \tilde{\mathcal{S}}_{CA},
\end{aligned}$$

where the first inequality comes from

$$\begin{aligned}
&\bigg[- iq_{0B} - \bigg(\ell \frac{\theta}{\omega_\theta(q_{1B})}\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)(u'(q_{1B}) - 1)\bigg)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) \\
&\quad + \ell(u(q_{0B}) - q_{0B}) + \ell\Big(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big)\bigg] \\
&- \bigg[- iq_{0B} - \bigg(\ell \frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B}) - 1)\bigg)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) \\
&\quad + \ell(u(q_{0B}) - q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big)\bigg] \\
=& \frac{\ell[(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}) - \pi\alpha^n_{CB}]\theta(q_{1B} - q_{0B})}{\theta + (1-\theta)u'(q_{1B})}\left(\frac{u(q_{1B}) - u(q_{0B})}{q_{1B} - q_{0B}} - u'(q_{1B})\right) < 0,
\end{aligned}$$

in which we used $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} < \pi\alpha^n_{CB}$ and the strict concavity of u; and the second inequality comes from that $\lim_{e_C \to 1} q_{1A} \leq \lim_{e_C \to 1} q_{1B} = q^*$ and (B.2). Therefore, $G(e_C) < 0$ as $e_C \to 1$. □

(e) From (c) and (d), we have $\lim_{e_C \to 0+} G(e_C) > 0 > \lim_{e_C \to 1} G(e_C)$ when $\pi \to 1$. The continuity of G immediately implies that there exists at least one robust interior equilibrium. □

(f) All we need to show is that one of α^n_{CA}, α^d_{CA} and α^n_{CB} is discontinuous at $e_C = 1$. Here, the discontinuity arises in α^d_{CA}. As $e_C \to 0+$,

$$\alpha^d_{CA} \to \lim_{e_C \to 0+} \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\eta}} = (1-\ell)^\eta.$$

On the other hand, $\alpha^d_{CA} = 0$ at $e_C = 0$. □

(g) All we need to show is that all α^n_{CA}, α^d_{CA} and α^n_{CB} continuously converge to their values at $e_C = 1$ as $e_C \to 1$. From the optimal entry decision by N-types (3),

$$e^n_N = \frac{-1 + e_C\ell + e_C\ell\left(\frac{(1-e_C)\mathcal{S}_{NB}/\mathcal{S}_{NA}}{e_C}\right)^{\frac{1}{1-\eta}}}{-(1-\ell)\left(1+\left(\frac{(1-e_C)\mathcal{S}_{NB}/\mathcal{S}_{NA}}{e_C}\right)^{\frac{1}{1-\eta}}\right)}.$$

Then, as $e_C \to 1$,

$$\begin{aligned}
\alpha^n_{CA} &= \frac{(1-\ell)e^n_N}{(\ell e_C + (1-\ell)e^n_N)^{1-\eta}} \\
&= \left(\frac{1}{1+\left(\left(-1+\frac{1}{e_C}\right)\frac{\mathcal{S}_{NB}}{\mathcal{S}_{NA}}\right)^{\frac{1}{1-\eta}}}\right)^{\eta}\left(1 - e_C\ell\left(1+\left(\left(-1+\frac{1}{e_C}\right)\frac{\mathcal{S}_{NB}}{\mathcal{S}_{NA}}\right)^{\frac{1}{1-\eta}}\right)\right) \to 1-\ell, \\
\alpha^d_{CA} &= \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\eta}} \to 1-\ell, \\
\alpha^n_{CB} &= \frac{(1-\ell)(1-e^n_N)}{(\ell(1-e_C) + (1-\ell)(1-e^n_N))^{1-\eta}} \\
&= \left(\frac{1}{1+\left(\left(-1+\frac{1}{e_C}\right)\frac{\mathcal{S}_{NB}}{\mathcal{S}_{NA}}\right)^{-\frac{1}{1-\eta}}}\right)^{\eta}\left(1 - (1-e_C)\ell\left(1+\left(\left(-1+\frac{1}{e_C}\right)\frac{\mathcal{S}_{NB}}{\mathcal{S}_{NA}}\right)^{-\frac{1}{1-\eta}}\right)\right) \to 0,
\end{aligned}$$

and, at $e_C = e^n_N = 1$,

$$\begin{aligned}
\alpha^n_{CA} &= \frac{(1-\ell)e^n_N}{(\ell e_C + (1-\ell)e^n_N)^{1-\eta}} = 1-\ell, \\
\alpha^d_{CA} &= \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\eta}} = 1-\ell, \\
\alpha^n_{CB} &= \frac{(1-\ell)(1-e^n_N)}{(\ell(1-e_C) + (1-\ell)(1-e^n_N))^{1-\eta}} = 0.
\end{aligned}$$

Therefore, α^n_{CA}, α^d_{CA} and α^n_{CB} continuously converge to their values at $e_C = 1$ as $e_C \to 1$, and $G(e_C)$ also continuously converges to its value at $e_C = 1$, which is greater than 0, as $e_C \to 1$. □

B.2 Proof of Proposition 2: When safety implies liquidity

(a) Guess-and-verify: at $\pi = 1$, all the equilibrium equations are symmetric between the A and B markets, so $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $L_A = L_B$ are satisfied. And $e_C = e_N = 0.5$ implies $\alpha^n_{CA} = \alpha^n_{CB}$ as well as $\alpha^n_{NA} = \alpha^n_{NB}$, so symmetry is complete. □

(b) In any interior equilibrium where $e_C \in (0,1)$ and both assets are scarce and valued for liquidity so that $q_{1A} < q^*$, $q_{1B} < q^*$, we can totally differentiate the equilibrium equations around the scarce-interior equilibrium.

Post-trade quantities (equations A.16 and A.17):

$$q_{1A} = \min\left\{ q^*,\, q_{0A} + \frac{\frac{S_A}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{e_C} - (1-\theta)\Big(u(q_{1A}) - u(q_{0A})\Big)}{\theta} \right\},$$

$$q_{1B} = \min\left\{ q^*,\, q_{0B} + \frac{\frac{S_B}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{1-e_C} - (1-\theta)\Big(u(q_{1B}) - u(q_{0B})\Big)}{\theta} \right\}.$$

Focusing on the scarce branch, total differentiate yields

$$\frac{w_\theta(q_{1A})}{\theta}\, dq_{1A} = \frac{S_A/M + w_\theta(q_{0A})}{\theta}\, dq_{0A} + \frac{S_A}{M}\frac{1-e_C}{e_C\theta}\, dq_{0B} - \frac{S_A}{M}\frac{q_{0B}}{e_C^2\theta}\, de_C, \tag{B.3}$$

$$\frac{w_\theta(q_{1B})}{\theta}\, dq_{1B} = \frac{S_B/M + w_\theta(q_{0B})}{\theta}\, dq_{0B} + \frac{S_B}{M}\frac{e_C}{(1-e_C)\theta}\, dq_{0A} + \frac{S_B}{M}\frac{q_{0A}}{(1-e_C)^2\theta}\, de_C. \tag{B.4}$$

Money demand (equations A.14 and A.15):

$$i = \ell(1-\theta\bar{\alpha}_{Cj})(u'(q_{0j}) - 1) + \ell\theta\frac{w_\theta(q_{0j})}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}(u'(q_{1j}) - 1),$$

which is equivalent to

$$i = \ell\left(1 - \frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}\right)(u'(q_{0j}) - 1) + \ell\frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}(u'(q_{1j}) - 1), \quad j = A, B,$$

where

$$\begin{aligned} w_\theta(q) &\equiv \theta + (1-\theta)u'(q), \\ \bar{\alpha}_{CA} &\equiv \pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}, \\ \bar{\alpha}_{CB} &\equiv \pi\alpha^n_{CB}. \end{aligned}$$

Total differentiation yields

$$
\begin{aligned}
0 = \ell \left(1 - \frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}\right) u''(q_{0j})\, dq_{0j} + \ell \frac{\theta}{w_\theta(q_{1j})}(u'(q_{1j}) - u'(q_{0j}))\, d\bar{\alpha}_{Cj} \\
+ \ell \frac{\theta}{w_\theta(q_{1j})^2}\bar{\alpha}_{Cj}\Big(u''(q_{1j})w_\theta(q_{1j}) - (u'(q_{1j}) - u'(q_{0j}))w'_\theta(q_{1j})\Big)dq_{1j}.
\end{aligned}
\tag{B.5}
$$

<u>*Liquidity premia (equation 6):*</u>

Define a new variable, for $j = A, B$:

$$
\bar{L}_j \equiv \ell \frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}(u'(q_{1j}) - 1),
$$

where $\bar{L}_A = L_A = (1+i)p_A - 1$ and $\bar{L}_B = \pi L_B = (1+i)p_B - \pi$. Total differentiation yields

$$
d\bar{L}_j = \ell \frac{\theta}{w_\theta(q_{1j})^2}\bar{\alpha}_{Cj}\Big(u''(q_{1j})w_\theta(q_{1j}) - (u'(q_{1j}) - 1)w'_\theta(q_{1j})\Big)dq_{1j} + \ell \frac{\theta}{w_\theta(q_{1j})}(u'(q_{1j}) - 1)\, d\bar{\alpha}_{Cj}.
$$

<u>*C's entry choice (equations 2, A.18, and A.19):*</u>

$$
\begin{aligned}
\mathcal{S}_{Cj} &= \theta\Big(u(q_{1j}) - u(q_{0j}) - q_{1j} + q_{0j}\Big), \\
\tilde{\mathcal{S}}_{Cj} &= -\, iq_{0j} - \bar{L}_j\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big) + \ell(u(q_{0j}) - q_{0j}) + \ell\bar{\alpha}_{Cj}\mathcal{S}_{Cj}.
\end{aligned}
$$

Total differentiation yields

$$
\begin{aligned}
d\mathcal{S}_{Cj} &= \theta(u'(q_{1j}) - 1)\, dq_{1j} - \theta(u'(q_{0j}) - 1)\, dq_{0j}, \\
d\tilde{\mathcal{S}}_{Cj} &= -\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big)d\bar{L}_j + \ell\mathcal{S}_{Cj}\, d\bar{\alpha}_{Cj} \\
&\quad + \Big(-i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1)\Big)dq_{0j} - \bar{L}_j w_\theta(q_{1j})\, dq_{1j} + \ell\bar{\alpha}_{Cj}\, d\mathcal{S}_{Cj},
\end{aligned}
$$

where

$$
\begin{aligned}
&\Big(-i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1)\Big)dq_{0j} - \bar{L}_j w_\theta(q_{1j})\, dq_{1j} + \ell\bar{\alpha}_{Cj}\, d\mathcal{S}_{Cj} \\
= &\Big(-i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1) - \ell\bar{\alpha}_{Cj}\theta(u'(q_{0j}) - 1)\Big)dq_{0j} \\
&+ \Big(-\bar{L}_j w_\theta(q_{1j}) + \ell\bar{\alpha}_{Cj}\theta(u'(q_{1j}) - 1)\Big)dq_{1j} = 0
\end{aligned}
$$

since the coefficient of dq_{0j} is equivalent to the first-version money demand. Thus,

$$
d\tilde{\mathcal{S}}_{Cj} = -\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big)d\bar{L}_j + \ell\mathcal{S}_{Cj}\, d\bar{\alpha}_{Cj}.
$$

Therefore, we have

$$
G(e_C) = \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB},
$$

and total differentiation yields

$$
\begin{aligned}
dG &= d\tilde{\mathcal{S}}_{CA} - d\tilde{\mathcal{S}}_{CB} \\
&= -\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big)d\bar{L}_A + \ell\mathcal{S}_{CA}\, d\bar{\alpha}_{CA} \\
&\quad + \Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big)d\bar{L}_B - \ell\mathcal{S}_{CB}\, d\bar{\alpha}_{CB}.
\end{aligned} \tag{B.6}
$$

N's entry choice (equations 3, A.20, and A.21):

$$
\alpha^n_{NA}\mathcal{S}_{NA} = \alpha^n_{NB}\mathcal{S}_{NB},
$$
$$
\mathcal{S}_{Nj} = (1-\theta)\Big(u(q_{1j}) - u(q_{0j}) - q_{1j} + q_{0j}\Big).
$$

Total differentiation yields

$$
\mathcal{S}_{NA}\, d\alpha^n_{NA} + \alpha^n_{NA}\, d\mathcal{S}_{NA} = \mathcal{S}_{NB}\, d\alpha^n_{NB} + \alpha^n_{NB}\, d\mathcal{S}_{NB},
$$
$$
d\mathcal{S}_{Nj} = (1-\theta)(u'(q_{1j}) - 1)\, dq_{1j} - (1-\theta)(u'(q_{0j}) - 1)\, dq_{0j}.
$$

Thus,

$$
\begin{aligned}
&\mathcal{S}_{NA}\, d\alpha^n_{NA} + \alpha^n_{NA}(1-\theta)(u'(q_{1A}) - 1)\, dq_{1A} - \alpha^n_{NA}(1-\theta)(u'(q_{0A}) - 1)\, dq_{0A} \\
&= \mathcal{S}_{NB}\, d\alpha^n_{NB} + \alpha^n_{NB}(1-\theta)(u'(q_{1B}) - 1)\, dq_{1B} - \alpha^n_{NB}(1-\theta)(u'(q_{0B}) - 1)\, dq_{0B}.
\end{aligned} \tag{B.7}
$$

Matching probabilities (Section 3.1):

$$
\begin{aligned}
\alpha^n_{CA} &= e^n_N(1-\ell)\,[e^n_N(1-\ell) + e_C\ell]^{\eta-1}, \\
\alpha^n_{CB} &= (1-e^n_N)(1-\ell)\,[(1-e^n_N)(1-\ell) + (1-e_C)\ell]^{\eta-1}, \\
\alpha^n_{NA} &= e_C\ell\,[e^n_N(1-\ell) + e_C\ell]^{\eta-1}, \\
\alpha^n_{NB} &= (1-e_C)\ell\,[(1-e^n_N)(1-\ell) + (1-e_C)\ell]^{\eta-1}, \\
\alpha^d_{CA} &= (1-\ell)\,[(1-\ell) + e_C\ell]^{\eta-1}, \\
\alpha^d_{NA} &= e_C\ell\,[(1-\ell) + e_C\ell]^{\eta-1}, \\
\alpha^d_{CB} &= \alpha^d_{NB} = 0.
\end{aligned}
$$

Total differentiation yields

$$
\begin{aligned}
d\alpha^n_{CA} = &-(1-\eta)\ell(1-\ell)e^n_N\,[e^n_N(1-\ell) + e_C\ell]^{\eta-2}\, de_C \\
&+ \left[\frac{\alpha^n_{CA}}{e_N} - (1-\eta)(1-\ell)^2 e^n_N\,[e^n_N(1-\ell) + e_C\ell]^{\eta-2}\right] de^n_N,
\end{aligned}
$$

$$
d\alpha^n_{CB} = (1-\eta)\ell(1-\ell)(1-e^n_N)\,[(1-e^n_N)(1-\ell) + (1-e_C)\ell]^{\eta-2}\, de_C
$$

$$
\begin{aligned}
&\quad - \left[\frac{\alpha^n_{CB}}{1-e^n_N} - (1-\eta)(1-\ell)^2(1-e^n_N)\left[(1-e^n_N)(1-\ell)+(1-e_C)\ell\right]^{\eta-2}\right] de^n_N,\\
d\alpha^n_{NA} &= \left[\frac{\alpha^n_{NA}}{e_C} - (1-\eta)\ell^2 e_C\left[e^n_N(1-\ell)+e_C\ell\right]^{\eta-2}\right] de_C\\
&\quad - (1-\eta)(1-\ell)\ell e_C\left[e^n_N(1-\ell)+e_C\ell\right]^{\eta-2} de^n_N,\\
d\alpha^n_{NB} &= -\left[\frac{\alpha^n_{NB}}{1-e_C} - (1-\eta)\ell^2(1-e_C)\left[(1-e^n_N)(1-\ell)+(1-e_C)\ell\right]^{\eta-2}\right] de_C\\
&\quad + (1-\eta)(1-\ell)\ell(1-e_C)\left[(1-e^n_N)(1-\ell)+(1-e_C)\ell\right]^{\eta-2} de^n_N,\\
d\alpha^d_{CA} &= -(1-\eta)\ell(1-\ell)\left[(1-\ell)+e_C\ell\right]^{\eta-2} de_C,\\
d\alpha^d_{NA} &= \left[\frac{\alpha^d_{NA}}{e_C} - (1-\eta)\ell^2 e_C\left[(1-\ell)+e_C\ell\right]^{\eta-2}\right] de_C,\\
d\alpha^d_{CB} &= d\alpha^d_{NB} = 0.
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
d\bar{\alpha}_{CA} &= \pi\, d\alpha^n_{CA} + (1-\pi)\, d\alpha^d_{CA} + (\alpha^n_{CA} - \alpha^d_{CA})\, d\pi,\\
d\bar{\alpha}_{CB} &= \pi\, d\alpha^n_{CB} + \alpha^n_{CB}\, d\pi.
\end{aligned}
$$

Now restrict attention to the symmetric equilibrium with CRS matching. If $S_A = S_B \equiv S$ and $\pi \to 1$, then a symmetric equilibrium exists where $e_C = e^n_N = 1/2$. When $\eta = 0$, the matching probabilities becomes

$$
\begin{aligned}
\bar{\alpha}_{CA} &= \bar{\alpha}_{CB} = \alpha^n_{CA} = \alpha^n_{CB} = 1-\ell,\\
\alpha^n_{NA} &= \alpha^n_{NB} = \ell,\\
\alpha^d_{CA} &= \frac{2(1-\ell)}{2-\ell},\\
\alpha^d_{NA} &= \frac{\ell}{2-\ell},\\
\alpha^d_{CB} &= \alpha^d_{NB} = 0\,,
\end{aligned}
$$

which in turn implies $q_{0A} = q_{0B} \equiv q_0$ and $q_{1A} = q_{1B} \equiv q_1$. Total differentiation yields

$$
\begin{aligned}
d\alpha^n_{CA} &= -d\alpha^n_{CB} = -d\alpha^n_{NA} = d\alpha^n_{NB} = -\,2\ell(1-\ell)\, de_C + 2\ell(1-\ell)\, de^n_N,\\
d\alpha^d_{CA} &= -d\alpha^d_{NA} = -(1-\ell)\ell\left(\frac{2-\ell}{2}\right)^{-2} de_C,\\
d\alpha^d_{CB} &= d\alpha^d_{NB} = 0.
\end{aligned}
$$

Assuming CRS matching ($\eta = 0$), put together (B.3), (B.4), (B.5), (B.6), (B.7) in matrix form:

$$\mathbf{A}\boldsymbol{u} = \boldsymbol{b}\,d\pi, \tag{B.8}$$

where:

$$\mathbf{A} = \begin{bmatrix} \mathfrak{a} & -\mathfrak{a} & -\mathfrak{b} & 0 & -\mathfrak{c} & 0 \\ -\mathfrak{a} & \mathfrak{a} & 0 & -\mathfrak{b} & 0 & -\mathfrak{c} \\ -\mathfrak{d} & 0 & -\mathfrak{e} & 0 & \mathfrak{f} & \mathfrak{g} \\ \mathfrak{d} & 0 & 0 & -\mathfrak{e} & \mathfrak{g} & \mathfrak{f} \\ \mathfrak{h} & -\mathfrak{h} & \mathfrak{j} & -\mathfrak{j} & -\mathfrak{k} & \mathfrak{k} \\ -\mathfrak{m} & \mathfrak{m} & \mathfrak{n} & -\mathfrak{n} & 0 & 0 \end{bmatrix}, \quad \boldsymbol{u} = \begin{bmatrix} de_C \\ de_N^n \\ dq_{1A} \\ dq_{1B} \\ dq_{0A} \\ dq_{0B} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} -\dfrac{\mathfrak{a}}{2(2-\ell)} \\ \dfrac{\mathfrak{a}}{2\ell} \\ 0 \\ 0 \\ 0 \\ \dfrac{\mathfrak{m}}{2\ell(2-\ell)} \end{bmatrix},$$

and:

$$\begin{aligned}
\mathfrak{a} &= \frac{2(1-\ell)\ell\theta[u'(q_0) - u'(q_1)]}{w_\theta(q_1)}, \\
\mathfrak{b} &= -\frac{(1-\ell)\theta w_\theta(q_0)}{w_\theta(q_1)^2} u''(q_1), \\
\mathfrak{c} &= -\frac{\ell\theta + (1-\ell)u'(q_1)}{w_\theta(q_1)} u''(q_0), \\
\mathfrak{d} &= \frac{4q_0 S/M}{\theta}, \\
\mathfrak{e} &= \frac{w_\theta(q_1)}{\theta}, \\
\mathfrak{f} &= \frac{S/M + w_\theta(q_0)}{\theta}, \\
\mathfrak{g} &= \frac{S/M}{\theta}, \\
\mathfrak{h} &= 4(1-\ell), \\
\mathfrak{j} &= \frac{u'(q_1) - 1}{u(q_1) - u(q_0) - q_1 + q_0}, \\
\mathfrak{k} &= \frac{u'(q_0) - 1}{u(q_1) - u(q_0) - q_1 + q_0}, \\
\mathfrak{m} &= \frac{4(1-\ell)\ell^2\theta[u(q_1) - u(q_0) - (q_1 - q_0)u'(q_1)]}{w_\theta(q_1)}, \\
\mathfrak{n} &= -\frac{(1-\ell)\ell\theta[(1-\theta)(u(q_1) - u(q_0)) + \theta(q_1 - q_0)]}{w_\theta(q_1)^2} u''(q_1).
\end{aligned}$$

Note that $\mathfrak{a}$ to $\mathfrak{n}$ are all positive. With a symbolic software package, it is easy to check that the solution is given by:

$$\boldsymbol{u} = \begin{bmatrix} -\dfrac{-\mathfrak{cehm}-\mathfrak{bfhm}+\mathfrak{bghm}+2\mathfrak{afhn}-2\mathfrak{aghn}}{4\mathfrak{d}(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex] \dfrac{\mathfrak{cehm}+\mathfrak{bfhm}-\mathfrak{bghm}-2\mathfrak{cdjm}-2\mathfrak{bdkm}-2\mathfrak{afhn}+2\mathfrak{aghn}+4\mathfrak{adkn}}{4\mathfrak{d}(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex] -\dfrac{\begin{pmatrix} \mathfrak{c}^2\mathfrak{ehm}+\mathfrak{bcfhm}+\mathfrak{bcghm}-2\mathfrak{acfjm}-2\mathfrak{acgjm}-2\mathfrak{abfkm}-2\mathfrak{abgkm} \\ +2\mathfrak{acfj}\ell\mathfrak{m}+2\mathfrak{acgj}\ell\mathfrak{m}+2\mathfrak{abfk}\ell\mathfrak{m}+2\mathfrak{abgk}\ell\mathfrak{m}-2\mathfrak{acfhn}-2\mathfrak{acghn}+4\mathfrak{a}^2\mathfrak{fkn} \\ +4\mathfrak{a}^2\mathfrak{gkn}+2\mathfrak{acfh}\ell\mathfrak{n}+2\mathfrak{acgh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{fk}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{gk}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex] -\dfrac{\begin{pmatrix} -\mathfrak{c}^2\mathfrak{ehm}-\mathfrak{bcfhm}-\mathfrak{bcghm}-2\mathfrak{acfjm}-2\mathfrak{acgjm}-2\mathfrak{abfkm}-2\mathfrak{abgkm} \\ +2\mathfrak{acfj}\ell\mathfrak{m}+2\mathfrak{acgj}\ell\mathfrak{m}+2\mathfrak{abfk}\ell\mathfrak{m}+2\mathfrak{abgk}\ell\mathfrak{m}-2\mathfrak{acfhn}-2\mathfrak{acghn}+4\mathfrak{a}^2\mathfrak{fkn} \\ +4\mathfrak{a}^2\mathfrak{gkn}+2\mathfrak{acfh}\ell\mathfrak{n}+2\mathfrak{acgh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{fk}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{gk}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex] -\dfrac{\begin{pmatrix} -\mathfrak{bcehm}-\mathfrak{b}^2\mathfrak{fhm}-\mathfrak{b}^2\mathfrak{ghm}-2\mathfrak{acejm}-2\mathfrak{abekm}+2\mathfrak{acej}\ell\mathfrak{m} \\ +2\mathfrak{abek}\ell\mathfrak{m}+2\mathfrak{abfhn}+2\mathfrak{abghn}+4\mathfrak{a}^2\mathfrak{ekn}+2\mathfrak{aceh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{ek}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex] \dfrac{\begin{pmatrix} -\mathfrak{bcehm}-\mathfrak{b}^2\mathfrak{fhm}-\mathfrak{b}^2\mathfrak{ghm}+2\mathfrak{acejm}+2\mathfrak{abekm}-2\mathfrak{acej}\ell\mathfrak{m}-2\mathfrak{abek}\ell\mathfrak{m} \\ +4\mathfrak{acehn}+2\mathfrak{abfhn}+2\mathfrak{abghn}-4\mathfrak{a}^2\mathfrak{ekn}-2\mathfrak{aceh}\ell\mathfrak{n}+4\mathfrak{a}^2\mathfrak{ek}\ell\mathfrak{n} \end{pmatrix}}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \end{bmatrix} d\pi. \quad \text{(B.9)}$$

Now, look at the liquidity premium:

$$\begin{aligned} L_A &= \ell \frac{\theta}{w_\theta(q_{1A})}(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA})(u'(q_{1A}) - 1), \\ L_B &= \ell \frac{\theta}{w_\theta(q_{1B})}\alpha^n_{CB}(u'(q_{1B}) - 1). \end{aligned}$$

Total differentiation, when $\pi \to 1$ in the symmetric equilibrium, yields

$$\begin{aligned} dL_A &= \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}\alpha^n_{CA}dq_{1A} + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}(\alpha^n_{CA} - \alpha^d_{CA})d\pi + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}d\alpha^n_{CA}, \\ dL_B &= \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}\alpha^n_{CB}dq_{1B} + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}d\alpha^n_{CB}. \end{aligned}$$

Therefore,

$$dL_A - dL_B = \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}(dq_{1A} - dq_{1B}) + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}(\alpha^n_{CA} - \alpha^d_{CA})d\pi + \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}(d\alpha^n_{CA} - d\alpha^n_{CB}).$$

Since:

$$\alpha_{CA}^n = 1-\ell, \quad \alpha_{CA}^d = \frac{2(1-\ell)}{2-\ell}, \quad \text{and} \quad d\alpha_{CA}^n = -d\alpha_{CB}^n = -\,2\ell(1-\ell)(de_C - de_N^n),$$

we get:

$$dL_A - dL_B = \ell\theta\frac{u''(q_1)}{w_\theta(q_1)^2}(dq_{1A}-dq_{1B}) - \ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}\frac{\ell(1-\ell)}{2-\ell}d\pi - 4\ell\theta\frac{u'(q_1)-1}{w_\theta(q_1)}\ell(1-\ell)(de_C - de_N^n) \tag{B.10}$$

In order to ensure $dL_A - dL_B < 0$, we want each term in $dL_A - dL_B$ to be negative. The second term is obviously negative. To determine the sign of the first term, look at $dq_{1A} - dq_{1B}$. From (B.9),

$$dq_{1A} - dq_{1B} = \frac{\mathfrak{chm}}{2(2-\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})}.$$

The sign of $dq_{1A} - dq_{1B}$ depends on that of $\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn}$ in the denominator. We define:

$$\begin{aligned}\mathfrak{D} &\equiv \mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn}\\ &= \left[4\ell(1-\ell)\frac{\theta}{w_\theta(q_1)}\right]\left[-u''(q_0)\left(1-\frac{(1-\ell)\theta}{w_\theta(q_1)}\right)\ell(u'(q_1)-1)\frac{S^1}{S}\ldots\right.\\ &\qquad\left.-u''(q_0)\left(1-\frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S}\right],\end{aligned}$$

where:

$$\begin{aligned}S &\equiv u(q_1)-u(q_0)-q_1+q_0 > 0,\\ S^\theta &\equiv (1-\theta)(u(q_1)-u(q_0)) + \theta(q_1-q_0) > 0,\\ S^1 &\equiv u(q_1)-u(q_0)-u'(q_1)(q_1-q_0) > 0,\\ S^0 &\equiv u(q_1)-u(q_0)-u'(q_0)(q_1-q_0) < 0.\end{aligned}$$

$S^1 > 0$ and $S^0 < 0$ due to the strict concavity of u. For the first term in $dL_A - dL_B$ to be negative, we want $\mathfrak{D} > 0$ so that $dq_{1A} - dq_{1B} > 0$. The first and the second terms in the second bracket in $\mathfrak{D}$ are positive, whereas the third term is negative. If $\theta \to 0$ or $\ell(1-\ell) \to 0$, then $\mathfrak{D} > 0$. In case of quadratic utility, $u(q) \equiv (1+\gamma)q - q^2/2$ with $q^* = \gamma$, we can show that $\mathfrak{D} > 0$ is always the case for all (ℓ, θ). First, observe the following from the sum of the second and the third terms in the second bracket in $\mathfrak{D}$:

$$-\,u''(q_0)\left(1-\frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S}$$

$$
\begin{aligned}
&> - u''(q_0)\ell(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S} \\
&= \frac{-u''(q_1)}{w_\theta(q_1)}\ell(1-\ell)\frac{1}{S}\left[-u''(q_0)S^\theta S + (u'(q_0)-1)\theta S^0\right],
\end{aligned}
$$

where the first inequality comes from $u'(q_1) = 1 + q^* - q_1 \geq 1 > \ell$. Denote $\Upsilon(\theta) \equiv -u''(q_0)S^\theta S + (u'(q_0)-1)\theta S^0$. Observe that $\Upsilon(\theta = 0) = -u''(q_0)(u(q_1) - u(q_0))S > 0$; $\Upsilon(\theta = 1) = (q_1 - q_0)^2(q^* - q_1)/2 > 0$; and $d\Upsilon/d\theta = (u'(q_0)-1)S^0 + S^2u''(q_0) < 0$. Therefore, $\Upsilon > 0$ and $\mathfrak{D} > 0$. For other cases, including log utility, we verified numerically and could not find any case where $\mathfrak{D} > 0$ is not satisfied. $\mathfrak{D} > 0$ implies that $dq_{1A} - dq_{1B} > 0$ and that the first term in $dL_A - dL_B$ is negative.

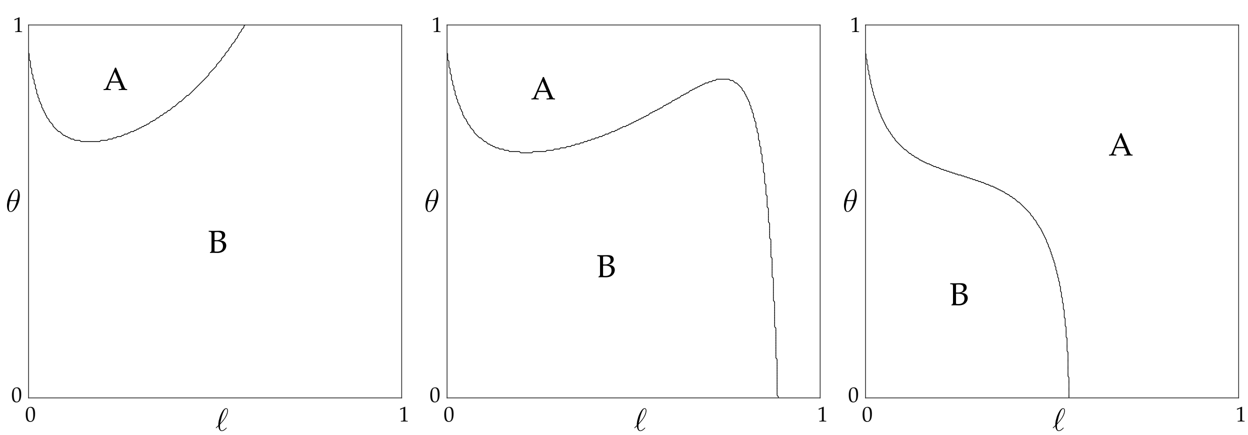
To determine the sign of the last term in Equation (B.10), look at $de_C - de_N^n$. From (B.9),

$$
de_C - de_N^n = -\frac{\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn}}{2(2-\ell)\ell\,\mathfrak{D}}.
$$

Since $\mathfrak{D} > 0$, the sign of $de_C - de_N^n$ depends on that of $\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn}$ in the numerator:

$$
\begin{aligned}
&\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn} \\
&= \left[4\ell^2(1-\ell)\frac{\theta}{w_\theta(q_1)}\right] \times \left[-u''(q_0)\left(1 - \frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(u'(q_1)-1)\frac{S^1}{S} - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}(u'(q_0)-1)\frac{S^0}{S}\right].
\end{aligned}
$$

For the third term in $dL_A - dL_B$ to be negative, we want $\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn} < 0$ so that $de_C - de_N^n > 0$. The first term in the second bracket is positive, whereas the second term is negative. From the equation, we can see that if $(1-\ell)\theta$ is sufficiently large, $\mathfrak{cjm}+\mathfrak{bkm}-2\mathfrak{akn}$ becomes negative, $de_C - de_N^n$ becomes positive, and the third term in $dL_A - dL_B$ becomes negative. Below is the figure that numerically shows in the (ℓ, θ) plane the parameter space where the third term in $dL_A - dL_B$ is negative (A) and where it is not (B):



Small asset supplies　Medium asset supplies　Large asset supplies

In region A, the third term in $dL_A - dL_B$ is negative, so all the components of $dL_A - dL_B$

are negative, while in region B the third term is positive. Under the sufficient condition that $(1-\ell)\theta$ is large enough, we will always be in region A so that *all* the components of $dL_A - dL_B$ become negative. Finally, $(dL_A - dL_B)/d\pi < 0$ in turn implies that for $d\pi < 1$ near $\pi = 1$, which is to say $\pi < 1$, we have $L_A > L_B$. □

B.3 Proof of Proposition 3: Liquidity premia in consolidated market

This proof assumes that asset supplies are jointly small enough so that assets are priced at a liquidity premium at the margin. In other words, we assume that asset supplies are such that q_1^n, q_1^d, implicitly defined in (C.11) and (C.12), satisfy $q^* > q_1^n > q_1^d$.

The liquidity premium of asset j, denoted by L_j, is given by the percentage difference between an asset's price and its fundamental value, and it solves

$$p_A = \frac{1}{1+i}(1+L_A), \qquad p_B = \frac{\pi}{1+i}(1+L_B),$$

where

$$\begin{aligned} L_A &= \ell\pi\alpha_C \frac{\theta}{\omega_\theta(q_1^n)}[u'(q_1^n)-1] + \ell(1-\pi)\alpha_C \frac{\theta}{\omega_\theta(q_1^d)}[u'(q_1^d)-1], \\ L_B &= \ell\alpha_C \frac{\theta}{\omega_\theta(q_1^n)}[u'(q_1^n)-1]. \end{aligned}$$

These can be rewritten as

$$\begin{aligned} L_A &= \ell\alpha_C\theta\left[\pi\frac{u'(q_1^n)-1}{\omega_\theta(q_1^n)} + (1-\pi)\frac{u'(q_1^d)-1}{\omega_\theta(q_1^d)}\right], \\ L_B &= \ell\alpha_C\theta\left[\frac{u'(q_1^n)-1}{\omega_\theta(q_1^n)}\right]. \end{aligned}$$

Notice that

$$\pi < 1, \quad 1-\pi > 0, \quad \frac{d\left(\dfrac{u'(q)-1}{\omega_\theta(q)}\right)}{dq} = \frac{u''(q)}{\omega_\theta(q)^2} < 0, \quad \text{and} \quad q_1^n > q_1^d.$$

Therefore, $L_A > L_B$. □

B.4 Result 1 with an alternative liquidity measure

In this appendix, we prove that Result 1 (safety implies liquidity) remains valid if we adopt a different measure of asset liquidity, namely, trade volume in the OTC market.

Before we state and prove the formal proposition, we need to define trade volume in the OTC markets. The OTC trade volumes in the normal state are defined by

$$\begin{aligned}\Delta_A^n &\equiv f(e_C\ell, e_N^n(1-\ell))\cdot\chi_A(m, d_A),\\ \Delta_B^n &\equiv f((1-e_C)\ell, (1-e_N^n)(1-\ell))\cdot\chi_B(m, d_B),\end{aligned}$$

where

$$\begin{aligned}\chi_j(m, d_j) = \min\{d_j^*, d_j\}, \quad d_j^* &\equiv \frac{z(m^*-m)}{\varphi},\\ z(\xi) \equiv (1-\theta)\Big(u(\varphi(m+\xi)) &- u(\varphi m)\Big) + \theta\varphi\xi,\\ d_A = \frac{S_A}{e_C}, \quad d_B &= \frac{S_B}{1-e_C}.\end{aligned}$$

These reduce as below:

$$\begin{aligned}\Delta_A^n &= e_C\ell\,\alpha_{CA}^n\cdot\min\left\{\frac{M\Big((1-\theta)(u(q^*)-u(q_{0A}))+\theta(q^*-q_{0A})\Big)}{e_Cq_{0A}+(1-e_C)q_{0B}}, \frac{S_A}{e_C}\right\},\\ \Delta_B^n &= (1-e_C)\ell\,\alpha_{CB}^n\cdot\min\left\{\frac{M\Big((1-\theta)(u(q^*)-u(q_{0B}))+\theta(q^*-q_{0B})\Big)}{e_Cq_{0A}+(1-e_C)q_{0B}}, \frac{S_B}{1-e_C}\right\}.\end{aligned}$$

The OTC trade volume of market A in the default state is defined by

$$\begin{aligned}\Delta_A^d &\equiv f(e_C\ell, 1-\ell)\cdot\chi_A(m, d_A)\\ &= e_C\ell\,\alpha_{CA}^d\cdot\min\left\{\frac{M\Big((1-\theta)(u(q^*)-u(q_{0A}))+\theta(q^*-q_{0A})\Big)}{e_Cq_{0A}+(1-e_C)q_{0B}}, \frac{S_A}{e_C}\right\}.\end{aligned}$$

Then, the OTC trade volumes, averaged across the normal and default states, are

$$\begin{aligned}\Delta_A &\equiv \pi\Delta_A^n + (1-\pi)\Delta_A^d,\\ \Delta_B &\equiv \pi\Delta_B^n.\end{aligned}\tag{B.11}$$

Proposition 4. *Assume that asset supplies S_A and S_B are equal and are low enough so that assets are scarce in OTC trade. Then:*

(a) At $\pi = 1$, there exists a symmetric equilibrium where $e_C = e_N = 0.5$, $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $\Delta_A = \Delta_B$.

(b) Assume $\eta = 0$ (CRS) and $(1-\ell)\theta$ is sufficiently large. Then, locally, $\pi < 1$ implies $\Delta_A > \Delta_B$: the safer asset is more liquid.

Proof. (a) Notice that when assets are scarce in OTC trade:

$$\Delta_A^n = \ell\, \alpha_{CA}^n S_A, \qquad \Delta_B^n = \ell\, \alpha_{CB}^n S_B, \qquad \Delta_A^d = \ell\, \alpha_{CA}^d S_A.$$

Therefore,

$$\begin{aligned}
\Delta_A &\equiv [\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d]\cdot \ell S_A, \\
\Delta_B &\equiv \pi\alpha_{CB}^n \cdot \ell S_B.
\end{aligned}$$

In Proposition 2 (a) we have proved that at $\pi = 1$, there exists a symmetric equilibrium where $e_C = e_N = 0.5$, $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $L_A = L_B$. In that symmetric equilibrium, $\Delta_A = \Delta_B$ since $\pi = 1$ and $\alpha_{CA}^n = \alpha_{CB}^n$. □

(b) In any interior equilibrium where $e_C \in (0, 1)$ and both assets are scarce and valued for liquidity so that $q_{1A} < q^*$, $q_{1B} < q^*$, we can totally differentiate the equilibrium equations around the scarce-interior equilibrium. We restrict attention to the symmetric equilibrium with CRS matching ($\eta = 0$). If $S_A = S_B \equiv S$ and $\pi \to 1$, then a symmetric equilibrium exists where $e_C = e_N^n = 1/2$. Total differentiation of trade volumes, when $\pi \to 1$ in the symmetric equilibrium, yields

$$\begin{aligned}
d\Delta_A &= \ell S(\alpha_{CA}^n - \alpha_{CA}^d)d\pi + \ell S d\alpha_{CA}^n, \\
d\Delta_B &= \ell S d\alpha_{CB}^n.
\end{aligned}$$

Therefore,

$$d\Delta_A - d\Delta_B = \ell S(\alpha_{CA}^n - \alpha_{CA}^d)d\pi + \ell S(d\alpha_{CA}^n - d\alpha_{CB}^n).$$

Since, in the symmetric equilibrium with CRS matching,

$$\alpha_{CA}^n = 1-\ell, \qquad \alpha_{CA}^d = \frac{2(1-\ell)}{2-\ell}, \qquad \text{and} \qquad d\alpha_{CA}^n = -d\alpha_{CB}^n = -\,2\ell(1-\ell)(de_C - de_N^n),$$

we get

$$d\Delta_A - d\Delta_B = -\ell S\frac{\ell(1-\ell)}{2-\ell}d\pi - 4\ell^2(1-\ell)S(de_C - de_N^n).$$

The first term is obviously negative. In the proof of Proposition 2 (b) in Appendix B.2, we have proved that $de_C - de_N^n > 0$ if $(1-\ell)\theta$ is sufficiently large, in which case $d\Delta_A - d\Delta_B < 0$ and, near $\pi = 1$, we have $\Delta_A > \Delta_B$. □

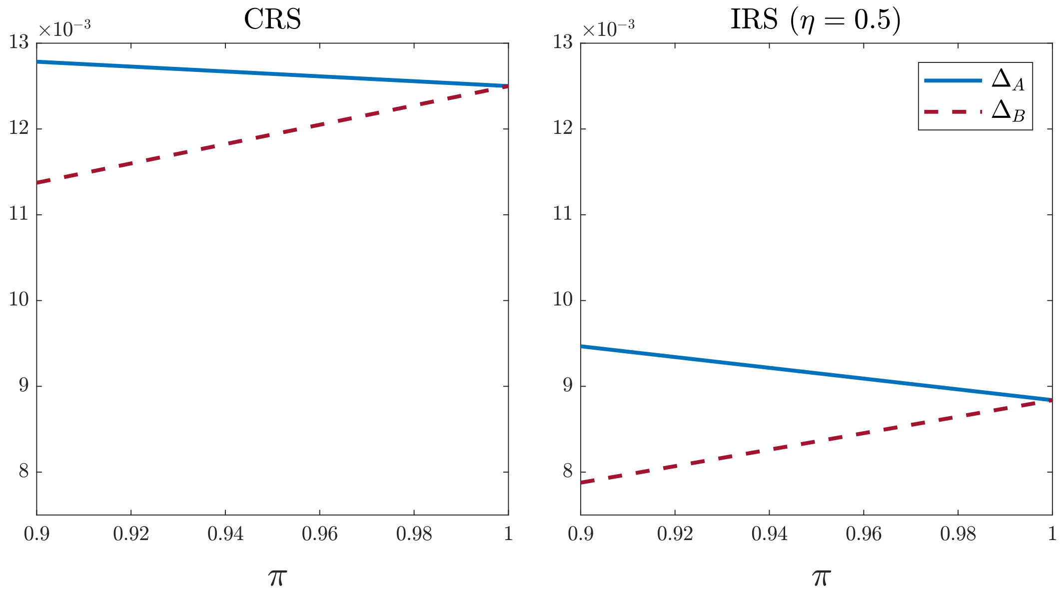


Figure 14. OTC trade volumes as functions of π

Notes: The figure depicts the trade volumes in OTC_A and OTC_B, Δ_A and Δ_B as defined in Appendix B.4, as functions of π, for $S_A = S_B$. In the left panel we assume the matching technology is CRS ($\eta = 0$), and in the right panel we have $\eta = 0.5$. The parameters used in the figure are: $M = 1$, $i = 0.1$, $\ell = 0.5$, $\theta = 0.5$, $S_A = S_B = 0.05$, with log utility.

Figure 14 graphs the trade volumes in the two OTC markets as functions of π for the cases of CRS and IRS. As seen in the figure, the trade volume is higher in the secondary market for the safer asset, and the difference in trade volumes between OTC_A and OTC_B is decreasing in π.[28] This is quite intuitive. The description of the mechanism behind Result 1 in Section 4.1 highlights that as π decreases, more agents will choose to coordinate in the market for asset A. Thus, it is not only the liquidity premium of asset A that increases in the default probability, but also the volume of trade in that market.

C Extensions and Microfoundations

C.1 Partial Default

Compared to the baseline model, there is one crucial difference. In the default state, which still happens with probability $1 - \pi$, asset B pays a recovery rate, $\rho \in (0, 1)$. As a result, in the event of default OTC_B does not shut down. This has two implications. First, in

[28] Within the context of a different model, Velioglu and Üslü (2019) obtain a result with similar flavor. They develop a multi-asset version of Duffie et al. (2005) and find that safer assets trade in larger quantities.

the event of default it is not the case any more that all N-types will rush to OTC$_A$ (see Equation C.9). Second, the entry decision of C-types (which is made ex-ante, i.e., in the CM of the previous period) now also depends on their payoff from visiting OTC$_B$ in the default state (see Equation C.1).

C.1.1 Analysis of value functions and terms of trade

First, we analyze the value functions in the CM. The value function of an agent who enters the CM with m units of money and d_j units of asset j, $j = \{A, B\}$ in state $s = \{n, d\}$ is given by

$$W_s(m, d_A, d_B) = \max_{\substack{X,H,\\ \hat{m},\hat{d}_A,\hat{d}_B}} \left\{ X - H + \beta\, \mathbb{E}_{s,i}\Big[\max\Big\{\Omega^s_{iA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^s_{iB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\}\Big] \right\}$$
$$\text{s.t. } X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A + d_B\cdot\mathbb{I}\{s = n\} + \rho\, d_B\cdot\mathbb{I}\{s = d\}),$$

where Ω^s_{ij} denotes the value function of an i-type agent, $i = \{C, N\}$, who enters the OTC market for asset j in state s. Compared to the baseline model, the only difference here is that asset B pays ρ dollars even in the state of default.

Next, the value functions in the OTC markets are given by

$$\begin{aligned}
\Omega^n_{CA}(m, d_A, d_B) &= \alpha^n_{CA}V_n(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha^n_{CA})V_n(m, d_A, d_B),\\
\Omega^d_{CA}(m, d_A, d_B) &= \alpha^d_{CA}V_d(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha^d_{CA})V_d(m, d_A, d_B),\\
\Omega^n_{CB}(m, d_A, d_B) &= \alpha^n_{CB}V_n(m + \xi^n_B, d_A, d_B - \chi^n_B) + (1 - \alpha^n_{CB})V_n(m, d_A, d_B),\\
\Omega^d_{CB}(m, d_A, d_B) &= \alpha^d_{CB}V_d(m + \xi^d_B, d_A, d_B - \chi^d_B) + (1 - \alpha^d_{CB})V_d(m, d_A, d_B),\\
\Omega^n_{NA}(m, d_A, d_B) &= \alpha^n_{NA}W_n(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha^n_{NA})W_n(m, d_A, d_B),\\
\Omega^d_{NA}(m, d_A, d_B) &= \alpha^d_{NA}W_d(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha^d_{NA})W_d(m, d_A, d_B),\\
\Omega^n_{NB}(m, d_A, d_B) &= \alpha^n_{NB}W_n(m - \xi^n_B, d_A, d_B + \chi^n_B) + (1 - \alpha^n_{NB})W_n(m, d_A, d_B),\\
\Omega^d_{NB}(m, d_A, d_B) &= \alpha^d_{NB}W_n(m - \xi^d_B, d_A, d_B + \chi^d_B) + (1 - \alpha^d_{NB})W_n(m, d_A, d_B),
\end{aligned}$$

where ξ_j is the amount of money that gets transferred to a C-type, and χ_j is the amount of asset j that gets transferred to an N-type in a typical match in OTC$_j$. For asset B, the aggregate state matters for the terms of trade, and thus ξ_B and χ_B have superscripts that indicate whether we are in the normal or the default state.

Finally, the value function in the DM is given by

$$V_s(m, d_A, d_B) = u(q) + W_s(m - \tau, d_A, d_B).$$

We now turn to the description of the terms of trade in the various markets, starting with the DM. Consider a meeting between a C-type agent with m units of money and a producer. Given that the C-type agent makes a take-it-or-leave-it offer and that the liquidity constraint will always bind due to the cost of carrying money, the bargaining solution is given by

$$q(m) = \varphi m, \quad \tau(m) = m.$$

Next, we turn to the terms of trade in the OTC markets. Consider a meeting in OTC$_j$ between a C-type asset seller with portfolio (m, d_A, d_B) and an N-type asset buyer with portfolio $(\tilde{m}, \tilde{d}_A, \tilde{d}_B)$. The bargaining surpluses of an i-type agent from an OTC$_j$ trading in state s, $\mathcal{S}_{ij}^s$, are given by

$$\begin{aligned}
\mathcal{S}_{CA} &= V_n(m + \xi_A, d_A - \chi_A, d_B) - V_n(m, d_A, d_B) = u(\varphi(m + \xi_A)) - u(\varphi m) - \varphi \chi_A, \\
\mathcal{S}_{NA} &= W_n(\tilde{m} - \xi_A, \tilde{d}_A + \chi_A, \tilde{d}_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi \xi_A + \varphi \chi_A, \\
\mathcal{S}_{CB}^n &= V_n(m + \xi_B^n, d_A, d_B - \chi_B^n) - V_n(m, d_A, d_B) = u(\varphi(m + \xi_B^n)) - u(\varphi m) - \varphi \chi_B^n, \\
\mathcal{S}_{NB}^n &= W_n(\tilde{m} - \xi_B^n, \tilde{d}_A, \tilde{d}_B + \chi_B^n) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi \xi_B^n + \varphi \chi_B^n, \\
\mathcal{S}_{CB}^d &= V_d(m + \xi_B^d, d_A, d_B - \chi_B^d) - V_d(m, d_A, d_B) = u(\varphi(m + \xi_B^d)) - u(\varphi m) - \varphi \rho \chi_B^d, \\
\mathcal{S}_{NB}^d &= W_d(\tilde{m} - \xi_B^d, \tilde{d}_A, \tilde{d}_B + \chi_B^d) - W_d(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi \xi_B^d + \varphi \rho \chi_B^d.
\end{aligned}$$

We now describe the bargaining solutions in the OTC markets. For (ξ_A, χ_A), we have

$$\begin{aligned}
\varphi \chi_A &= (1-\theta)[u(\varphi(m + \xi_A)) - u(\varphi m)] + \theta \varphi \xi_A, \\
\xi_A &= \min\{m^* - m, \tilde{\xi}_A\}, \\
\tilde{\xi}_A &= \{\xi : \varphi d_A = (1-\theta)[u(\varphi(m + \xi)) - u(\varphi m)] + \theta \varphi \xi\}.
\end{aligned}$$

For (ξ_B^n, χ_B^n), we have

$$\begin{aligned}
\varphi \chi_B^n &= (1-\theta)[u(\varphi(m + \xi_B^n)) - u(\varphi m)] + \theta \varphi \xi_B^n, \\
\xi_B^n &= \min\{m^* - m, \tilde{\xi}_B^n\}, \\
\tilde{\xi}_B^n &= \{\xi : \varphi d_B = (1-\theta)[u(\varphi(m + \xi)) - u(\varphi m)] + \theta \varphi \xi\}.
\end{aligned}$$

Lastly, for (ξ_B^d, χ_B^d), we have

$$\begin{aligned}
\varphi \rho \chi_B^d &= (1-\theta)[u(\varphi(m + \xi_B^d)) - u(\varphi m)] + \theta \varphi \xi_B^d, \\
\xi_B^d &= \min\{m^* - m, \tilde{\xi}_B^d\}, \\
\tilde{\xi}_B^d &= \{\xi : \varphi \rho d_B = (1-\theta)[u(\varphi(m + \xi)) - u(\varphi m)] + \theta \varphi \xi\}.
\end{aligned}$$

Compared to the baseline model, only the terms of trade (ξ_B^d, χ_B^d) are affected, since now asset B pays $\rho < 1$ dollars in the default state.

We can now derive the objective function of an agent in the CM, which is given by

$$J(\hat{m}, \hat{d}_A, \hat{d}_B) = -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) + \beta\hat{\varphi}[\hat{m} + \hat{d}_A + (\pi + (1-\pi)\rho)\hat{d}_B] \tag{C.1}$$
$$+ \beta\ell\Big[u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \max\Big\{\pi\alpha_{CA}^n\mathcal{S}_{CA} + (1-\pi)\alpha_{CA}^d\mathcal{S}_{CA}, \pi\alpha_{CB}^n\mathcal{S}_{CB}^n + (1-\pi)\alpha_{CB}^d\mathcal{S}_{CB}^d\}\Big\}\Big].$$

There are two main ways in which allowing for partial default affects the objective function compared to the baseline model. First, the new term ρ appears in the first line because an agent who walks into the next period's CM with some asset B will be entitled to the recovery rate in the default state, which happens with probability $1-\pi$. Second, the term $(1-\pi)\alpha_{CB}^d\mathcal{S}_{CB}^d$ now appears inside the maximum operator because OTC$_B$ does not shut down, and the agent's entry choice is also driven by her payoff from visiting OTC$_B$ in the default state.

C.1.2 Equilibrium

We now describe the steady state equilibrium of the model with partial default. The core variables are $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}^n, q_{1B}^d, e_C, e_N^n, e_N^d\}$, and we will describe the derivation of equilibrium following the same methodology as in the baseline model. For this analysis recall that we have defined $\omega_\theta(q) \equiv \theta + (1-\theta)u'(q) \geq 1$.

First, the money demand equations are given by

$$i = \ell\left[1 - (\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\frac{\theta}{\omega_\theta(q_{1A})}\right][u'(q_{0A}) - 1] \tag{C.2}$$
$$+ \ell(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\frac{\theta}{\omega_\theta(q_{1A})}[u'(q_{1A}) - 1],$$
$$i = \ell\left[1 - \pi\alpha_{CB}^n\frac{\theta}{\omega_\theta(q_{1B}^n)} - (1-\pi)\alpha_{CB}^d\frac{\theta}{\omega_\theta(q_{1B}^d)}\right][u'(q_{0B}) - 1] \tag{C.3}$$
$$+ \ell\pi\alpha_{CB}^n\frac{\theta}{\omega_\theta(q_{1B}^n)}[u'(q_{1B}^n) - 1] + \ell(1-\pi)\alpha_{CA}^d\frac{\theta}{\omega_\theta(q_{1B}^d)}[u'(q_{1B}^d) - 1].$$

The OTC trading protocols,

$$q_{1A} = \min\{q^*, q_{0A} + \varphi\tilde{\xi}_A\},$$
$$q_{1B}^n = \min\{q^*, q_{0B} + \varphi\tilde{\xi}_B^n\},$$
$$q_{1B}^d = \min\{q^*, q_{0B} + \varphi\tilde{\xi}_B^d\},$$

combined with the OTC bargaining solutions,

$$\varphi d_A = (1-\theta)[u(q_{1A}) - u(q_{0A})] + \theta\varphi\tilde{\xi}_A,$$

$$\varphi d_B = (1-\theta)[u(q_{1B}^n) - u(q_{0B})] + \theta\varphi\tilde{\xi}_B^n,$$
$$\varphi\rho d_B = (1-\theta)[u(q_{1B}^d) - u(q_{0B})] + \theta\varphi\tilde{\xi}_B^d,$$

and the market clearing conditions,

$$S_A = e_C d_A, \quad S_B = (1-e_C)d_B, \quad \varphi M = e_C q_{0A} + (1-e_C)q_{0B},$$

yield

$$q_{1A} = \min\left\{q^*, q_{0A} + \frac{\frac{e_C q_{0A} + (1-e_C)q_{0B}}{M}\frac{S_A}{e_C} - (1-\theta)[u(q_{1A}) - u(q_{0A})]}{\theta}\right\}, \tag{C.4}$$

$$q_{1B}^n = \min\left\{q^*, q_{0B} + \frac{\frac{e_C q_{0A} + (1-e_C)q_{0B}}{M}\frac{S_B}{1-e_C} - (1-\theta)[u(q_{1B}^n) - u(q_{0B})]}{\theta}\right\}, \tag{C.5}$$

$$q_{1B}^d = \min\left\{q^*, q_{0B} + \frac{\frac{e_C q_{0A} + (1-e_C)q_{0B}}{M}\rho\frac{S_B}{1-e_C} - (1-\theta)[u(q_{1B}^d) - u(q_{0B})]}{\theta}\right\}. \tag{C.6}$$

The next important task is to describe the agents' entry decisions. For that, it is useful to start by describing the liquidity premia of the various assets. Define $\bar{\rho} = \pi + (1-\pi)\rho$. Then, the liquidity premium of asset j, denoted by L_j, is given by the percentage difference between an asset's price and its fundamental value. In other words, L_j solves

$$p_A = \frac{1}{1+i}(1+L_A), \qquad p_B = \frac{\bar{\rho}}{1+i}(1+L_B),$$

where

$$L_A = \ell(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\frac{\theta}{\omega_\theta(q_{1A})}[u'(q_{1A}) - 1],$$
$$L_B = \frac{1}{\bar{\rho}}\left(\ell\pi\alpha_{CB}^n\frac{\theta}{\omega_\theta(q_{1B}^n)}[u'(q_{1B}^n) - 1] + \ell(1-\pi)\rho\alpha_{CB}^d\frac{\theta}{\omega_\theta(q_{1B}^d)}[u'(q_{1B}^d) - 1]\right).$$

The optimal entry of C-type buyers is characterized by

$$e_C = \begin{cases} 1, & \tilde{S}_{CA} > \tilde{S}_{CB} \\ 0, & \tilde{S}_{CA} < \tilde{S}_{CB} \\ \in [0,1], & \tilde{S}_{CA} = \tilde{S}_{CB} \end{cases}, \tag{C.7}$$

where

$$\tilde{S}_{CA} = -iq_{0A} - L_A\Big[(1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big]$$

$$
\begin{aligned}
&\qquad + \ell\Big[u(q_{0A}) - q_{0A} + (\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA})S_{CA}\Big], \\
S_{CA} &= \theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big),
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{S}_{CB} =& - iq_{0B} - \bar{\rho}L_B\Big[(1-\theta)(u(q^n_{1B}) - u(q_{0B})) + \theta(q^n_{1B} - q_{0B})\Big] \\
&+ \ell\Big[u(q_{0B}) - q_{0B} + \pi\alpha^n_{CB}S^n_{CB} + (1-\pi)\alpha^d_{CB}S^d_{CB}\Big], \\
S^n_{CB} =& \theta\Big(u(q^n_{1B}) - u(q_{0B}) - q^n_{1B} + q_{0B}\Big), \\
S^d_{CB} =& \theta\Big(u(q^d_{1B}) - u(q_{0B}) - q^d_{1B} + q_{0B}\Big).
\end{aligned}
$$

The optimal entry of N-type buyers is characterized by

$$
e^n_N = \begin{cases} 1, & \alpha^n_{NA}S_{NA} > \alpha^n_{NB}S^n_{NB} \\ 0, & \alpha^n_{NA}S_{NA} < \alpha^n_{NB}S^n_{NB} \\ \in [0,1], & \alpha^n_{NA}S_{NA} = \alpha^n_{NB}S^n_{NB} \end{cases}, \tag{C.8}
$$

and

$$
e^d_N = \begin{cases} 1, & \alpha^d_{NA}S_{NA} > \alpha^d_{NB}S^d_{NB} \\ 0, & \alpha^d_{NA}S_{NA} < \alpha^d_{NB}S^d_{NB} \\ \in [0,1], & \alpha^d_{NA}S_{NA} = \alpha^d_{NB}S^d_{NB} \end{cases}, \tag{C.9}
$$

where

$$
\begin{aligned}
S_{NA} &= (1-\theta)\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big), \\
S^n_{NB} &= (1-\theta)\Big(u(q^n_{1B}) - u(q_{0B}) - q^n_{1B} + q_{0B}\Big), \\
S^d_{NB} &= (1-\theta)\Big(u(q^d_{1B}) - u(q_{0B}) - q^d_{1B} + q_{0B}\Big).
\end{aligned}
$$

Definition 2. For given asset supplies $\{A, B\}$, the steady-state equilibrium for the core variables of the model consists of the equilibrium quantities and entry choices, $\{q_{0A}, q_{1A}, q_{0B}, q^n_{1B}, q^d_{1B}, e_C, e^n_N, e^d_N\}$, such that (C.2), (C.3), (C.4), (C.5), (C.6), (C.7), (C.8), and (C.9) hold. The remaining variables follow directly from the core variables as in the baseline model.

C.2 Consolidated Secondary Asset Market

C.2.1 Analysis of value functions and terms of trade

First, we analyze the value functions in the CM. The value function of an agent who enters the CM with m units of money and d_j units of asset j, $j = \{A, B\}$ in state $s = \{n, d\}$

is given by

$$W_s(m, d_A, d_B) = \max_{\substack{X,H,\\ \hat{m},\hat{d}_A,\hat{d}_B}} \left\{ X - H + \beta\,\mathbb{E}_{s,i}\Big[\Omega_i^s\Big(\hat{m}, \hat{d_A}, \hat{d_B}\Big)\Big] \right\},$$
$$\text{s.t. } X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A + d_B\cdot\mathbb{I}\{s = n\}),$$

where

$$\mathbb{E}_{s,i}\Big[\Omega_i^s\Big(\hat{m}, \hat{d_A}, \hat{d_B}\Big)\Big] = \pi\ell\,\Omega_C^n\Big(\hat{m}, \hat{d_A}, \hat{d_B}\Big) + \pi(1-\ell)\Omega_N^n\Big(\hat{m}, \hat{d_A}, \hat{d_B}\Big) + (1-\pi)\ell\,\Omega_C^d\Big(\hat{m}, \hat{d_A}, \hat{d_B}\Big) + (1-\pi)(1-\ell)\Omega_N^d\Big(\hat{m}, \hat{d_A}, \hat{d_B}\Big),$$

and Ω_i^s denotes the value function of an i-type agent, $i = \{C, N\}$, who enters the consolidated OTC market in state s.

Next, the value functions in the consolidated OTC market are given by

$$\begin{aligned}
\Omega_C^n(m, d_A, d_B) &= \alpha_C V_n(m + \xi^n, d_A - \chi_A^n, d_B - \chi_B^n) + (1-\alpha_C)V_n(m, d_A, d_B),\\
\Omega_C^d(m, d_A, d_B) &= \alpha_C V_d(m + \xi^d, d_A - \chi_A^d, d_B) + (1-\alpha_C)V_d(m, d_A, d_B),\\
\Omega_N^n(m, d_A, d_B) &= \alpha_N W_n(m - \xi^n, d_A + \chi_A^n, d_B + \chi_B^n) + (1-\alpha_N)W_n(m, d_A, d_B),\\
\Omega_N^d(m, d_A, d_B) &= \alpha_N W_d(m - \xi^d, d_A + \chi_A^d, d_B) + (1-\alpha_N)W_d(m, d_A, d_B),
\end{aligned}$$

where ξ^s is the amount of money that gets transferred to a C-type, and χ_j^s is the amount of asset j that gets transferred to an N-type in state s.

Finally, the value function in the DM is given by

$$V_s(m, d_A, d_B) = u(q) + W_s(m - \tau, d_A, d_B).$$

We now turn to the description of the terms of trade in the various markets, starting with the DM. Since this version of the model has no differences regarding the bargaining protocol in the DM, the bargaining solution is still given by

$$q(m) = \varphi m, \quad \tau(m) = m.$$

Next, we turn to the terms of trade in the OTC market. Consider a meeting between a C-type with portfolio (m, d_A, d_B) and an N-type with portfolio $(\tilde{m}, \tilde{d}_A, \tilde{d}_B)$. The bargaining surpluses of an i-type agent in state s, $\mathcal{S}_i^s$, are given by

$$\begin{aligned}
\mathcal{S}_C^n &= V_n(m + \xi^n, d_A - \chi_A^n, d_B - \chi_B^n) - V_n(m, d_A, d_B) = u(\varphi(m + \xi^n)) - u(\varphi m) - \varphi\chi_A^n - \varphi\chi_B^n,\\
\mathcal{S}_N^n &= W_n(\tilde{m} - \xi^n, \tilde{d}_A + \chi_A^n, \tilde{d}_B + \chi_B^n) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi^n + \varphi\chi_A^n + \varphi\chi_B^n,\\
\mathcal{S}_C^d &= V_d(m + \xi^d, d_A - \chi_A^d, d_B - \chi_B^d) - V_d(m, d_A, d_B) = u(\varphi(m + \xi^d)) - u(\varphi m) - \varphi\chi_A^d,
\end{aligned}$$

$$\mathcal{S}_N^d = W_d(\tilde{m} - \xi^d, \tilde{d}_A + \chi_A^d, \tilde{d}_B + \chi_B^d) - W_d(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi^d + \varphi\chi_A^d.$$

We now describe the bargaining solutions. For $(\xi^n, \chi_A^n, \chi_B^n)$, we have

$$\begin{aligned}
\varphi\chi_A^n + \varphi\chi_B^n &= (1-\theta)[u(\varphi(m+\xi^n)) - u(\varphi m)] + \theta\varphi\xi^n, \\
\xi^n &= \min\{m^* - m, \tilde{\xi}^n\}, \\
\tilde{\xi}^n &= \{\xi : \varphi d_A + \varphi d_B = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta\varphi\xi\},
\end{aligned}$$

For (ξ^d, χ_A^d), we have

$$\begin{aligned}
\varphi\chi_A^d &= (1-\theta)[u(\varphi(m+\xi^d)) - u(\varphi m)] + \theta\varphi\xi^d, \\
\xi^d &= \min\{m^* - m, \tilde{\xi}^d\}, \\
\tilde{\xi}^d &= \{\xi : \varphi d_A = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta\varphi\xi\}.
\end{aligned}$$

We can now derive the objective function of an agent in the CM, which is given by[29]

$$\begin{aligned}
J(\hat{m}, \hat{d}_A, \hat{d}_B) = &- \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) + \beta\hat{\varphi}[\hat{m} + \hat{d}_A + \pi\hat{d}_B] \\
&+ \beta\ell[u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi\alpha_C\mathcal{S}_C^n + (1-\pi)\alpha_C\mathcal{S}_C^d].
\end{aligned}$$

C.2.2 Equilibrium

We now describe the steady state equilibrium of the model with the consolidated OTC market. The core variables are $\{q_0, q_1^n, q_1^d\}$, and we will describe the derivation of equilibrium following the same methodology as in the baseline model. For this analysis recall that we have defined $\omega_\theta(q) \equiv \theta + (1-\theta)u'(q) \geq 1$.

First, the money demand equation is given by

$$\begin{aligned}
i = \ell&\left[1 - \pi\alpha_C\frac{\theta}{\omega_\theta(q_1^n)} - (1-\pi)\alpha_C\frac{\theta}{\omega_\theta(q_1^d)}\right][u'(q_0) - 1] \\
&+ \ell\pi\alpha_C\frac{\theta}{\omega_\theta(q_1^n)}[u'(q_1^n) - 1] + \ell(1-\pi)\alpha_C\frac{\theta}{\omega_\theta(q_1^d)}[u'(q_1^d) - 1].
\end{aligned} \tag{C.10}$$

The OTC trading protocols,

$$\begin{aligned}
q_1^n &= \min\{q^*, q_0 + \varphi\tilde{\xi}^n\}, \\
q_1^d &= \min\{q^*, q_0 + \varphi\tilde{\xi}^d\},
\end{aligned}$$

[29] Given that there is one consolidated OTC market, the matching probabilities are simply given by $\alpha_C = 1 - \ell$ and $\alpha_N = \ell$.

combined with the OTC bargaining solutions,

$$\begin{aligned}\varphi d_A + \varphi d_B &= (1-\theta)[u(q_1^n) - u(q_0)] + \theta\varphi\tilde{\xi}^n,\\ \varphi d_A &= (1-\theta)[u(q_1^d) - u(q_0)] + \theta\varphi\tilde{\xi}^d,\end{aligned}$$

and the market clearing conditions,

$$S_A = d_A, \quad S_B = d_B, \quad \varphi M = q_0,$$

yield

$$q_1^n = \min\left\{q^*, q_0 + \frac{\frac{q_0}{M}S_A + \frac{q_0}{M}S_B - (1-\theta)[u(q_1^n) - u(q_0)]}{\theta}\right\}, \tag{C.11}$$

$$q_1^d = \min\left\{q^*, q_0 + \frac{\frac{q_0}{M}S_A - (1-\theta)[u(q_1^d) - u(q_0)]}{\theta}\right\}. \tag{C.12}$$

Of course, with one consolidated market, we do not have to worry about entry decisions, which is why this version is so much simpler than the baseline model.

Definition 3. For given asset supplies $\{A, B\}$, the steady-state equilibrium for the core variables of the model consists of the equilibrium quantities, $\{q_0, q_1^n, q_1^d\}$, such that (C.10), (C.11), and (C.12) hold. The remaining variables follow directly from the core variables as in the baseline model.

C.3 Endogenous Asset Specialization

C.3.1 Analysis of value functions and terms of trade

As explained in the main text, we will use the term A-specialist (B-specialist) to describe the agent who visits OTC$_A$ (OTC$_B$) in the normal state. Recall that an A-specialist does not have an incentive to carry any asset B, but a B-specialist may very well choose to bring some asset A for precautionary motives, i.e., in order to liquidate some asset A if asset B defaults. Recall that in the baseline model this was not an option, since agents had to specialize *exclusively* in one asset.

First, we analyze the value functions in the CM. The value function of an agent who enters the CM is given by:

$$
\begin{aligned}
W_s(m, d_A, d_{BA}, d_B) = \max_{\substack{X,H,\hat{m},\\ \hat{d}_A,\hat{d}_{BA},\hat{d}_B}} \Bigg\{ & X - H \\
& + \beta\ell \max\Big\{ \pi\Omega^n_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B\Big) + (1-\pi)\Omega^d_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B\Big), \\
& \qquad \pi\Omega^n_{CB}\Big(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B\Big) + (1-\pi)\Omega^d_{CBA}\Big(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B\Big)\Big\} \\
& + \beta(1-\ell)\pi \max\Big\{ \Omega^n_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B\Big), \Omega^n_{NB}\Big(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B\Big)\Big\} \\
& + \beta(1-\ell)(1-\pi)\, \Omega^d_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B\Big) \Bigg\}
\end{aligned}
$$

$$
\text{s.t. } X + \varphi(\hat{m} + p_A\hat{d}_A + p_A\hat{d}_{BA} + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A + d_{BA} + d_B \cdot \mathbb{I}\{s = n\}).
$$

This value function is very similar to the one in the baseline model, but there is one important difference. We have now introduced the new key variable d_{BA} which stands for the amount of asset A carried by a B-specialist.

We now describe the value functions in the OTC markets. Like before, Ω^s_{ij} denotes the value function of an i-type agent, $i = \{C, N\}$, who enters the OTC market for asset j in state s. What is new is that now we have a new term, Ω^d_{CBA}, that stands for the value function of the B-specialist (who enters OTC$_A$) in the default state. We have:

$$
\begin{aligned}
\Omega^n_{CA}(m, d_A, d_{BA}, d_B) &= \alpha^n_{CA} V_n(m + \xi_A, d_A - \chi_A, d_{BA}, d_B) + (1 - \alpha^n_{CA}) V_n(m, d_A, d_{BA}, d_B), \\
\Omega^d_{CA}(m, d_A, d_{BA}, d_B) &= \alpha^d_{CA} V_d(m + \xi_A, d_A - \chi_A, d_{BA}, d_B) + (1 - \alpha^d_{CA}) V_d(m, d_A, d_{BA}, d_B), \\
\Omega^n_{CB}(m, d_A, d_{BA}, d_B) &= \alpha^n_{CB} V_n(m + \xi^n_B, d_A, d_{BA}, d_B - \chi^n_B) + (1 - \alpha^n_{CB}) V_n(m, d_A, d_{BA}, d_B), \\
\Omega^d_{CBA}(m, d_A, d_{BA}, d_B) &= \alpha^d_{CA} V_d(m + \xi^d_{BA}, d_A, d_{BA} - \chi^d_{BA}, d_B) + (1 - \alpha^d_{CA}) V_d(m, d_A, d_{BA}, d_B), \\
\Omega^n_{NA}(m, d_A, d_{BA}, d_B) &= \alpha^n_{NA} W_n(m - \xi_A, d_A + \chi_A, d_{BA}, d_B) + (1 - \alpha^n_{NA}) W_n(m, d_A, d_{BA}, d_B), \\
\Omega^n_{NB}(m, d_A, d_{BA}, d_B) &= \alpha^n_{NB} W_n(m - \xi^n_B, d_A, d_{BA}, d_B + \chi^n_B) + (1 - \alpha^n_{NB}) W_n(m, d_A, d_{BA}, d_B), \\
\Omega^d_{NA}(m, d_A, d_{BA}, d_B) &= \alpha^d_{NA} \begin{bmatrix} e_C W_d(m - \xi_A, d_A + \chi_A, d_{BA}, d_B) \\ + (1 - e_C) W_d(m - \xi^d_{BA}, d_A, d_{BA} + \chi^d_{BA}, d_B) \end{bmatrix} \\
&\quad + (1 - \alpha^d_{NA}) W_d(m, d_A, d_{BA}, d_B).
\end{aligned}
$$

Again, there is an important difference compared to the baseline model. In the default state, an N-type who visits OTC$_A$ realizes that she may match with an A-specialist (as was the case in the baseline model), but she may also match with a B-specialist who visited OTC$_A$ because asset B defaulted (assuming that this agent chose $d_{BA} > 0$). Thus, in this version of the model we need to define two sets of terms of trade in OTC$_A$: ξ_A and χ_A denote the amounts of money and asset A, receptively, traded between an N-type and a C-type A-specialist, and ξ^d_{BA} and χ^d_{BA} denote the amounts of money and asset A, respectively, traded between an N-type and a C-type B-specialist. Of course, this type of meeting can occur only in the default state. Also, ξ_B^n and χ_B^n are the amounts of money and asset B, receptively, traded between an N-type and a C-type B-specialist in OTC$_B$ in the normal state.

Finally, the value function in the DM is given by

$$V_s(m, d_A, d_{BA}, d_B) = u(q) + W_s(m - \tau, d_A, d_{BA}, d_B).$$

We now turn to the description of the terms of trade in the various markets, starting with the DM. Since this version of the model has no differences regarding the bargaining protocol in the DM, the bargaining solution is still given by

$$q(m) = \varphi m, \quad \tau(m) = m.$$

Next, we turn to the terms of trade in the OTC markets. Consider a meeting in OTC$_j$ between a C-type asset seller with portfolio (m, d_A, d_{BA}, d_B) and an N-type asset buyer with portfolio $(\tilde{m}, \tilde{d}_A, \tilde{d}_{BA}, \tilde{d}_B)$. As already explained, for a C-type A-specialist $d_{BA} = d_B = 0$, and for a C-type B-specialist $d_A = 0$ and d_{BA} may or may not be zero. The bargaining surpluses in the various types of meetings are given by

$$\begin{aligned}
\mathcal{S}_{CA} &= V_n(m + \xi_A, d_A - \chi_A, d_{BA}, d_B) - V_n(m, d_A, d_{BA}, d_B) = u(\varphi(m + \xi_A)) - u(\varphi m) - \varphi \chi_A, \\
\mathcal{S}_{CB}^n &= V_n(m + \xi_B^n, d_A, d_{BA}, d_B - \chi_B^n) - V_n(m, d_A, d_{BA}, d_B) = u(\varphi(m + \xi_B^n)) - u(\varphi m) - \varphi \chi_B^n, \\
\mathcal{S}_{CBA}^d &= V_d(m + \xi_{BA}^d, d_A, d_{BA} - \chi_{BA}^d, d_B) - V_d(m, d_A, d_{BA}, d_B) = u(\varphi(m + \xi_{BA}^d)) - u(\varphi m) - \varphi \chi_{BA}^d, \\
\mathcal{S}_{NA} &= W_n(\tilde{m} - \xi_A, \tilde{d}_A + \chi_A, d_{BA}, \tilde{d}_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_{BA}, \tilde{d}_B) = -\varphi \xi_A + \varphi \chi_A, \\
\mathcal{S}_{NB}^n &= W_n(\tilde{m} - \xi_B^n, \tilde{d}_A, \tilde{d}_{BA}, \tilde{d}_B + \chi_B^n) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_{BA}, \tilde{d}_B) = -\varphi \xi_B^n + \varphi \chi_B^n, \\
\mathcal{S}_{NBA}^d &= W_d(\tilde{m} - \xi_{BA}^d, \tilde{d}_A, \tilde{d}_{BA} + \chi_{BA}^d, \tilde{d}_B) - W_d(\tilde{m}, \tilde{d}_A, \tilde{d}_{BA}, \tilde{d}_B) = -\varphi \xi_{BA}^d + \varphi \chi_{BA}^d.
\end{aligned}$$

The notation used in the expressions above is standard, with the exception of the new terms, $\mathcal{S}_{CBA}^d$ and $\mathcal{S}_{NBA}^d$, which are now introduced because we have a new type of meeting in OTC$_A$ in the default state. The former is the surplus of a C-type B-specialist selling asset A in OTC$_A$ in the default state, and the latter is the surplus of a N-type who matched with a B-specialist in OTC$_A$ in the default state.

We now describe the bargaining solutions in the OTC markets. For (ξ_A, χ_A), we have

$$\begin{aligned}
\varphi \chi_A &= (1 - \theta)[u(\varphi(m + \xi_A)) - u(\varphi m)] + \theta \varphi \xi_A, \\
\xi_A &= \min\{m^* - m, \tilde{\xi}_A\}, \\
\tilde{\xi}_A &= \{\xi : \varphi d_A = (1 - \theta)[u(\varphi(m + \xi)) - u(\varphi m)] + \theta \varphi \xi\}.
\end{aligned}$$

For (ξ_B^n, χ_B^n), we have

$$\varphi \chi_B^n = (1 - \theta)[u(\varphi(m + \xi_B^n)) - u(\varphi m)] + \theta \varphi \xi_B^n,$$

$$
\begin{aligned}
\xi_B^n &= \min\{m^* - m, \tilde{\xi}_B^n\}, \\
\tilde{\xi}_B^n &= \{\xi : \varphi d_B = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta \varphi \xi\}.
\end{aligned}
$$

Lastly, for $(\xi_{BA}^d, \chi_{BA}^d)$, we have

$$
\begin{aligned}
\varphi \chi_{BA}^d &= (1-\theta)[u(\varphi(m+\xi_{BA}^d)) - u(\varphi m)] + \theta \varphi \xi_{BA}^d, \\
\xi_{BA}^d &= \min\{m^* - m, \tilde{\xi}_{BA}^d\}, \\
\tilde{\xi}_{BA}^d &= \{\xi : \varphi d_{BA} = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta \varphi \xi\}.
\end{aligned}
$$

We can now derive the objective function of an agent in the CM, which is given by

$$
\begin{aligned}
&J(\hat{m}, \hat{d}_A, \hat{d}_{BA}, \hat{d}_B) = -\varphi(\hat{m} + p_A \hat{d}_A + p_A \hat{d}_{BA} + p_B \hat{d}_B) + \beta \hat{\varphi}[\hat{m} + \hat{d}_A + \hat{d}_{BA} + \pi \hat{d}_B] \\
&+ \beta \ell \Big[u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \max\Big\{ \pi \alpha_{CA}^n \mathcal{S}_{CA} + (1-\pi)\alpha_{CA}^d \mathcal{S}_{CA}, \pi \alpha_{CB}^n \mathcal{S}_{CB}^n + (1-\pi)\alpha_{CA}^d \mathcal{S}_{CBA}^d \} \Big\} \Big].
\end{aligned}
$$

Notice that the maximum operator in the second line represents the choice of the agent to become an A- or a B-specialist. Unlike in the baseline model, where B-specialists do not trade in the OTC in the case of default, here these agents can visit OTC_A and sell asset A, assuming they chose to bring $d_{BA} > 0$.[30]

C.3.2 Equilibrium

We now describe the steady state equilibrium of the model. The core variables are $\{q_{0A}$, q_{1A}, q_{0B}, q_{1B}, q_{BA}, d_{BA}, e_C, $e_N^n\}$, and we will describe the derivation of equilibrium following the same methodology as in the baseline model. For this analysis recall that we have defined $\omega_\theta(q) \equiv \theta + (1-\theta)u'(q) \geq 1$.

First, the money demand equations are given by

$$
\begin{aligned}
i &= \ell \left[1 - (\pi \alpha_{CA}^n + (1-\pi)\alpha_{CA}^d) \frac{\theta}{\omega_\theta(q_{1A})} \right] [u'(q_{0A}) - 1] \\
&\quad + \ell (\pi \alpha_{CA}^n + (1-\pi)\alpha_{CA}^d) \frac{\theta}{\omega_\theta(q_{1A})} [u'(q_{1A}) - 1],
\end{aligned} \tag{C.13}
$$

$$
\begin{aligned}
i &= \ell \left[1 - \pi \alpha_{CB}^n \frac{\theta}{\omega_\theta(q_{1B})} - (1-\pi)\alpha_{CA}^d \frac{\theta}{\omega_\theta(q_{BA})} \right] [u'(q_{0B}) - 1] \\
&\quad + \ell \pi \alpha_{CB}^n \frac{\theta}{\omega_\theta(q_{1B})} [u'(q_{1B}) - 1] + \ell (1-\pi)\alpha_{CA}^d \frac{\theta}{\omega_\theta(q_{BA})} [u'(q_{BA}) - 1].
\end{aligned} \tag{C.14}
$$

[30] Notice that the term α_{CA}^d appears in both arguments of the maximum operator. Why is the matching probability of a C-type in the default state the same regardless of her specialization choice? The reason is that here in the case of default *all* agents visit OTC_A. Therefore, the matching probability of a C-type does not depend on which asset she chose to specialize in, and it is given by $\alpha_{CA}^d = f(\ell, 1-\ell)/\ell = 1-\ell$.

The OTC trading protocols,

$$
\begin{aligned}
q_{1A} &= \min\{q^*, q_{0A} + \varphi\tilde{\xi}_A\}, \\
q_{1B} &= \min\{q^*, q_{0B} + \varphi\tilde{\xi}_B^n\}, \\
q_{BA} &= \min\{q^*, q_{0B} + \varphi\tilde{\xi}_{BA}^d\},
\end{aligned}
$$

combined with the OTC bargaining solutions,

$$
\begin{aligned}
\varphi d_A &= (1-\theta)[u(q_{1A}) - u(q_{0A})] + \theta\varphi\tilde{\xi}_A, \\
\varphi d_B &= (1-\theta)[u(q_{1B}) - u(q_{0B})] + \theta\varphi\tilde{\xi}_B^n, \\
\varphi d_{BA} &= (1-\theta)[u(q_{BA}) - u(q_{0B})] + \theta\varphi\tilde{\xi}_{BA}^d,
\end{aligned}
$$

and the market clearing conditions,

$$
S_A = e_C d_A + (1-e_C)d_{BA}, \quad S_B = (1-e_C)d_B, \quad \varphi M = e_C q_{0A} + (1-e_C)q_{0B},
$$

yield

$$
q_{1A} = \min\left\{ q^*, q_{0A} + \frac{\frac{e_C q_{0A} + (1-e_C)q_{0B}}{M}\frac{S_A - (1-e_C)d_{BA}}{e_C} - (1-\theta)[u(q_{1A}) - u(q_{0A})]}{\theta} \right\}, \tag{C.15}
$$

$$
q_{1B} = \min\left\{ q^*, q_{0B} + \frac{\frac{e_C q_{0A} + (1-e_C)q_{0B}}{M}\frac{S_B}{1-e_C} - (1-\theta)[u(q_{1B}) - u(q_{0B})]}{\theta} \right\}, \tag{C.16}
$$

$$
q_{BA} = \min\left\{ q^*, q_{0B} + \frac{\frac{e_C q_{0A} + (1-e_C)q_{0B}}{M} d_{BA} - (1-\theta)[u(q_{BA}) - u(q_{0B})]}{\theta} \right\}. \tag{C.17}
$$

The next task is to describe the agents' entry decisions. To that end, it is useful to start by describing the liquidity premia of the various assets. The liquidity premium of asset j, denoted by L_j, is given by the percentage difference between an asset's price and its fundamental value, and it solves:

$$
p_A = \frac{1}{1+i}(1+L_A), \qquad p_B = \frac{\pi}{1+i}(1+L_B),
$$

where:

$$
L_A = \ell(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\frac{\theta}{\omega_\theta(q_{1A})}[u'(q_{1A}) - 1], \quad \text{(implied by FOC of } A\text{-specialist)}
$$

$$L_A = \ell(1-\pi)\alpha^d_{CA}\frac{\theta}{\omega_\theta(q_{BA})}[u'(q_{BA})-1], \qquad \text{(implied by FOC of } B\text{-specialist)}$$
$$L_B = \ell\alpha^n_{CB}\frac{\theta}{\omega_\theta(q_{1B})}[u'(q_{1B})-1].$$

The first equation for L_A comes from the first-order condition of A-specialists, and the second comes from the first-order condition of B-specialists. Of course, in equilibrium, the right-hand sides of these two expressions must be equal to each other, and this is the additional equilibrium condition that we need in order to solve for the additional equilibrium variable, namely, d_{BA}. Hence, we have:

$$\ell(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA})\frac{\theta}{\omega_\theta(q_{1A})}[u'(q_{1A})-1] = \ell(1-\pi)\alpha^d_{CA}\frac{\theta}{\omega_\theta(q_{BA})}[u'(q_{BA})-1]. \tag{C.18}$$

Moving on to the entry decisions, the optimal entry of C-types is characterized by:

$$e_C = \begin{cases} 1, & \tilde{\mathcal{S}}_{CA} > \tilde{\mathcal{S}}_{CB} \\ 0, & \tilde{\mathcal{S}}_{CA} < \tilde{\mathcal{S}}_{CB} \\ \in [0,1], & \tilde{\mathcal{S}}_{CA} = \tilde{\mathcal{S}}_{CB}\ , \end{cases} \tag{C.19}$$

where:

$$\begin{aligned} \tilde{\mathcal{S}}_{CA} =& -iq_{0A} - L_A\Big[(1-\theta)(u(q_{1A})-u(q_{0A}))+\theta(q_{1A}-q_{0A})\Big] \\ &+\ell\Big[u(q_{0A})-q_{0A}+(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA})\mathcal{S}_{CA}\Big], \\ \mathcal{S}_{CA} =& \,\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big), \end{aligned}$$

and

$$\begin{aligned} \tilde{\mathcal{S}}_{CB} =& -iq_{0B} - L_A\Big[(1-\theta)(u(q_{BA})-u(q_{0B}))+\theta(q_{BA}-q_{0B})\Big] \\ &-\pi L_B\Big[(1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big] \\ &+\ell\Big[u(q_{0B})-q_{0B}+\pi\alpha^n_{CB}\mathcal{S}^n_{CB}+(1-\pi)\alpha^d_{CA}\mathcal{S}^d_{CBA}\Big], \\ \mathcal{S}^n_{CB} =& \,\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big), \\ \mathcal{S}^d_{CBA} =& \,\theta\Big(u(q_{BA})-u(q_{0B})-q_{BA}+q_{0B}\Big). \end{aligned}$$

The optimal entry of N-types is always simpler and characterized by:

$$e^n_N = \begin{cases} 1, & \alpha^n_{NA}\mathcal{S}_{NA} > \alpha^n_{NB}\mathcal{S}^n_{NB} \\ 0, & \alpha^n_{NA}\mathcal{S}_{NA} < \alpha^n_{NB}\mathcal{S}^n_{NB} \\ \in [0,1], & \alpha^n_{NA}\mathcal{S}_{NA} = \alpha^n_{NB}\mathcal{S}^n_{NB}\ , \end{cases} \tag{C.20}$$

where

$$\mathcal{S}_{NA} = (1-\theta)\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big),$$
$$\mathcal{S}^n_{NB} = (1-\theta)\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big).$$

Definition 4. For given asset supplies $\{A, B\}$, the steady-state equilibrium for the core variables of the model consists of the equilibrium quantities and entry choices, $\{q_{0A}$, q_{1A}, q_{0B}, q_{1B}, q_{BA}, d_{BA}, e_C, $e^n_N\}$, such that (C.13), (C.14), (C.15), (C.16), (C.17), (C.18), (C.19), and (C.20) hold. The remaining variables follow directly from the core variables as in the baseline model.

C.4 Model with Two Marketplaces

C.4.1 Analysis of value functions and terms of trade

First, we analyze the value functions in the CM. The representative agent has five state variables. As always m represents money holdings, and d_A, d_B denote the amounts of asset A and B, respectively, carried by the agent for trade in the segmented marketplace. The new states d_{A2}, d_{B2} represent the amounts of asset A and B, receptively, carried by the agent for trade in the consolidated marketplace.[31] The value function of an agent who enters the CM is given by:

$$\begin{aligned} W_s(m, d_A, d_B, d_{A2}, d_{B2}) = \max_{\substack{X,H,\hat{m},\\ \hat{d}_A,\hat{d}_B,\hat{d}_{A2},\hat{d}_{B2}}} &\Big\{X - H + \beta \max\Big\{\mathcal{M}_0(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}), \\ &\mathcal{M}_1(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}) - \kappa_1,\ \mathcal{M}_2(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}) - \kappa_2\Big\}\Big\} \end{aligned}$$

$$\begin{aligned} \text{s.t. } & X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B + p_A\hat{d}_{A2} + p_B\hat{d}_{B2}) \\ &= H + \varphi(m + \mu M + d_A + d_B\cdot\mathbb{I}\{s=n\} + d_{A2} + d_{B2}\cdot\mathbb{I}\{s=n\}, \end{aligned}$$

where:

$$\begin{aligned} \mathcal{M}_0(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}) &= V_s(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}), \\ \mathcal{M}_1(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}) &= \mathbb{E}_{s,i}\Big[\max\Big\{\Omega^s_{iA}(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}), \Omega^s_{iB}(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2})\Big\}\Big], \\ \mathcal{M}_2(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}) &= \mathbb{E}_{s,i}\Big[\Omega^s_{i2}(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2})\Big]. \end{aligned}$$

[31] As a mnemonic rule, the "2" is meant to remind the reader that these variables pertain to the market where *two* assets can be traded, namely, the consolidated market.

Notice that $\mathcal{M}_0$ is the value function of not participating in any marketplace. (Since entering either of the two marketplaces entails a cost, the agent should always the option to not participate.) $\mathcal{M}_1$ is the value function of participating in the segmented marketplace, and κ_1 is the associated entry cost. $\mathcal{M}_2$ is the value function of participating in the consolidated marketplace, and κ_2 is the associated entry cost. (A mnemonic rule similar to the one described in footnote 31 also applies here.)

Next, we move to the value functions in the OTC markets. We start with the segmented marketplace. Let Ω^s_{ij} denote the value function of an i-type agent, $i = \{C, N\}$, who enters OTC$_j$, $j = \{A, B\}$, in state s. These value functions are given by:

$$
\begin{aligned}
\Omega^n_{CA}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha^n_{CA} V_n(m + \xi_A, d_A - \chi_A, d_B, d_{A2}, d_{B2}) \\
&\quad + (1 - \alpha^n_{CA}) V_n(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^d_{CA}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha^d_{CA} V_d(m + \xi_A, d_A - \chi_A, d_B, d_{A2}, d_{B2}) \\
&\quad + (1 - \alpha^d_{CA}) V_d(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^n_{CB}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha^n_{CB} V_n(m + \xi^n_B, d_A, d_B - \chi^n_B, d_{A2}, d_{B2}) \\
&\quad + (1 - \alpha^n_{CB}) V_n(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^d_{CB}(m, d_A, d_B, d_{A2}, d_{B2}) &= V_d(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^n_{NA}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha^n_{NA} W_n(m - \xi_A, d_A + \chi_A, d_B, d_{A2}, d_{B2}) \\
&\quad + (1 - \alpha^n_{NA}) W_n(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^d_{NA}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha^d_{NA} W_d(m - \xi_A, d_A + \chi_A, d_B, d_{A2}, d_{B2}) \\
&\quad + (1 - \alpha^d_{NA}) W_d(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^n_{NB}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha^n_{NB} W_n(m - \xi^n_B, d_A, d_B + \chi^n_B, d_{A2}, d_{B2}) \\
&\quad + (1 - \alpha^n_{NB}) W_n(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^d_{NB}(m, d_A, d_B, d_{A2}, d_{B2}) &= W_n(m, d_A, d_B, d_{A2}, d_{B2}),
\end{aligned}
$$

where ξ_j is the amount of money transferred to a C-type, and χ_j is the amount of asset j transferred to an N-type in a typical match in OTC$_j$. (As usual, for OTC$_B$ the superscript n emphasizes that trade in that market only takes place in the normal state.)

Now we turn to the value functions in the consolidated marketplace. Let Ω^s_{i2} denote the value function of an i-type agent who enters this marketplace in state s. These value functions are given by:

$$
\begin{aligned}
\Omega^n_{C2}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha_{C2} V_n(m + \xi^n_2, d_A, d_B, d_{A2} - \chi^n_{A2}, d_{B2} - \chi^n_{B2}) \\
&\quad + (1 - \alpha_{C2}) V_n(m, d_A, d_B, d_{A2}, d_{B2}), \\
\Omega^d_{C2}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha_{C2} V_d(m + \xi^d_2, d_A, d_B, d_{A2} - \chi^d_{A2}, d_{B2})
\end{aligned}
$$

$$
\begin{aligned}
&+ (1-\alpha_{C2})V_d(m, d_A, d_B, d_{A2}, d_{B2}),\\
\Omega^n_{N2}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha_{N2}W_n(m-\xi^n_2, d_A, d_B, d_{A2}+\chi^n_{A2}, d_{B2}+\chi^n_{B2})\\
&+ (1-\alpha_{N2})W_n(m, d_A, d_B, d_{A2}, d_{B2}),\\
\Omega^d_{N2}(m, d_A, d_B, d_{A2}, d_{B2}) &= \alpha_{N2}W_d(m-\xi^d_2, d_A, d_B, d_{A2}+\chi^d_{A2}, d_{B2})\\
&+ (1-\alpha_{N2})W_d(m, d_A, d_B, d_{A2}, d_{B2}),
\end{aligned}
$$

where ξ^s_2 is the amount of money transferred to a C-type, and χ^s_{j2} is the amount of asset j transferred to an N-type in a typical match in state s. (Notice that the term χ^d_{B2} is not defined since asset B fully defaults in the d-state.)

Finally, the value function in the DM is given by:

$$
V_s(m, d_A, d_B, d_{A2}, d_{B2}) = u(q) + W_s(m-\tau, d_A, d_B, d_{A2}, d_{B2}).
$$

We now turn to the description of the terms of trade in the various markets, starting with the DM. Since this version of the model has no differences regarding the bargaining protocol in the DM, the bargaining solution is still given by:

$$
q(m) = \varphi m, \quad \tau(m) = m.
$$

Next, we turn to the terms of trade in the OTC markets. First consider a meeting in OTC$_j$ in the segmented the marketplace between a C-type with portfolio $(m, d_A, d_B, d_{A2}, d_{B2})$ and an N-type with portfolio $(\tilde{m}, \tilde{d}_A, \tilde{d}_B, \tilde{d}_{A2}, \tilde{d}_{B2})$. By definition, a C-type who enters this marketplace has $d_{A2} = d_{B2} = 0$, and either $d_A = 0$ or $d_B = 0$ (because an agent who goes to the segmented marketplace specializes only in one asset). Let $\mathcal{S}_{ij}$ denote the surplus of an i-type in OTC$_j$. (As usual, for OTC$_B$ the superscript n emphasizes that trade in that market only takes place in the normal state.) These surpluses are given by:

$$
\begin{aligned}
\mathcal{S}_{CA} &= V_n(m+\xi_A, d_A-\chi_A, d_B, d_{A2}, d_{B2}) - V_n(m, d_A, d_B, d_{A2}, d_{B2})\\
&= u(\varphi(m+\xi_A)) - u(\varphi m) - \varphi\chi_A,\\
\mathcal{S}_{NA} &= W_n(\tilde{m}-\xi_A, \tilde{d}_A+\chi_A, \tilde{d}_B, \tilde{d}_{A2}, \tilde{d}_{B2}) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B, \tilde{d}_{A2}, \tilde{d}_{B2})\\
&= -\varphi\xi_A + \varphi\chi_A,\\
\mathcal{S}^n_{CB} &= V_n(m+\xi^n_B, d_A, d_B-\chi^n_B, d_{A2}, d_{B2}) - V_n(m, d_A, d_B, d_{A2}, d_{B2})\\
&= u(\varphi(m+\xi^n_B)) - u(\varphi m) - \varphi\chi^n_B,\\
\mathcal{S}^n_{NB} &= W_n(\tilde{m}-\xi^n_B, \tilde{d}_A, \tilde{d}_B+\chi^n_B, \tilde{d}_{A2}, \tilde{d}_{B2}) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B, \tilde{d}_{A2}, \tilde{d}_{B2})\\
&= -\varphi\xi^n_B + \varphi\chi^n_B.
\end{aligned}
$$

We now describe the bargaining solutions. For (ξ_A, χ_A), we have:

$$
\varphi\chi_A = (1-\theta)[u(\varphi(m+\xi_A)) - u(\varphi m)] + \theta\varphi\xi_A,
$$

$$
\begin{aligned}
\xi_A &= \min\{m^* - m, \tilde{\xi}_A\}, \\
\tilde{\xi}_A &= \{\xi : \varphi d_A = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta\varphi\xi\},
\end{aligned}
$$

For (ξ_B^n, χ_B^n), we have:

$$
\begin{aligned}
\varphi\chi_B^n &= (1-\theta)[u(\varphi(m+\xi_B^n)) - u(\varphi m)] + \theta\varphi\xi_B^n, \\
\xi_B^n &= \min\{m^* - m, \tilde{\xi}_B^n\}, \\
\tilde{\xi}_B^n &= \{\xi : \varphi d_B = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta\varphi\xi\}.
\end{aligned}
$$

Next consider a meeting in the consolidated marketplace between a C-type with portfolio $(m, d_A, d_B, d_{A2}, d_{B2})$ and an N-type with portfolio $(\tilde{m}, \tilde{d}_A, \tilde{d}_B, \tilde{d}_{A2}, \tilde{d}_{B2})$. By definition, a C-type agent who enters this marketplace has $d_A = d_B = 0$. Let $\mathcal{S}_{i2}^s$ denote the surplus of an i-type in this marketplace in state s. These surpluses are given by:

$$
\begin{aligned}
\mathcal{S}_{C2}^n &= V_n(m+\xi_2^n, d_A, d_B, d_A - \chi_{A2}^n, d_B - \chi_{B2}^n) - V_n(m, d_A, d_B, d_{A2}, d_{B2}) \\
&= u(\varphi(m+\xi_2^n)) - u(\varphi m) - \varphi\chi_{A2}^n - \varphi\chi_{B2}^n, \\
\mathcal{S}_{N2}^n &= W_n(\tilde{m} - \xi_2^n, \tilde{d}_A, \tilde{d}_B, \tilde{d}_A + \chi_{A2}^n, \tilde{d}_B + \chi_{B2}^n) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B, \tilde{d}_{A2}, \tilde{d}_{B2}) \\
&= -\varphi\xi_2^n + \varphi\chi_{A2}^n + \varphi\chi_{B2}^n, \\
\mathcal{S}_{C2}^d &= V_d(m+\xi_2^d, d_A, d_B, d_A - \chi_{A2}^d, d_B) - V_d(m, d_A, d_B, d_{A2}, d_{B2}) \\
&= u(\varphi(m+\xi_2^d)) - u(\varphi m) - \varphi\chi_{A2}^d, \\
\mathcal{S}_{N2}^d &= W_d(\tilde{m} - \xi_2^d, \tilde{d}_A, \tilde{d}_B, \tilde{d}_A + \chi_{A2}^d, \tilde{d}_B) - W_d(\tilde{m}, \tilde{d}_A, \tilde{d}_B, \tilde{d}_{A2}, \tilde{d}_{B2}) \\
&= -\varphi\xi_2^d + \varphi\chi_{A2}^d.
\end{aligned}
$$

We now describe the bargaining solutions. For $(\xi_2^n, \chi_{A2}^n, \chi_{B2}^n)$,

$$
\begin{aligned}
\varphi\chi_{A2}^n + \varphi\chi_{B2}^n &= (1-\theta)[u(\varphi(m+\xi_2^n)) - u(\varphi m)] + \theta\varphi\xi_2^n, \\
\xi_2^n &= \min\{m^* - m, \tilde{\xi}_2^n\}, \\
\tilde{\xi}_2^n &= \{\xi : \varphi d_A + \varphi d_B = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta\varphi\xi\}.
\end{aligned}
$$

For (ξ_2^d, χ_{A2}^d), we have:

$$
\begin{aligned}
\varphi\chi_{A2}^d &= (1-\theta)[u(\varphi(m+\xi_2^d)) - u(\varphi m)] + \theta\varphi\xi_2^d, \\
\xi_2^d &= \min\{m^* - m, \tilde{\xi}_2^d\}, \\
\tilde{\xi}_2^d &= \{\xi : \varphi d_A = (1-\theta)[u(\varphi(m+\xi)) - u(\varphi m)] + \theta\varphi\xi\}.
\end{aligned}
$$

We can now derive the objective function of an agent in the CM, which is given by:

$$
J(\hat{m}, \hat{d}_A, \hat{d}_B, \hat{d}_{A2}, \hat{d}_{B2}) = -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B + p_A\hat{d}_{A2} + p_B\hat{d}_{B2})
$$

$$
\begin{aligned}
&+ \beta\hat{\varphi}[\hat{m} + \hat{d}_A + \pi\hat{d}_B + \hat{d}_{A2} + \pi\hat{d}_{B2}] + \beta\ell[u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m}] \\
&+ \beta\max\Big\{0, \max\Big\{\max\big\{\ell(\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA})\mathcal{S}_{CA}, \ell\pi\alpha^n_{CB}\mathcal{S}^n_{CB}\big\} \\
&\qquad\qquad + (1-\ell)\pi\max\big\{\alpha^n_{NA}\mathcal{S}_{NA}, \alpha^n_{NB}\mathcal{S}^n_{NB}\big\} + (1-\ell)(1-\pi)\alpha^d_{NA}\mathcal{S}_{NA} - \kappa_1, \\
&\qquad \ell\big[\pi\alpha_{C2}\mathcal{S}^n_{C2} + (1-\pi)\alpha_{C2}\mathcal{S}^d_{C2}\big] + (1-\ell)\big[\pi\alpha_{N2}\mathcal{S}^n_{N2} + (1-\pi)\alpha_{N2}\mathcal{S}^d_{N2}\big] - \kappa_2\Big\}\Big\}.
\end{aligned}
$$

This objective function is quite different and more complicated compared to the baseline model, therefore a couple of comments are in order. Notice that the agent's objective has three layers of maximum operators. First, the agent chooses whether to participate in any marketplace at all, or walk away and obtain a zero surplus from OTC trade. Second, conditional on participating, then the agent chooses whether she will go to the segmented or the consolidated marketplace. Third, if the agent chose to visit the segmented marketplace, she must then decide whether to visit OTC$_A$ or OTC$_B$. Another, more subtle difference is that, unlike the baseline model, here the surplus terms of an N-type appear in the objective function. This is because agents choose their marketplace before the idiosyncratic liquidity shock has been revealed. As a result, the agent's optimal decision must take into account the surplus she will make as an N-type.[32]

C.4.2 Matching probabilities

With agents' choosing between two marketplaces, matching probabilities change drastically compared to the baseline model. Denote the *measure* of agents who do not participate in any marketplace by e_0. Also, let e_2 denote the *fraction* of marketplace participants who choose to go to the consolidated marketplace.[33] Let e_C denote the fraction of C-types who go to OTC$_A$ among the segmented marketplace participants. Similarly, let e^n_N denote

[32] Throughout the paper we have assumed that the aggregate and idiosyncratic shocks are revealed concurrently. We maintain this assumption here, and this implies that when agents make their marketplace entry decision, they do not yet know whether they are a C- or an N-type. However, all the results of this section would go through if agents made their entry choice after the idiosyncratic shock has been revealed. The crucial assumption is that the *aggregate* shock has not been revealed when the entry choice is made. More precisely, in the main text we explain that agents are attracted to the segmented marketplace because that marketplace gives them the *ex-post* flexibility to avoid OTC$_B$ in the default state, if they turn out to be an N-type. If agents chose their marketplace knowing their type, it would still be true that N-types are attracted to the segmented marketplace for the very same reason, i.e., the ex-post flexibility to avoid OTC$_B$ in the default state. (The key here is that the term "ex-post" refers to the aggregate shock, not the idiosyncratic.) In this alternative specification, agents who already know they are N-types would be drawn to the segmented marketplace, and, for the reasons analyzed in Section 3.4, C-types would follow.

[33] Notice that while the term e_0 is a measure, the term e_2 is a fraction. We do not explicitly define the measure of the segmented marketplace participants, since we can always write it as $(1 - e_0)(1 - e_2)$.

the fraction of N-types who visit OTC$_A$, conditional on having chosen the segmented marketplace, in the normal state. In the default state, all N-types go to OTC$_A$.

For the segmented marketplace, α_{ij}^s is the matching probability of an i-type who enters OTC$_j$ in state s. These matching probabilities are given by:

$$
\begin{aligned}
\alpha_{CA}^n &= \frac{f[(1-e_0)(1-e_2)e_C\ell,(1-e_0)(1-e_2)e_N^n(1-\ell)]}{(1-e_0)(1-e_2)e_C\ell}, \\
\alpha_{CB}^n &= \frac{f[(1-e_0)(1-e_2)(1-e_C)\ell,(1-e_0)(1-e_2)(1-e_N^n)(1-\ell)]}{(1-e_0)(1-e_2)(1-e_C)\ell}, \\
\alpha_{NA}^n &= \frac{f[(1-e_0)(1-e_2)e_C\ell,(1-e_0)(1-e_2)e_N^n(1-\ell)]}{(1-e_0)(1-e_2)e_N^n(1-\ell)}, \\
\alpha_{NB}^n &= \frac{f[(1-e_0)(1-e_2)(1-e_C)\ell,(1-e_0)(1-e_2)(1-e_N^n)(1-\ell)]}{(1-e_0)(1-e_2)(1-e_N^n)(1-\ell)}, \\
\alpha_{CA}^d &= \frac{f[(1-e_0)(1-e_2)e_C\ell,(1-e_0)(1-e_2)(1-\ell)]}{(1-e_0)(1-e_2)e_C\ell}, \\
\alpha_{NA}^d &= \frac{f[(1-e_0)(1-e_2)e_C\ell,(1-e_0)(1-e_2)(1-\ell)]}{(1-e_0)(1-e_2)(1-\ell)},
\end{aligned}
$$

which are equal to:

$$
\begin{aligned}
\alpha_{CA}^n &= [(1-e_0)(1-e_2)]^\eta \, \frac{f[e_C\ell,e_N^n(1-\ell)]}{e_C\ell}, \\
\alpha_{CB}^n &= [(1-e_0)(1-e_2)]^\eta \, \frac{f[(1-e_C)\ell,(1-e_N^n)(1-\ell)]}{(1-e_C)\ell}, \\
\alpha_{NA}^n &= [(1-e_0)(1-e_2)]^\eta \, \frac{f[e_C\ell,e_N^n(1-\ell)]}{e_N^n(1-\ell)}, \\
\alpha_{NB}^n &= [(1-e_0)(1-e_2)]^\eta \, \frac{f[(1-e_C)\ell,(1-e_N^n)(1-\ell)]}{(1-e_N^n)(1-\ell)}, \\
\alpha_{CA}^d &= [(1-e_0)(1-e_2)]^\eta \, \frac{f[e_C\ell,(1-\ell)]}{e_C\ell}, \\
\alpha_{NA}^d &= [(1-e_0)(1-e_2)]^\eta \, \frac{f[e_C\ell,(1-\ell)]}{(1-\ell)}.
\end{aligned}
$$

For the consolidated marketplace, α_{i2} is the matching probability of an i-type. These matching probabilities are given by:

$$
\alpha_{C2} = \frac{f[(1-e_0)e_2\ell,(1-e_0)e_2(1-\ell)]}{(1-e_0)e_2\ell}, \qquad \alpha_{N2} = \frac{f[(1-e_0)e_2\ell,(1-e_0)e_2(1-\ell)]}{(1-e_0)e_2(1-\ell)},
$$

which are equal to:

$$
\alpha_{C2} = [(1-e_0)e_2]^\eta \,(1-\ell), \qquad \alpha_{N2} = [(1-e_0)e_2]^\eta \,\ell.
$$

C.4.3 Equilibrium

We now describe the steady state equilibrium of the model with two marketplaces. We proceed as follows. First, we describe the equilibrium conditions (importantly, the demand for the various assets) implied by agents who (i) do not participate in any marketplaces; (ii) participate in the segmented marketplace; and (iii) participate in the consolidated marketplace. We do so taking as given the measures of agents in the various marketplaces (including the non-participants). Second, we endogenize these measures by studying agents' optimal entry decisions in the various marketplaces.

Equilibrium conditions implied by non-participants Here there is only one core variable, q. Non-participants rely only on their own money holdings, and their money demand equation is given by:

$$i = \ell(u'(q) - 1). \tag{C.21}$$

Since in the last step of equilibrium characterization we will describe the agents' entry decisions, it is useful to define the non-participant agent's surplus, which is given by:

$$\mathcal{S}_0 = -iq + \ell[u(q) - q].$$

Equilibrium conditions implied by segmented marketplace participants Here the core variables are $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}^n, e_C, e_N^n\}$, as in the baseline model. Recall that we have defined $\omega_\theta(q) \equiv \theta + (1-\theta)u'(q) \geq 1$.

First, the money demand equations are given by

$$\begin{aligned} i &= \ell\left[1 - (\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\frac{\theta}{\omega_\theta(q_{1A})}\right][u'(q_{0A}) - 1] \\ &\quad + \ell(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\frac{\theta}{\omega_\theta(q_{1A})}[u'(q_{1A}) - 1], \end{aligned} \tag{C.22}$$

$$i = \ell\left[1 - \pi\alpha_{CB}^n\frac{\theta}{\omega_\theta(q_{1B}^n)}\right][u'(q_{0B}) - 1] + \ell\pi\alpha_{CB}^n\frac{\theta}{\omega_\theta(q_{1B}^n)}[u'(q_{1B}^n) - 1]. \tag{C.23}$$

The OTC trading protocols,

$$\begin{aligned} q_{1A} &= \min\{q^*, q_{0A} + \varphi\tilde{\xi}_A\}, \\ q_{1B}^n &= \min\{q^*, q_{0B} + \varphi\tilde{\xi}_B^n\}, \end{aligned}$$

combined with the OTC bargaining solutions,

$$\varphi d_A = (1-\theta)[u(q_{1A}) - u(q_{0A})] + \theta\varphi\tilde{\xi}_A,$$

$$\varphi d_B = (1-\theta)[u(q_{1B}^n) - u(q_{0B})] + \theta\varphi\tilde{\xi}_B^n,$$

yield

$$q_{1A} = \min\left\{q^*, q_{0A} + \frac{\varphi d_A - (1-\theta)[u(q_{1A}) - u(q_{0A})]}{\theta}\right\}, \tag{C.24}$$

$$q_{1B}^n = \min\left\{q^*, q_{0B} + \frac{\varphi d_B - (1-\theta)[u(q_{1B}^n) - u(q_{0B})]}{\theta}\right\}. \tag{C.25}$$

Next, we describe the liquidity premia implied by the demand of segmented marketplace participants, denoted by L_j. We define these premia as the percentage difference between an asset's price and its fundamental value. Assuming that the measure of agents visiting the segmented marketplace is positive, then L_j solves

$$p_A = \frac{1}{1+i}(1+L_A), \qquad p_B = \frac{\pi}{1+i}(1+L_B),$$

where

$$L_A = \ell(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\frac{\theta}{\omega_\theta(q_{1A})}[u'(q_{1A}) - 1], \tag{C.26}$$

$$L_B = \ell\alpha_{CB}^n\frac{\theta}{\omega_\theta(q_{1B}^n)}[u'(q_{1B}^n) - 1]. \tag{C.27}$$

If the measure of agents visiting this marketplace is zero, the liquidity premia will be exclusively determined by the demand of the consolidated marketplace participants.

The optimal entry of C-types (who have chosen the segmented marketplace) is characterized by

$$e_C = \begin{cases} 1, & \tilde{\mathcal{S}}_{CA} > \tilde{\mathcal{S}}_{CB} \\ 0, & \tilde{\mathcal{S}}_{CA} < \tilde{\mathcal{S}}_{CB} \\ \in [0,1], & \tilde{\mathcal{S}}_{CA} = \tilde{\mathcal{S}}_{CB} \end{cases}, \tag{C.28}$$

where

$$\tilde{\mathcal{S}}_{CA} = -iq_{0A} - L_A\varphi d_A + \ell\Big[u(q_{0A}) - q_{0A} + (\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d)\mathcal{S}_{CA}\Big],$$
$$\mathcal{S}_{CA} = \theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big),$$

and

$$\tilde{\mathcal{S}}_{CB} = -iq_{0B} - \pi L_B\varphi d_B + \ell\Big[u(q_{0B}) - q_{0B} + \pi\alpha_{CB}^n\mathcal{S}_{CB}^n\Big],$$
$$\mathcal{S}_{CB}^n = \theta\Big(u(q_{1B}^n) - u(q_{0B}) - q_{1B}^n + q_{0B}\Big).$$

Similarly, for the N-types' decision, we have

$$e_N^n = \begin{cases} 1, & \alpha_{NA}^n \mathcal{S}_{NA} > \alpha_{NB}^n \mathcal{S}_{NB}^n \\ 0, & \alpha_{NA}^n \mathcal{S}_{NA} < \alpha_{NB}^n \mathcal{S}_{NB}^n \\ \in [0,1], & \alpha_{NA}^n \mathcal{S}_{NA} = \alpha_{NB}^n \mathcal{S}_{NB}^n \ , \end{cases} \tag{C.29}$$

where

$$\begin{aligned} \mathcal{S}_{NA} &= (1-\theta)\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big), \\ \mathcal{S}_{NB}^n &= (1-\theta)\Big(u(q_{1B}^n) - u(q_{0B}) - q_{1B}^n + q_{0B}\Big). \end{aligned}$$

Equilibrium conditions implied by consolidated marketplace participants Here the core variables are $\{q_0, q_1^n, q_1^d\}$, as in Section C.2. Recall that we have defined $\omega_\theta(q) \equiv \theta + (1-\theta)u'(q) \geq 1$.

First, the money demand equation is given by

$$\begin{aligned} i = \ell\left[1 - \pi\alpha_{C2}\frac{\theta}{\omega_\theta(q_1^n)} - (1-\pi)\alpha_{C2}\frac{\theta}{\omega_\theta(q_1^d)}\right][u'(q_0) - 1] \\ + \ell\pi\alpha_{C2}\frac{\theta}{\omega_\theta(q_1^n)}[u'(q_1^n) - 1] + \ell(1-\pi)\alpha_{C2}\frac{\theta}{\omega_\theta(q_1^d)}[u'(q_1^d) - 1]. \end{aligned} \tag{C.30}$$

The OTC trading protocols,

$$\begin{aligned} q_1^n &= \min\{q^*, q_0 + \varphi\tilde{\xi}_2^n\}, \\ q_1^d &= \min\{q^*, q_0 + \varphi\tilde{\xi}_2^d\}, \end{aligned}$$

combined with the OTC bargaining solutions,

$$\begin{aligned} \varphi d_{A2} + \varphi d_{B2} &= (1-\theta)[u(q_1^n) - u(q_0)] + \theta\varphi\tilde{\xi}^n, \\ \varphi d_{A2} &= (1-\theta)[u(q_1^d) - u(q_0)] + \theta\varphi\tilde{\xi}^d, \end{aligned}$$

yield

$$q_1^n = \min\left\{q^*, q_0 + \frac{\varphi d_{A2} + \varphi d_{B2} - (1-\theta)[u(q_1^n) - u(q_0)]}{\theta}\right\}, \tag{C.31}$$

$$q_1^d = \min\left\{q^*, q_0 + \frac{\varphi d_{A2} - (1-\theta)[u(q_1^d) - u(q_0)]}{\theta}\right\}. \tag{C.32}$$

Next, we describe the liquidity premia implied by the demand of consolidated marketplace participants, denoted by L_{j2}. As usual, we define these premia as the percentage difference between an asset's price and its fundamental value. Assuming that the measure of agents visiting the consolidated marketplace is positive, then L_{j2} solves

$$p_{A2} = \frac{1}{1+i}(1 + L_{A2}), \qquad p_{B2} = \frac{\pi}{1+i}(1 + L_{B2}),$$

where

$$L_{A2} = \ell\pi\alpha_{C2}\frac{\theta}{\omega_\theta(q_1^n)}[u'(q_1^n) - 1] + \ell(1-\pi)\alpha_{C2}\frac{\theta}{\omega_\theta(q_1^d)}[u'(q_1^d) - 1], \tag{C.33}$$

$$L_{B2} = \ell\alpha_{C2}\frac{\theta}{\omega_\theta(q_1^n)}[u'(q_1^n) - 1]. \tag{C.34}$$

If the measure of agents visiting this marketplace is zero, the liquidity premia will be exclusively determined by the demand of the segmented marketplace participants.

Since in the last step of equilibrium characterization we will describe the agents' entry decisions, it is useful to define the consolidated marketplace participants' surpluses. For the C-types, we have

$$\begin{aligned}
\tilde{\mathcal{S}}_{C2} &= -iq_0 - L_A\varphi d_{A2} - \pi L_B\varphi d_{B2} + \ell\Big[u(q_0) - q_0 + \pi\alpha_{C2}\mathcal{S}_{C2}^n + (1-\pi)\alpha_{C2}\mathcal{S}_{C2}^d\Big],\\
\mathcal{S}_{C2}^n &= \theta\Big(u(q_1^n) - u(q_0) - q_1^n + q_0\Big),\\
\mathcal{S}_{C2}^d &= \theta\Big(u(q_1^d) - u(q_0) - q_1^d + q_0\Big).
\end{aligned}$$

For the N-types, we have

$$\begin{aligned}
\mathcal{S}_{N2}^n &= (1-\theta)\Big(u(q_1^n) - u(q_0) - q_1^n + q_0\Big),\\
\mathcal{S}_{N2}^d &= (1-\theta)\Big(u(q_1^d) - u(q_0) - q_1^d + q_0\Big).
\end{aligned}$$

Market clearing and no-arbitrage conditions The money market clearing condition is

$$\varphi M = (1-e_0)[(1-e_2)[e_C q_{0A} + (1-e_C)q_{0B}] + e_2 q_0] + e_0 q. \tag{C.35}$$

The asset market clearing conditions are

$$S_A = (1-e_0)[(1-e_2)e_C d_A + e_2 d_{A2}], \tag{C.36}$$

$$S_B = (1-e_0)[(1-e_2)(1-e_C)d_B + e_2 d_{B2}]. \tag{C.37}$$

Finally, assuming positive measures of agents in both marketplaces, no-arbitrage requires the liquidity premia implied by both types of marketplace participants to be equal:

$$L_A = L_{A2}, \tag{C.38}$$

$$L_B = L_{B2}. \tag{C.39}$$

Entry choices of marketplaces Recall that the surplus of a non-participant agent was defined in Section C.4.3 and denoted by $\mathcal{S}_0$. The surplus of an agent in the segmented marketplace is given by

$$\begin{aligned}\mathcal{S}_1 = &\tilde{\mathcal{S}}_{CA}\cdot\mathbb{I}\{e_C > 0\} + \tilde{\mathcal{S}}_{CB}\cdot\mathbb{I}\{e_C = 0\} \\ &+ (1-\ell)\pi\Big[\alpha^n_{NA}\mathcal{S}_{NA}\cdot\mathbb{I}\{e^n_N > 0\} + \alpha^n_{NB}\mathcal{S}^n_{NB}\cdot\mathbb{I}\{e^n_N = 0\}\Big] \\ &+ (1-\ell)(1-\pi)\alpha^d_{NA}\mathcal{S}_{NA}.\end{aligned} \tag{C.40}$$

Finally, the surplus of an agent choosing the consolidated marketplace is given by

$$\mathcal{S}_2 = \tilde{\mathcal{S}}_{C2} + (1-\ell)\Big[\pi\alpha_{N2}\mathcal{S}^n_{N2} + (1-\pi)\alpha_{N2}\mathcal{S}^d_{N2}\Big]. \tag{C.41}$$

The optimal marketplace choice is then characterized by

$$e_0 = \begin{cases} 1, & \mathcal{S}_0 > \max\{\mathcal{S}_1 - \kappa_1, \mathcal{S}_2 - \kappa_2\} \\ 0, & \mathcal{S}_0 < \max\{\mathcal{S}_1 - \kappa_1, \mathcal{S}_2 - \kappa_2\} \\ \in [0,1], & \mathcal{S}_0 = \max\{\mathcal{S}_1 - \kappa_1, \mathcal{S}_2 - \kappa_2\} \end{cases} \tag{C.42}$$

and, conditional on $e_0 < 1$,

$$e_2 = \begin{cases} 1, & \mathcal{S}_2 - \kappa_2 > \mathcal{S}_1 - \kappa_1 \\ 0, & \mathcal{S}_2 - \kappa_2 < \mathcal{S}_1 - \kappa_1 \\ \in [0,1], & \mathcal{S}_2 - \kappa_2 = \mathcal{S}_1 - \kappa_1\,. \end{cases} \tag{C.43}$$

Definition of equilibrium

Definition 5. For given asset supplies $\{A, B\}$, the steady-state equilibrium of the model consists of 21 variables: the equilibrium quantity $\{q\}$ implied by the agents not participating in any marketplace; the equilibrium quantities $\{q_{0A}, q_{1A}, q_{0B}, q^n_{1B}\}$, the entry choices $\{e_C, e^n_N\}$, the liquidity premia $\{L_A, L_B\}$, and the asset holdings $\{d_A, d_B\}$, implied by the agents participating in the segmented marketplace; the equilibrium quantities $\{q_0, q^n_1, q^d_1\}$, the liquidity premia $\{L_{A2}, L_{B2}\}$, and the asset holdings $\{d_{A2}, d_{B2}\}$, implied by agents participating in the consolidated marketplace; the price of money $\{\varphi\}$; and the entry decisions of marketplaces $\{e_0, e_2\}$. These equilibrium variables are determined by (C.21), (C.22), (C.23), (C.24), (C.25), (C.26), (C.27), (C.28), (C.29), (C.30) (C.31), (C.32), (C.33), (C.34), (C.35), (C.36), (C.37), (C.38), (C.39), (C.42), and (C.43).

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