Medium–Term Business Cycles and Labor–Market Search*

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Abstract

We study short and medium–term dynamics of the labor market. We first empirically document fluctuations and co-movements of some key macroeconomic and labor market variables. We show that while the business-cycle dynamics are important, a substantial part of the labor market adjustments happens also at lower "medium-term" frequencies. We ask what role interactions between labor market frictions and the determinants of growth play for explaining these medium–term labor market dynamics. We build a macroeconomic model in which long-term economic growth is endogenously determined by R&D investment. Our results suggest that endogenous growth is crucial for quantitatively explaining the medium–term dynamics of labor market variables. In particular, we show that the R&D sector amplifies the responses of the labor market variables to shocks of plausible magnitude at both business–cycle frequencies and medium–term cycles.

Keywords: Labor market dynamics, Medium–term cycles, Endogenous growth, R&D.

JEL Codes: E24, E32, J64, O30.

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1 Introduction

How do research and development activities, innovation, and the resulting technological progress affect the short and medium-term dynamics of the labor market? To answer this question, we first empirically document fluctuations and co-movements of R&D investment and the key labor market and macroeconomic variables. We then build a model of semi-endogenous growth with search and matching frictions in the labor market that helps us understand the role of interactions between these frictions, the determinants of long-term economic growth, and total factor productivity (TFP) shocks for explaining the labor market dynamics.

In contrast to most of the existing macroeconomics-labor literature, we do not focus only on the business-cycle fluctuations, usually defined as those with frequencies between 6 and 32 quarters, but we follow Comin and Gertler (2006) and study also the medium-term fluctuations with frequencies between 32 and 200 quarters.[1] Our results show that while the short-term dynamics are important, a substantial part of the labor market adjustments happens also at these medium-term frequencies. Several of the patterns we find for labor market variables are consistent with those found by Comin and Gertler (2006) for a set of other standard macroeconomic variables. Labor market variables are twice as volatile in the medium-term frequencies than in the business-cycle frequencies. R&D, labor productivity, job vacancies, labor market tightness, and wages are pro-cyclical in both business-cycle and medium-term frequencies, while unemployment and job separations are counter-cyclical. R&D displays statistically and economically significant contemporaneous correlation with some key labor market variables including unemployment, vacancies, labor market tightness, and wages in both business-cycle and medium-term frequencies. There is a lead of R&D over labor productivity, wages, and unemployment in the medium-term frequencies. This lead is almost absent in the business-cycle frequencies. Our empirical evidence sheds some new light on the analysis of labor market fluctuations and suggests that limiting the focus only to fluctuations up to 32 quarters may miss a substantial part of the dynamics linking the labor market and aggregate economic activity. It also provides support to the idea that the sources of economic fluctuations are interconnected with the sources of long-term growth in line with Comin (2009).

[1] In practice the differentiation between business-cycle and medium-term fluctuations is obtained by filtering the time series with two band pass filters, each enabling us to isolate different frequencies of the data. See Section 2 for details.

Our model extends Romer's (1990) model of endogenous growth through expanding variety of inputs to allow for interactions between R&D activity and the labor market. Specifically, we assume that final good producers purchase intermediate inputs, and hire workers on a labor market characterized by search and matching frictions. New varieties of intermediate goods are generated by innovators, who combine final good and specialized labor as inputs to R&D activity. Innovators hire workers on a separate labor market that is not subject to search and matching frictions. However, in order to enter this labor market, workers must become specialists in R&D which is costly. In absence of shocks the model exhibits a deterministic balanced growth path along which the final output growth rate, the share of workers specializing in R&D, and the unemployment rate are constant and endogenously determined. Compared to a model in which growth is exogenous, the introduction of the market for R&D-specialized labor raises the elasticity of labor market tightness with respect to productivity in the final good sector. This allows the stochastic version of the model to better propagate and substantially amplify the effects of TFP shocks to labor market quantities - namely unemployment - and other macroeconomic variables.

In addition to standard transmission mechanisms studied in the macro-labor literature, our model features a new channel: A current increase in TFP in the final good sector raises the future productivity of intermediate goods and thus the value of innovation. This in turn pushes the innovators to demand more specialized labor today. As a result, in equilibrium more workers will become R&D specialists instead of searching for a job in the final good sector and the current unemployment falls. The resulting higher discovery of new varieties of intermediate inputs further enhances the productivity in the final good sector in the subsequent periods, triggering a powerful feedback effect that leads to persistent responses to productivity fluctuations. Thanks to this channel, a calibrated version of our model is able to generate quantitatively adequate volatility and persistence in labor market and other standard macro variables in both business-cycle and medium-term frequencies in response to TFP shocks of realistic magnitude.

Our paper is related to a growing body of literature that stresses the importance of considering the short-term behavior of the economy in connection to its longer-term dynamics for understanding economic fluctuations (see e.g. Blanchard 1997, Evans et al. 1998, Solow 2000, Comin and Gertler 2006, Comin et al. 2014, Schwark 2014, Anzoategui et al. 2019, Beaudry, Galizia, and Portier 2020, Schüler 2020). In contrast to these papers, we specifically focus on labor market dynamics and the interaction between labor market frictions and R&D activity.

Labor market fluctuations have been the interest of extensive empirical and theoretical literature (see e.g. Mortensen and Pissarides 1994, Merz 1995, Andolfatto 1996, Shimer 2005, Hagedorn and Manovskii 2008, and the subsequent literature). Papers in this literature somewhat differ in terms of the frequencies at which the labor market fluctuations are measured, but we are not aware of a study systematically documenting the differences and patterns across the business-cycle and medium-term frequencies. Our work thus complements the existing evidence by providing such an overview. One of the challenges for models relying on the search and matching framework has been to reproduce the observed magnitude of volatility of key labor market variables, such as unemployment and vacancies, in response to productivity shocks of realistic size. The existing papers proposed several solutions which, as shown by Ljungqvist and Sargent (2017), mainly come down to reducing the fundamental surplus a job creates.[2] This in turn increases the elasticity of labor market tightness with respect to productivity and thus amplifies fluctuations in unemployment. We explore a different, complementary mechanism. In our model, the elasticity of labor market tightness with respect to productivity is high not because of low fundamental surplus, but because of existence of an alternative labor market to which unemployed workers can turn, albeit at a cost. Crucially, the opportunities on this alternative labor market are procyclical and persistent, leading to amplified and persistent responses of unemployment to productivity shocks.

Finally, our framework shares some features with news shocks literature on perceptions and revisions of beliefs about the future as an important source of business-cycle fluctuations (see Beaudry and Portier 2006, 2007, and the subsequent literature). Our work differs from these papers by connecting the beliefs to determinants of long-term economic growth. In our framework, the future innovation possibilities frontier increases in responses to persistent positive TFP shocks. This in turn spurs R&D activity that further enhances the expectations of future productivity. Accordingly, a current TFP shock induces also effects similar to news about future technologies.

[2]These modifications of the matching model range from different calibration strategies (Hagedorn and Manovskii, 2008), introduction of nominal rigidities (Hall, 2005), financing frictions (Wasmer, 2004; Petrosky-Nadeau and Wasmer, 2013), fixed costs (Pissarides, 2009), and different bargaining protocols (Hall and Milgrom, 2008) to including firm heterogeneity and decreasing returns to scale (Elsby and Michaels, 2013).

2 Medium-term Cycles - Empirical Evidence

This section presents some empirical evidence about medium-term cycles in the U.S., which are defined as the sum of the high- and medium-frequency fluctuations. High-frequency fluctuations are usually referred to as conventional business cycles (i.e. cycles with periods smaller than 32 quarters as in Burns and Mitchell 1946).[3] The medium-frequency fluctuations correspond to cycles with periods between 32 and 200 quarters.

In practice the high- and medium-frequency components of the medium-term cycle are obtained by applying two band pass filters with different bandwidths to the data. The first band pass filter allows us to identify the *medium-term cycle*, i.e. fluctuations with periodicity of 6-200 quarters. The second band pass filter allows us to isolate the *medium-frequency component*, i.e. fluctuations with periodicity of 32-200 quarters. The difference between the medium-term cycle and its medium-frequency component then corresponds to the *high-frequency component* or the conventional measure of the business cycle, i.e. fluctuations with periodicity of 6-32 quarters. Band pass filters are a particularly appropriate method for identifying stochastic trends in our context because they specify with precision the frequencies in which the trends are isolated. Other popular methods, such as the Hodrick and Prescott's (1997) filter, allow to control the degree of smoothing of the trend, but the mapping into the frequency domain is less straightforward.

As in Comin and Gertler (2006), before applying the band pass filters we transform all raw data series into growth rates by taking log differences. Our implementation of band pass filters follows Baxter and King (1999). We use approximate band pass filters that are constrained to produce stationary outcomes when applied to growing time series and allow to specify frequencies of the data that one wishes to isolate. Because we are using band pass filters that are optimal approximations to the ideal filter but for finite series, we pad the series by forecasting and back-casting. The padding allows to minimize biases that are likely to arise at sample endpoints. By applying the filters to the growth rate data we obtain measures of trend growth rates in the specified frequencies of interest. We accumulate these trend growth rates to obtain measures of trends in log levels. The medium-term cycle is then computed as deviations of the log-level data with respect to

[3] Stock and Watson (1999) are pointing out that the shortest full cycle (peak to peak) from NBER business cycle reference dates is 6 quarters, and about 90% of these cycles are no longer than 32 quarters. Baxter and King (1999) recommend using a band pass filter that admits frequency components between 6 and 32 quarters for isolating business cycle fluctuations.

the baseline log-level trend in the 6-200 quarters periodicity. The medium-frequency component is computed as deviations of the log-level data with respect to the log-level trend in the 32-200 quarters periodicity.

Comin and Gertler (2006) and Comin (2009) derived a series of stylized facts on medium-term cycles for a set of usual macro variables that appear in the business-cycle analysis. We update their empirical evidence and extend it with a set of labor market variables. Our data are in quarterly frequencies from 1951:01 to 2006:01, except when noted otherwise. The data include two sets of variables.The first set includes: gross domestic product (GDP), consumption, investment, total factor productivity (TFP), and research and development (R&D) investment. GDP, consumption and investment are from the Bureau of Economic Analysis (BEA) National Income and Product Account (NIPA) tables. These series are deflated by the GDP deflator and expressed in per capita terms after dividing by the civilian non-institutionalized population aged 16 and over. Consumption includes non-durables and services. Investment is nonresidential. We use the quarterly TFP series from Fernald (2014).[4] The quarterly series of R&D investment is from BEA and is deflated using the R&D chain-type price index.

The second set of variables characterizes the labor market dynamics. It includes labor productivity, job vacancies, unemployment, labor market tightness, and wages. Following Shimer (2005), labor productivity is measured as the real average output per person in the non-farm business sector. This series is directly available from the Bureau of Labor Statistics (BLS). Job vacancies are measured by the Index of Help-Wanted Advertising in Newspapers from the Conference Board divided by labor force from BLS based on Current Population Survey (CPS). Unemployment is quarterly rate directly constructed from averages of monthly BLS series. Wages are non-farm business compensation divided by number of employed from the BLS. Labor market tightness is the ratio of job vacancies and unemployment.

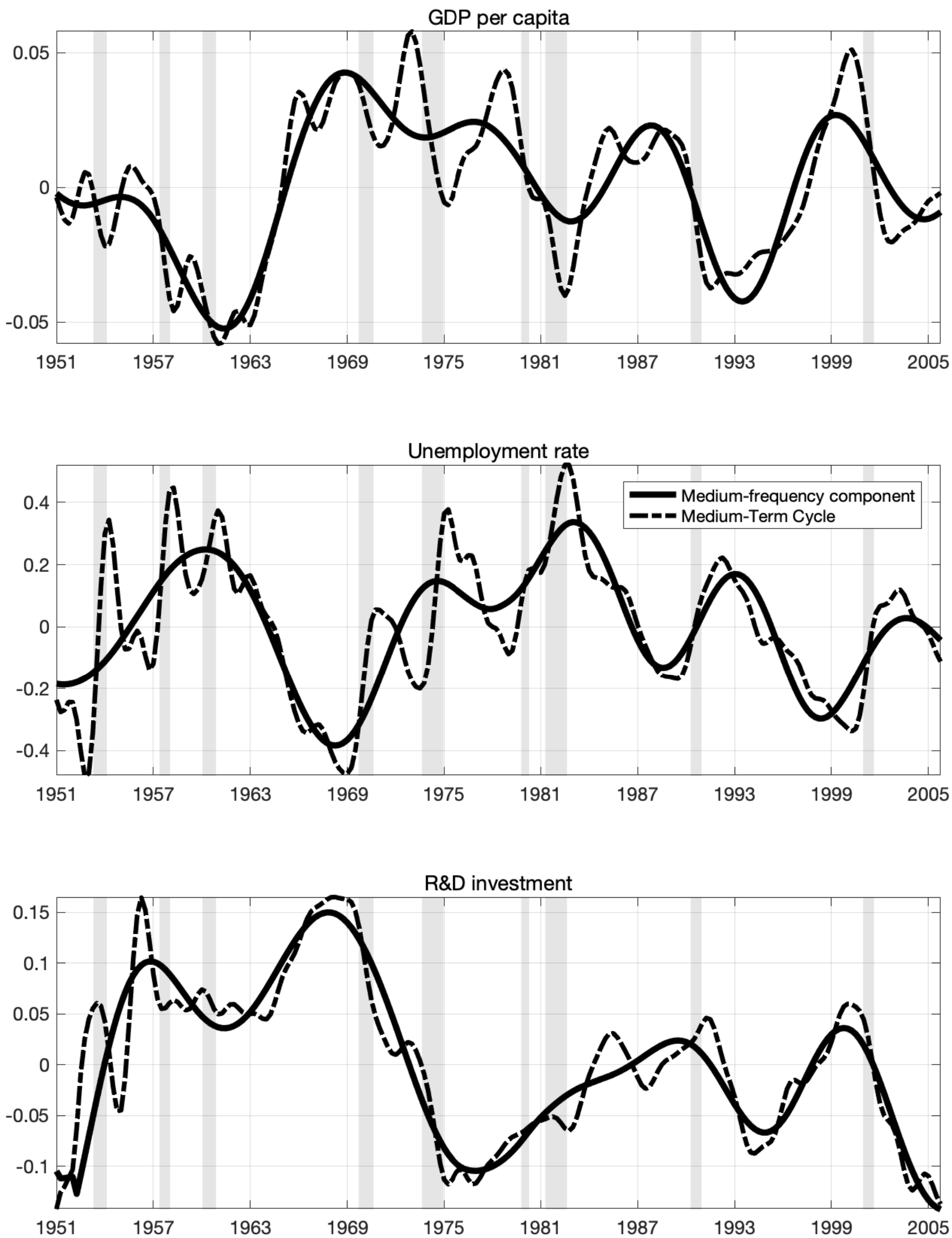
Following Comin and Gertler (2006), we also consider annual data because medium-term fluctuations seem considerably more persistent than are typically considered. Annual data for per capita GDP, consumption, investment, TFP, labor productivity, wages and, unemployment are obtained from St. Louis Fed database. The annual series of R&D is non-federally funded R&D

[4]Fernald's (2014) TFP series is the quarterly version of the annual series developed by Basu, Fernald, and Kimball (2006).

expenditures as reported by the National Science Foundation (NSF). The NSF R&D series is deflated using the implicit GDP deflator. The annual job vacancies series is an average calculated from Robert Shimer's data. Finally, the annual job finding and job separation rates are obtained from the observed average quarterly job finding and job separation rates provided by Shimer by following the transition probability tree over four quarterly sub-periods. Annual data span the period 1953 to 2006. The definition of the medium-term cycle in annual data corresponds to the fluctuations with periodicity of 0-50 years, with high- and medium-frequency components corresponding respectively to the fluctuations with periodicity of 0-8 years and 8-50 years.

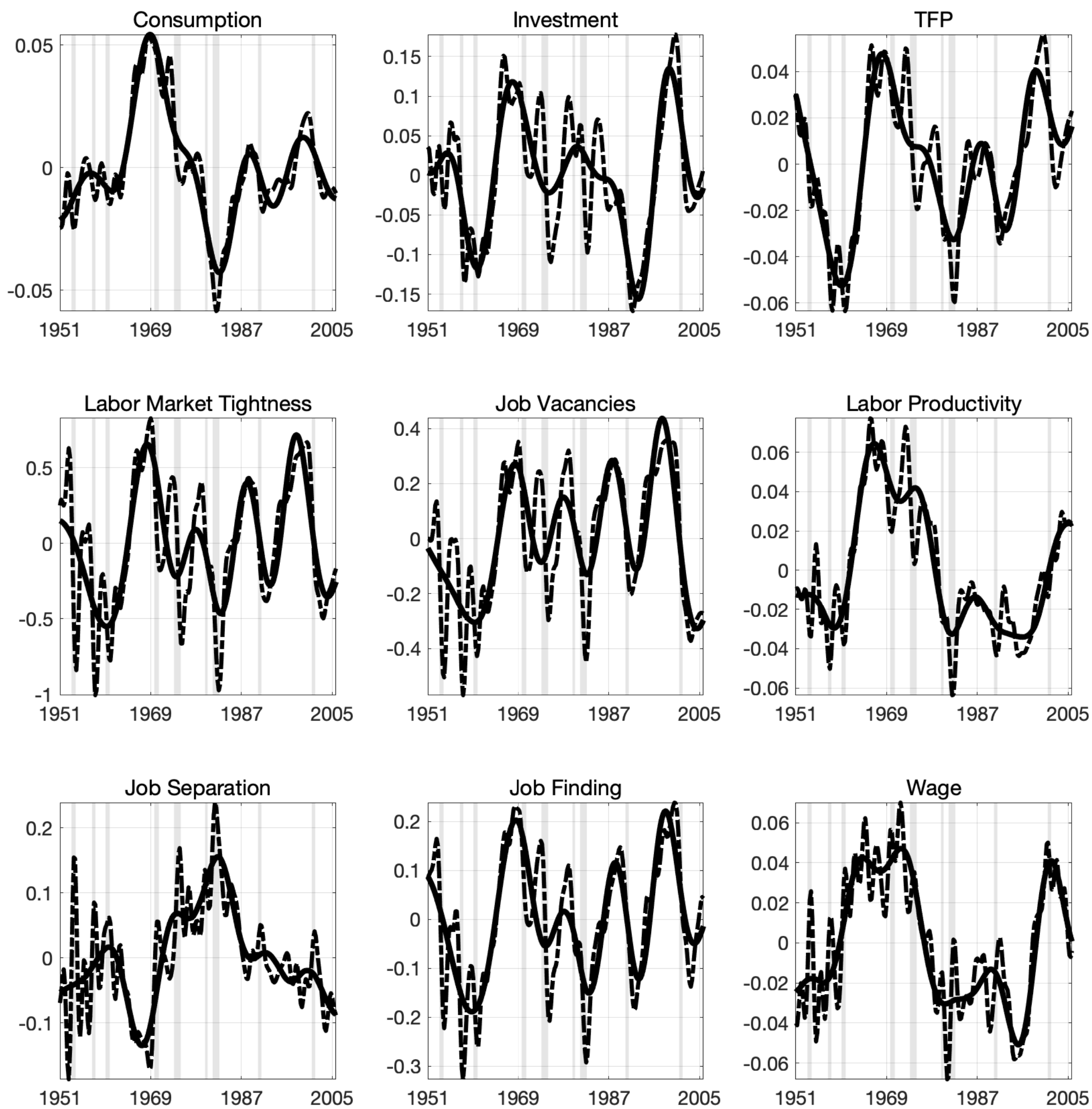
Figure 1 gives a first look at the medium-term cycle in three important variables in the U.S. data, per capita GDP, unemployment rate, and R&D investment. For each variable the dashed line depicts the medium-term cycle expressed as percentage deviation from the baseline trend at periodicity between 6 and 200 quarters. The solid line depicts the medium-frequency component, i.e. the percentage deviation from the stochastic trend at periodicity between 32 and 200 quarters. The difference between the two lines is the high-frequency component. We observe that, for the three variables, the medium-frequency component accounts for a substantial part of the fluctuations between 6 and 200 quarters. Clearly, studying the medium-term fluctuations is important for understanding the dynamics of goods and labor markets. Moreover, it is crucial for the dynamics of innovation activities, where the medium-frequency component accounts for almost the entirety of the fluctuations. Unemployment displays medium-term fluctuations which move in the opposite direction to the medium-term fluctuations of GDP per capita. Conversely, the medium-term fluctuations of R&D investment are procyclical. In the light of this evidence, it appears that by limiting the horizon of fluctuations to 32 quarters the typical business-cycle analysis reveals only a small part of the relationship between the GDP and the labor market and it seems virtually unable to link these fluctuations to the R&D dynamics. Figure 2 confirms that the medium-frequency component also accounts for a substantial part of fluctuations of other important macroeconomic variables. This is particularly true for the labor market variables such as labor market tightness, vacancies, labor productivity, and wages. We now present a set of formal statistics characterizing the medium-term business cycles for a number of key macro variables.

Figure 1: The Medium-Term Cycle. U.S. Quarterly Data 1951:01–2006:01



Note: Figure 1 plots the medium-term cycle along with the associated medium-frequency component for U.S. GDP per capita, unemployment rate, and R&D investment between 1951:01 and 2006:01. Dashed lines depict the medium-term cycle. Solid lines depict the medium-frequency component. The shaded areas represent recession periods as determined by the National Bureau of Economic Research (NBER).

Figure 2: Medium-Term Variations for some Key Macro Variables. U.S. Quarterly Data: 1951:01–2006:01



Note: Figure 2 plots the medium-term cycle along with the associated medium-frequency component for consumption, investment, TFP, labor market tightness, job vacancies, labor productivity, job separation, job finding, and wages between 1951:01 and 2006:01. Dashed lines depict the medium-term cycle. Solid lines depict the medium-frequency component. The shaded areas represent recession periods as determined by the National Bureau of Economic Research (NBER).

Table 1: Standard Deviations. U.S. Data: 1951–2006

	Medium-term cycle	High-frequency component	Medium-frequency component
U.S. Quarterly Data	6–200	6–32	32–200
GDP	3.30	1.49	2.97
Consumption	2.25	0.75	2.14
Investment	8.07	4.15	6.94
TFP	2.90	1.26	2.63
R&D	8.87	3.28	8.57
Labor productivity	3.27	1.25	3.07
Wage	2.95	1.03	2.81
Unemployment	21.99	12.16	18.24
Job vacancies	23.77	13.45	20.01
Labor market tightness	42.44	25.20	34.34
Job finding	13.16	7.23	11.01
Job separation	8.12	4.66	6.77
U.S. Annual Data	0–50	0–8	8–50
GDP	3.50	1.38	3.21
Consumption	2.33	0.69	2.22
Investment	8.42	3.96	7.38
TFP	3.00	1.16	2.76
R&D	7.66	2.37	7.16
Labor productivity	3.31	1.12	3.11
Wage	3.12	0.98	2.96
Unemployment	21.50	11.26	18.40
Job vacancies	25.52	14.00	20.96
Labor market tightness	43.37	24.30	35.83
Job finding	3.86	1.96	3.31
Job separation	19.09	8.86	16.91

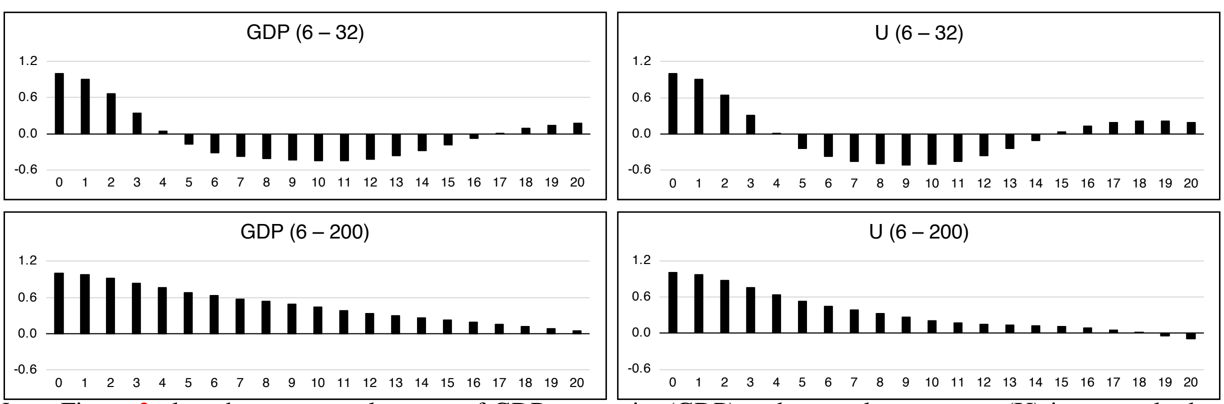
Note: The standard deviations are expressed in terms of percentage deviations from trend.

Volatility Table 1 reports the standard deviations of macroeconomic variables in the medium-term cycle and of its high- and medium-frequency components expressed in terms of percentage deviations from trend. The medium-frequency component is systematically more volatile than the high-frequency component, resulting in the medium-term cycle displaying higher standard deviations than the conventionally measured business cycle. The labor market variables, in particular, are on average twice as volatile in the medium-term cycle than in its high-frequency component. One comment is in order concerning the trend-cycle decomposition of the labor market variables.

The existing labor literature generally employed a HP-filter to obtain the trend and the cyclical component of these variables. However, the papers vary in the chosen value for the smoothing parameter λ of the filter. While some studies (e.g. Hagedorn and Manovskii, 2008) use the standard value in business-cycle analysis ($\lambda = 1,600$), others (e.g. Shimer, 2005; Costain and Reiter, 2008) use much higher value ($\lambda = 10^5$). This is akin to including in the cyclical fluctuations those with a longer periodicity than 32 quarters. Nonetheless, we are not aware of a paper systematically investigating the patterns and differences across the fluctuations at the business-cycle and medium-term frequencies.

Persistence Table 2 reports the first-order auto-correlations. All variables are very persistent over the medium-term cycle with more persistence concentrated in the medium-frequency component. This is particularly visible with the annual data, where the auto-correlation of the high-frequency component is lower than 1/3 of that of the medium-term cycle.[5] Looking at more complete auto-correlograms for GDP and unemployment in Figure 3 reveals that the auto-correlation of the high-frequency component decays much faster than that of the medium-term cycle. The pattern is consistently present across the majority of macro and labor variables.

Figure 3: Auto-correlograms. U.S. Quarterly Data: 1951:01–2006:01



Note: Figure 3 plots the auto-correlograms of GDP per capita (GDP) and unemployment rate (U) in quarterly data for the high-frequency component (6-32) and the medium-term cycle (6-200).

[5] The pattern is consistent with the quarterly data. When we take quarterly series and compute the annual auto-correlation (auto-correlation of order $t-4$) over medium-term cycles, we obtain the numbers of the same magnitude as for the annual series.

Table 2: First-Order Auto-correlations. U.S. Data: 1951–2006

	Medium–term cycle	Business–cycle component	Medium–frequency component
U.S. Quarterly Data	6–200	6–32	32–200
GDP	0.978	0.908	0.996
Consumption	0.989	0.922	0.997
Investment	0.976	0.927	0.994
TFP	0.983	0.930	0.996
R&D	0.984	0.913	0.996
Labor productivity	0.983	0.898	0.998
Wage	0.989	0.928	0.998
Unemployment	0.967	0.903	0.994
Job vacancies	0.970	0.919	0.994
Labor market tightness	0.965	0.913	0.993
Job finding	0.969	0.913	0.994
Job separation	0.944	0.834	0.997
U.S. Annual Data	0–50	0–8	8–50
GDP	0.822	0.118	0.944
Consumption	0.889	0.216	0.953
Investment	0.766	0.227	0.919
TFP	0.836	0.105	0.965
R&D	0.849	0.049	0.940
Labor productivity	0.862	0.071	0.966
Wage	0.894	0.268	0.964
Unemployment	0.726	0.121	0.916
Job vacancies	0.691	0.173	0.897
Labor market tightness	0.683	0.145	0.897
Job finding	0.719	0.182	0.902
Job separation	0.752	0.118	0.920

Cyclicality Table 3 reports contemporaneous correlations between GDP per capita and other variables. In line with the recent literature (Barlevy 2007, Ouyang 2011, Fabrizio and Tsolmon 2014, Anzoategui et al. 2019), our results show that R&D is pro-cyclical. We show that this is the case at both business-cycle and medium-term frequencies. Consumption, investment, TFP, labor productivity, job vacancies, labor market tightness, and job finding are also pro-cyclical at both business-cycle and medium-term frequencies. However, unemployment and job separation are counter-cyclical.

Table 3: Contemporaneous Correlation with GDP. U.S. Data: 1951–2006

	Medium-term cycle	High-frequency component	Medium-frequency component
U.S. Quarterly Data	6–200	6–32	32–200
Consumption	0.898	0.856	0.921
Investment	0.669	0.817	0.641
TFP	0.855	0.882	0.830
R&D	0.484	0.594	0.489
Labor productivity	0.700	0.723	0.698
Wage	0.686	0.663	0.701
Unemployment	-0.878	-0.858	-0.895
Job vacancies	0.625	0.936	0.509
Labor market tightness	0.804	0.914	0.772
Job finding	0.830	0.877	0.814
Job separation	-0.710	-0.678	-0.718
U.S. Annual Data	0–50	0–8	8–50
Consumption	0.931	0.857	0.944
Investment	0.697	0.858	0.665
TFP	0702	0.821	0.677
R&D	0.597	0.374	0.635
Labor productivity	0.754	0.729	0.755
Wage	0.747	0.673	0.757
Unemployment	-0.932	-0.863	-0.953
Job vacancies	0.561	0.812	0.512
Labor market tightness	0.792	0.869	0.788
Job finding	0.774	0.800	0.784
Job separation	-0.880	-0.820	-0.898

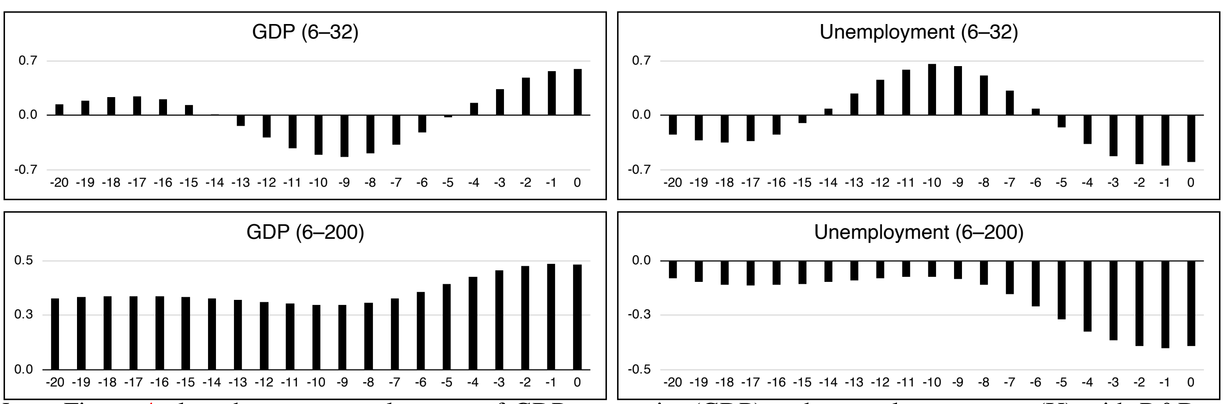
Lead of R&D Table 4 reports the correlation between R&D at year $t-5$ and other variables at year t. Our results confirm the finding of Comin and Gertler (2006) that accounting for medium-frequency component in the analysis modifies the timing of the cross-correlogram for a number of key macro variables including GDP per capita, consumption, and TFP. In particular, there is a lead of R&D in the medium-frequency component, but this lead is almost absent in the high-frequency component. We extend these findings to a set of labor market variables including wages, and unemployment. In line with Comin and Gertler (2006), we focus in the table on the annual data as the pattern is not evident in a single lead common to all variables in the quarterly data. For

Table 4: Cross-correlation with R&D at $t-5$. Data: 1951–2006

	Medium-term cycle	High-frequency component	Medium-frequency component
U.S. Annual Data	0–50	0–8	8–50
Per capita GDP	0.494	0.110	0.549
Consumption	0.590	0.055	0.657
Investment	0.074	0.202	0.054
TFP	0.518	-0.002	0.590
Labor productivity	0.616	-0.070	0.698
Wage	0.585	-0.185	0.684
Unemployment	-0.308	-0.087	-0.352
Job vacancies	-0.115	0.027	-0.111
Labor market tightness	0.089	0.059	0.121
Job finding	0.169	0.149	0.194
Job separation	-0.253	-0.125	-0.285

illustration, Figure 4 gives the detail of the cross-correlogram between two key variables (GDP and Unemployment) and R&D in the quarterly data.

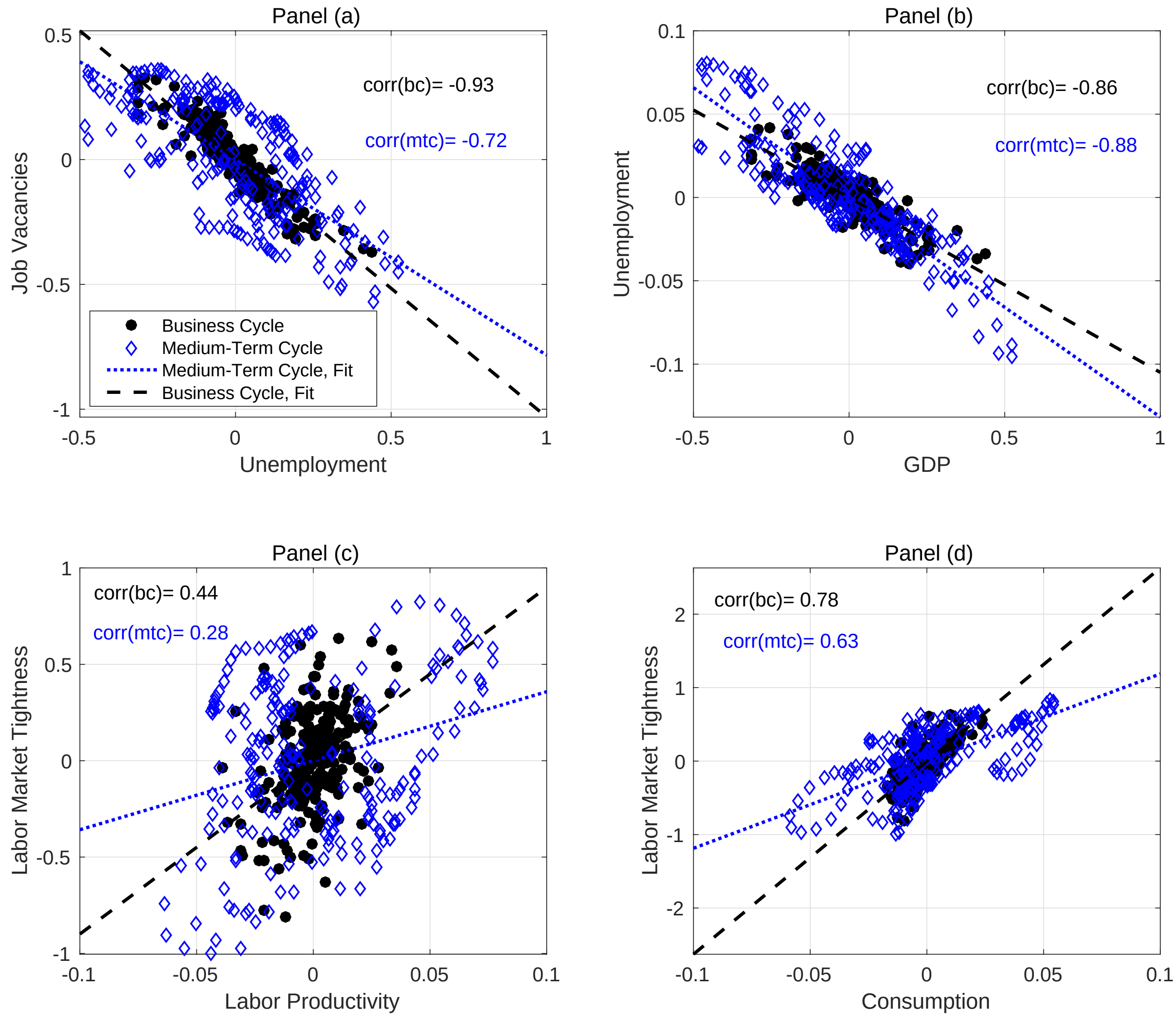
Figure 4: Cross-correlograms with R&D. U.S. Quarterly Data: 1951:01–2006:01



Note: Figure 4 plots the cross-correlograms of GDP per capita (GDP) and unemployment rate (U) with R&D at different leads in quarterly data for the high-frequency component (6-32) and the medium-term cycle (6-200).

Labor market Figure 5 shows scatter plots of the key relationships in the macro-labor literature. In each panel, every point corresponds to a quarter between 1951 and 2006. We plot data for the medium-term cycle and its high-frequency component (business cycle). Panel (a) displays the Beveridge curve, Panel (b) the the Okun's law, Panel (c) the relationship between labor market

Figure 5: Correlation Patterns for some Key Macro Variables. Data: 1951:01–2006:01



Note: Figure 5 plots correlation patterns for some key labor market variables at both business cycle frequencies and medium-term cycles. Each point corresponds to a quarter between 1951:01 and 2006:01.

tightness and labor productivity, and Panel (d) the relationship between labor market tightness and consumption. The general conclusion that emerges is that the same correlation patterns are present in the medium-term cycle as in the business cycle, but they are weaker, except for the Okun's law.[6]

[6]All correlations displayed in Figure 5 are statistically significant.

3 The Model

In this section, we develop a model to understand the documented key empirical facts on labor market dynamics over medium-term business cycles. The basic setup is an endogenous growth model in which growth happens through expanding variety of intermediate inputs along the lines of Romer (1990). We explicitly build in a frictional labor market with firms posting vacancies and unemployed searching for jobs as in Mortensen and Pissarides (1994). To generate economic fluctuations we introduce shocks to the total factor productivity (TFP) in the final good production as in real business cycle literature (see e.g. Kydland and Prescott, 1995). We study the propagation and amplification mechanisms and we particularly focus on the short and medium–term fluctuations of the labor market variables.

There are four types of agents: final good producers, intermediate goods producers, households, and government. There are three sectors: final good, intermediate goods, and research and development (R&D). Time is discrete and goes from zero to infinity. In anticipation of the recursive formulation of agents' problems we denote with prime the next period variables. The aggregate state of the economy at the beginning of the period is given by vector $\Omega \equiv \left(\zeta, N, K, L, L_R^{(-1)}, A\right)$, where ζ is an aggregate TFP shock, N is the number of varieties of intermediate goods available to be used in production of the final good in the current period, K is the aggregate stock of physical capital in the current period, L is the aggregate number of workers hired by the final good producers in the current period, $L_R^{(-1)}$ is the aggregate number of workers hired by the R&D sector in the previous period, and A is the aggregate households loans to innovators in the current period. We now formally describe the problems of agents and the market structure in each of the sectors of the economy. The final good is taken as the numeraire and its price is normalized to one.

3.1 Final good sector

The final good is produced by continuum of measure one of identical and perfectly competitive firms according to production function

$$y = \zeta z^{1-\psi} \left(k^{\alpha} l^{1-\alpha}\right)^{\psi}. \tag{1}$$

ζ is the aggregate TFP shock following an AR(1) process in logs, k and l are respectively inputs of capital and labor services, and z is a constant elasticity of substitution (CES) aggregate of intermediate goods

$$z = \left[\int_0^N x(i)^{1-\psi} \, \mathrm{d}i\right]^{\frac{1}{1-\psi}}, \tag{2}$$

where $x(i)$ is the quantity of intermediate good of variety i. The parameters governing the elasticity of final output with respect to different inputs and the elasticity of substitution between the varieties of intermediate goods are such that $0 < \alpha < 1$ and $0 < \psi < 1$.

A representative final good producer contracts the quantities of production inputs as to maximize its profits. Intermediate goods and physical capital are available on spot markets, whereas workers must be hired on the labor market one period in advance by posting job vacancies because of search and matching frictions. When making these decisions, the firm takes the rental rate for physical capital R, and the price of each variety i of intermediate good $p(i)$ as given.[7] In contrast, the wage rate ω is determined by bargaining with the workers who were matched to the vacancies posted in the previous period. Capital depreciates at rate $\delta_k \in (0, 1)$.

Denote $V(l, \Omega)$ the value function of the final good producer that has hired l workers, v the number of vacancies to be posted for the next period, and κ the parameter that determines per vacancy posting cost. Let also denote $\beta\Lambda(\Omega, \Omega')$ the firm's appropriate discount factor.[8] The problem of the final good producer can be written recursively as

$$\begin{aligned} V(l, \Omega) = \max_{x(i),k,l',v} \Big\{ &\zeta z^{1-\psi} \left(k^\alpha l^{1-\alpha}\right)^\psi - \int_0^N p(i)\, x(i)\, di - Rk \\ &-\omega l - \kappa (N')^{\frac{1}{1-\alpha}} v + \beta E\left[\Lambda(\Omega, \Omega') V(l', \Omega')\right] \Big\}, \end{aligned} \tag{3}$$

subject to the CES aggregate of intermediate goods (2), the law of motion of the firm's employment

$$l' = (1 - \delta_l)\, l + q(\theta)\, v, \tag{4}$$

where δ_l is exogenous job separation rate and $q(\theta)$ is the probability of filling a vacancy expressed as a function of the labor market tightness θ, and subject to the law of motion of the aggregate state

[7] Equation (1) implies that in equilibrium, the final good firm will have incentives to use all available N varieties.

[8] Since we assume households are proprietors of the firms, in equilibrium $\Lambda(\Omega, \Omega')$ will be equal to the marginal rate of substitution $u'(c')/u'(c)$.

and pricing functions

$$\Omega' = G_{\Omega}(\Omega), \tag{5}$$

$$p(i) = P(i, \Omega), \tag{6}$$

$$R = R(\Omega), \tag{7}$$

$$\omega = \omega(\Omega), \tag{8}$$

$$\theta = \theta(\Omega). \tag{9}$$

In equation (3) the firm's total vacancy posting cost, $\kappa (N')^{1/1-\alpha} v$, is assumed to be proportional to the innovation frontier given by the number of the intermediate good varieties that become available in the next period to an appropriate power. This assumption is akin to Pissarides (2000, *ch. 3*), who assumes vacancy posting cost to be proportional to labor productivity. As will be seen later, this specification ensures existence of a well defined deterministic balanced growth path.[9] The corresponding decision rules for the final good producer are functions

$$x(i) = f_{x(i)}(l, \Omega), \tag{10}$$

$$k = f_k(l, \Omega), \tag{11}$$

$$l' = f_l(l, \Omega), \tag{12}$$

$$v = f_v(l, \Omega). \tag{13}$$

3.2 Intermediate goods sector

A continuum of differentiated varieties of intermediate goods is produced by monopolistically competitive firms. Production function for each variety of intermediate good is identical and linear. Namely, one unit of variety i can be produced at constant marginal cost $\mu > 0$ units of the final good. Each intermediate good producer maximizes the value of expected discounted monopolistic profits by choosing the price of its output subject to the final good sector demand schedule. Denoting

[9]Along the deterministic balanced growth path the vacancy posting cost in our model is effectively proportional to the combined multi-factor productivity of capital and labor.

$W(i;\Omega)$ the value of discounted profits of producer i, the maximization problem can be written recursively as

$$W(i;\Omega) = \max_{p(i)} \{p(i)\,x(i) - \mu x(i) + (1-\delta_N)\,\beta E\left[\Lambda(\Omega,\Omega')\,W(i;\Omega')\right]\}, \tag{14}$$

subject to $x(i) = f_{x(i)}(l,\Omega)$. The decision rule for the intermediate good producer is a pricing function

$$p(i) = g_p(i,\Omega). \tag{15}$$

3.3 R&D sector

New varieties of intermediate goods can be invented by combining labor input (l_R) and final goods (a). We assume that labor for the R&D sector is hired on a separate competitive spot market that is free from search and matching frictions. The newly invented varieties become available for use as production inputs in the final good sector in the following period. The aggregate level of knowledge (the number of existing varieties) evolves according to

$$\frac{N'}{N} = (1-\delta_N) + \chi\,(l_R)^{\iota}\left(N^{-1/1-\alpha}a\right)^{1-\iota}, \tag{16}$$

where δ_N is the rate of obsolescence of technologies, ι is the elasticity of new intermediate goods with respect to R&D labor input, $(1-\iota)$ is the elasticity of new intermediate goods with respect to R&D final goods investment, and χ is a congestion externality taken as given by the individual innovators. The productivity of a is inversely proportional to N to an appropriate power. This feature of the R&D production function implies that on balance growth path the R&D final goods investment (a) will be proportional to $N^{1/(1-\alpha)}$. As in Comin and Gertler (2006), we assume that while the congestion externality χ is regarded as given by the individual innovators, it depends on aggregate conditions in the economy

$$\chi = \eta\left[{L_R}^{-\iota}\left(N^{-1/1-\alpha}A\right)^{\iota-1}\right]^{1-\phi}, \tag{17}$$

where $\eta > 0$ is a technology coefficient and $0 < \phi \leq 1$ is a curvature parameter. Equation (17) implies a congestion externality of R&D on economic growth. It is more difficult to come up with successful innovations as the aggregate level of R&D activity increases.[10]

There is free entry into the R&D sector, any firm or individual can initiate research. A successful inventor of a new variety receives a fully enforced perpetual patent on production of this variety. The value of owning the patent is the value of expected discounted monopolistic profits of the intermediate good producer given by equation (14). Free entry implies that the research sector will attract additional hires and investment as long as the expected future marginal benefit from developing new intermediate goods in R&D sector is higher than the associated current marginal cost. As a result, free entry equilibrium condition implies

$$\iota \chi N \left(\frac{N^{-1/1-\alpha} a}{l_R} \right)^{1-\iota} (1 - \delta_N) \beta E \left\{ \Lambda \left(\Omega, \Omega' \right) W \left(i; \Omega' \right) | \Omega \right\} = \omega_R, \tag{18}$$

$$(1 - \iota) \chi N^{1 - \frac{1}{1-\alpha}} \left(\frac{N^{-1/1-\alpha} a}{l_R} \right)^{-\iota} (1 - \delta_N) \beta E \left\{ \Lambda \left(\Omega, \Omega' \right) W \left(i; \Omega' \right) | \Omega \right\} = 1. \tag{19}$$

The left hand side of equation (18) is the expected future benefit of hiring one more unit of R&D labor. This additional unit of R&D labor leads to invention of $\iota \chi N \left(N^{-1/1-\alpha} a / l_R \right)^{1-\iota}$ new varieties of intermediate goods, each with a net present discounted value of profits given by (14). The right hand side of equation (18) is the current cost of hiring one more unit of R&D labor, which is simply given by the wage rate in on the R&D labor market ω_R. The interpretation of equation (19) is analogous for marginal benefit and cost of R&D final goods investment. Equations (18) and (19) show how the model generates pro-cyclical R&D. For instance, during a boom, the value of a new intermediate good, W, increases. Since the profit flow from intermediate goods rises, the benefit from creating new varieties of these goods goes up, labor and investment demand of the research sector will increase in response. The hiring and investment decision rules for the innovators are functions

$$l_R = f_{l_R} \left(\Omega \right), \tag{20}$$

$$a = f_a \left(\Omega \right). \tag{21}$$

[10]See Eicher and Turnovsky (2000) for more details on the importance of congestion effects in endogenous growth models.

3.4 Households

The representative household consists of a continuum of measure one of members, each of whom can be either employed in the final good production sector, employed in the R&D sector, or unemployed. Because of search and matching frictions on the final good sector labor market, household members have to secure their jobs in that sector one period in advance. The R&D labor is hired on a spot market. However, in order to be able to supply R&D labor, household members need to go through a training and become specialists, which is costly. Once trained, they remain competent in R&D labor as long as they stay employed in the R&D sector. Once they quit that sector, their R&D skill fully depreciates.

As a result, at the beginning of period fraction l of household members have a job in the final good sector, fraction l_R take jobs in the R&D sector, and the remaining fraction $1 - l - l_R$ are unemployed and spend the period searching for a new job. Among the members who are employed by the R&D sector, mass n are newly entering this sector and must go through the training. The distinction between employed and unemployed members of the household is important for the characterization of the labor market equilibrium. However, we assume that the household provides perfect insurance to its members in terms of consumption. Consequently, we can write the utility maximization problem at the household level and abstract from the intra-household transfers among the individual members.

The representative household consumes, saves by either investing into physical capital or lending to innovators (investing in R&D), and supplies labor services to the final good and R&D sectors. The household maximizes its expected inter-temporal utility subject to the budget constraint, the laws of motion of the household's capital stock, employment, and aggregate variables. The per period utility function $u(c)$ is strictly increasing, strictly concave, and satisfies the usual Inada conditions. Denote c the consumption, k the stock of physical capital, l labor supply to the final good sector, l_R labor supply to the R&D sector, n the mass of workers newly entering the R&D sector, and a the one-period household loans to innovators. Let $Q\left(k, l, l_R^{(-1)}, a, \Omega\right)$ be the household's value function. In order to lighten the notation let $\Omega^\star \equiv \left\{k, l, l_R^{(-1)}, a, \Omega\right\}$ the vector of household's state variables. The household's problem can be written recursively as

$$Q\left(\Omega^{\star}\right) = \max_{\left\{\substack{c,k',l,\\ l_R,n,a'}\right\}} \left\{u\left(c\right) + \beta E\left\{Q\left(\Omega^{\star\prime}\right)|\Omega^{\star}\right\}\right\} \tag{22}$$

subject to

$$\begin{aligned} c + k' + a' + \frac{1}{2}c_R n^2 = \omega l + \omega_R l_R + b\left(1 - l - l_R\right) \\ + \left[R + (1-\delta_k)\right]k + (1+r)a + T + \Pi^i + \Pi^f, \end{aligned} \tag{23}$$

$$l' = (1-\delta_l)l + f\left(\theta\right)\left(1 - l - l_R\right), \tag{24}$$

$$l_R = (1-\delta_{lR})l_R^{(-1)} + n, \tag{25}$$

$$c, k' \geq 0, c_R = \overline{c}_R N^{\frac{1}{1-\alpha}}, \tag{26}$$

$$r = r\left(\Omega\right), \tag{27}$$

and subject to the law of motion of the aggregate state (5), and the pricing equations (6)–(9), and (27).

Equation (23) is the budget constraint, where the left hand side corresponds to household's expenditures and the right hand side to the revenues. The last term on the expenditure side is the cost of training new workers to become R&D specialists. We assume the cost quadratic in the quantity of workers undertaking the training and $c_R > 0$ is a parameter governing the its magnitude.[11] On the revenue side, ω is the wage in the final good sector, ω_R is the wage in the R&D sector, b is unemployment benefit, R is the capital rental rate, r is the payoff on the loans, and T are lump-sum taxes. We assume the household is the residual claimant of the profits generated by the firms in the intermediate and final good sectors, (Π^i, Π^f).

Equation (24) is the law of motion of employment in the final good sector, where $\delta_l \in (0, 1)$ is the exogenous job destruction rate in the sector and $f\left(\theta\right)$ is the probability of finding a new job for the unemployed expressed as a function of labor market tightness. The sunk cost nature of the

[11]While, in principle, this formulation would imply a positive retraining cost also if the chosen net flow of workers into the R&D sector was negative, the sunk cost nature of the training cost means that in equilibrium it is always optimal for the household to choose $n \geq 0$.

training cost implies that the fraction of workers specialized in R&D labor evolves dynamically according to the law of motion that is given by Equation (25), even though the market for R&D labor is spot. $\delta_{lR} \in (0, 1)$ is the exogenous job destruction rate in the R&D sector. The decision rules that solve household's problem are functions

$$c = h_c\left(\Omega^{\star}\right), \tag{28}$$

$$k' = h_k\left(\Omega^{\star}\right), \tag{29}$$

$$l' = h_l\left(\Omega^{\star}\right), \tag{30}$$

$$l_R = h_{l_R}\left(\Omega^{\star}\right), \tag{31}$$

$$n = h_n\left(\Omega^{\star}\right), \tag{32}$$

$$a' = h_a\left(\Omega^{\star}\right). \tag{33}$$

3.5 Government

The government finances unemployment benefits by collecting lump-sum taxes as to maintain balanced budget in every period

$$(1 - L - L_R)\, b = T, \tag{34}$$

where unemployment benefits, b, are assumed proportional to the current innovation frontier given by the number of intermediate good varieties that are available as inputs in production to an appropriate power

$$b = \bar{b} N^{\frac{1}{1-\alpha}}, \tag{35}$$

where $\bar{b} > 0$ is a parameter. This assumption makes unemployment benefits proportional to the job productivity and insures existence of a well-defined balanced growth path.

3.6 Frictional labor market and wage determination in the final good sector

The market for labor in the final good sector is characterized by search and matching frictions. Employment relationship consists of a worker and a final good producer who engage in production until the relationship is severed. As in Cahuc and Wasmer (2001) and Elsby and Michaels (2013), final good producers need to post job vacancies one period ahead of production in order to recruit workers. Wages, however, are determined by bargaining between the firms and the workers only after production (and the TFP shock) is realized. In such framework, firm's employment is a predetermined variable but the wage rate and firm's demand for renting capital are not predetermined.

Each job can be in one of two states, filled or vacant. On the household side, workers can be employed in the final good or R&D sector or unemployed and searching for a job. The total number of vacancies posted by the firms is v. Let $u \equiv 1 - L - L_R$ be the measure of the pool of unemployed workers searching for a job. Following Mortensen and Pissarides (1994) we model the number of new hires, m, as a function of the number of vacancies and the number of searching workers

$$m = m(v, u), \tag{36}$$

where $m(\cdot)$ is the matching function that is homogeneous of degree one, increasing in each of its arguments, concave, and continuously differentiable. Let $\theta \equiv v/u$ be the labor market tightness. Then, the probability for a final good producer to fill a vacancy is

$$q(\theta) \equiv m\left(1, \frac{1}{\theta}\right), \tag{37}$$

with $\partial q/\partial \theta < 0$. Similarly, the probability for an unemployed worker to find a job is

$$f(\theta) \equiv m(\theta, 1), \tag{38}$$

with $\partial f/\partial \theta > 0$. As is clear from problems (3) and (22), both final good producers and workers take functions f and q as given and form anticipation on θ in function of the aggregate state according to equation (9).

A realized job match yields economic surplus that is shared between the firm and workers. We assume that the wage that pins down the surplus sharing is determined by a Nash bargain. The bargaining takes place after the realization of the current-period aggregate TFP shock. In particular, let $J(\cdot)$ and $H(\cdot)$ be, respectively, the firm's final surplus and worker's final surplus as a function of the bargained wage and the aggregate state, and let $\xi \in (0, 1)$ be the worker's bargaining power.

Then the bargained wage is

$$\omega = \arg\max_{\omega} G\left\{H(\omega,\Omega), J(\omega,\Omega); \xi\right\}, \tag{39}$$

where $G(\cdot)$ is the joint surplus written as a Nash product.[12] Given the firm's problem (3), $J(\cdot)$ can be expressed as the Lagrange multiplier of the law of motion for the labor input (4). Similarly, $H(\cdot)$ can be expressed as the Lagrange multiplier of the law of motion of employment (24) in the household's problem (22).[13] A solution to problem (39) is the negotiated wage that is a weighted arithmetic average of reservation wages

$$\omega = \xi\overline{\overline{\omega}}(\Omega) + (1-\xi)\underline{\underline{\omega}}(\Omega), \tag{40}$$

where $\overline{\overline{\omega}}(\Omega)$ is the highest wage that the final good producer is willing to pay for each additional worker and $\underline{\underline{\omega}}(\Omega)$ is the lowest wage at which an unemployed would be willing to accept a new job in state of nature Ω.

3.7 Timing

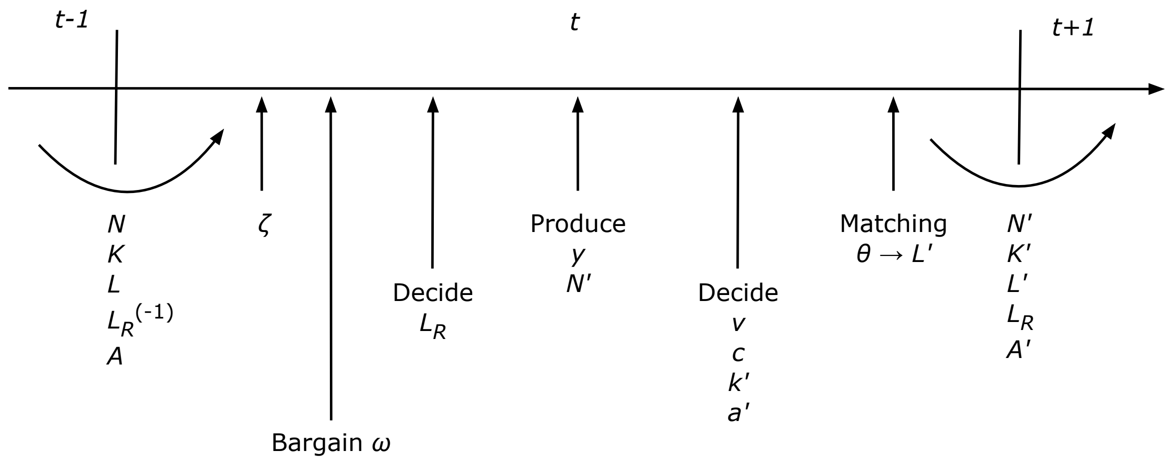
Figure 6 illustrates the timing of the model. The economy starts period t with the vector of predetermined endogenous state variables $\left(N, K, L, L_R^{(-1)}, A\right)$ and a realization of the exogenous aggregate TFP shock ζ. Then final good sector firms and their employees bargain over the current wage ω. Simultaneously, firms rent the available physical capital and purchase the available intermediate goods for inputs in production. Innovators hire labor and use the available loans to finance final good expenditures as inputs in R&D. Next, production y is realized and the R&D activity determines the number of intermediate good varieties available for use as inputs in production in the next period N'. Once production is determined and production factors are paid, firms decide the number of vacancies to post v. The currently unemployed workers search for jobs and the number of new realized hires depends on the current labor market tightness θ. The matching process together with exogenous destruction of a part of the already existing jobs,

[12] See Ljungqvist and Sargent (2012, p. 1194).

[13] See Gertler and Trigari (2009) for a similar treatment of frictional labor market in a dynamic stochastic general equilibrium model.

determines the next period employment level L'. Finally, households decide the desired levels of consumption, investment in physical capital, and investment in R&D. The investment determines stock of physical capital and the quantity of final good for R&D input available for renting in the next period (K',A').

Figure 6: Model Timing



3.8 Equilibrium

We are now ready to define an equilibrium.

Definition. A recursive equilibrium is collection of value functions $V(l,\Omega), W(i;\Omega), Q(k,a,\Omega)$ and decision rules $f = \{f_{x(i)}(l,\Omega), f_k(l,\Omega), f_l(l,\Omega), f_v(l,\Omega), f_{l_R}(\Omega), f_a(\Omega)\}$, $g = \{g_p(i;\Omega)\}$, and $h = \{h_c(\Omega^\star), \ h_k(\Omega^\star), h_l(\Omega^\star), h_{l_R}(\Omega^\star), h_n(\Omega^\star), h_a(\Omega^\star)\}$, government fiscal policy $T = \{b(\Omega), \ \tau(\Omega)\}$, collection of pricing and aggregator functions $P = \{P(i,\Omega), R(\Omega), r(\Omega), \omega(\Omega), \omega_R(\Omega), \theta(\Omega), L_R(\Omega)\}$, and law of motion for the aggregate state $G_\Omega(\Omega)$ such that

- given P, $G_\Omega(\Omega)$, and T, the final good producers and the intermediate goods producers maximize their profits, and the households maximize their utility i.e. the value functions solve problems (3), (14), and (22) and f, g, and h are the associated decision rules,
- given P, and $G_\Omega(\Omega)$, the labor and final good input demand in the R&D sector is determined by the free entry equilibrium conditions (18) and (19),

- given P, $G_\Omega(\Omega)$, and T, the fiscal policy balances government's budged, i.e. equation (34) holds,

- the wage rate in the final good sector is determined by the bargaining between the firms and workers, i.e. $\omega(\Omega)$ solves (39),

- aggregate consistency holds:

  (i) the law of motion of aggregate state is consistent with the individual decision rules, the markets for labor in the R&D sector and for capital clear, and the mass of filled vacancies in the production sector corresponds to the mass of workers with job in that sector

$$G_\Omega(\Omega) = \begin{pmatrix} \exp\left[(1-\rho)\ln\bar{\zeta} + \rho\ln\zeta + \epsilon_\zeta\right] \\ (1-\delta_N) N + \chi N h_{l_R}\left(\Omega^\star\right) A(\Omega) \\ h_k\left(\Omega^\star\right) \\ h_l\left(\Omega^\star\right) \\ h_{l_R}\left(\Omega^\star\right) \\ h_a\left(\Omega^\star\right) \end{pmatrix} = \begin{pmatrix} \exp\left[\left[(1-\rho)\ln\bar{\zeta} + \rho\ln\zeta + \epsilon_\zeta\right]\right. \\ (1-\delta_N) N + \chi N L_R(\Omega) A(\Omega) \\ f_k(L,\Omega) \\ f_l(L,\Omega) \\ f_{l_R}(\Omega) \\ f_a(\Omega) \end{pmatrix}, \tag{41}$$

  (ii) the pricing function for intermediate goods is consistent with the intermediate goods producer's decision rule and the market tightness is consistent with the hiring decisions

$$P(i;\Omega) = g_p(i;\Omega), \tag{42}$$
$$\theta(\Omega) = \frac{f_v(L,\Omega)}{1-L-L_R(\Omega)}, \tag{43}$$

  (iii) the market for final good clears

$$\zeta\left[\int_0^N f_{x(i)}(L,\Omega)\,di\right]^{1-\psi}\left(K^\alpha L^{1-\alpha}\right)^\psi - \int_0^N g_p(i;\Omega)\, f_{x(i)}(L,\Omega)\,di = h_c\left(K,L,L_R^{(-1)},A,\Omega\right) + h_k\left(K,L,L_R^{(-1)},A,\Omega\right) - (1-\delta_K)K + h_a\left(K,L,L_R^{(-1)},A,\Omega\right) + \kappa\left[(1-\delta_N)N + \chi N L_R(\Omega) A(\Omega)\right]^{\frac{1}{1-\alpha}} f_v(L,\Omega). \tag{44}$$

3.9 Balanced growth path

The non-stochastic version of the model features a well-defined balanced growth path along which the mass of available varieties of intermediate inputs N grows at a constant endogenous rate $\bar{g}_N$. The final good output, consumption, stock of capital, goods expenditures in R&D investment, and wages in both final good and R&D sectors grow at constant rate $\bar{g}_y = \bar{g}_N^{\frac{1}{1-\alpha}}$. The share of workers employed in the R&D sector, unemployment rate, labor market tightness, prices of intermediate goods, rental price of capital, and interest rate are constant. We use extensively the balanced growth path in calibration by requiring that, along the balanced growth path, the model matches a set of empirical moments.

Moreover, by dividing any growing variable x by an appropriate factor, the model rewritten in terms of normalized variables $\tilde{x} \equiv x/N^{\frac{1}{1-\alpha}}$ becomes difference-stationary.[14] This facilitates solving the model. When conducting the simulations, we accumulate and add back all effects of changes in N in order to obtain simulated data in levels before calculating any statistics.

4 Calibration

We solve the model numerically using perturbation methods.[15] In this section we describe the calibration of the model's parameters. In the next section we proceed to numerical simulations and discuss the ability of the model to generate medium-term cycles. We use two distinct calibrations. The baseline calibration considers model time period length of one quarter as in much of the business cycle literature. The second parameterization (annual calibration) is based on model time period length of one year. We opt for using two different calibration strategies because fluctuations we are modeling are considerably more persistent than typically considered in the macroeconomics-labor literature and the time aggregation of the quarterly calibrated model is not straightforward. Comparing the results of the yearly simulated model to those of the quarterly model also provides a consistency check of the mechanism of interaction of R&D investment and labor market frictions over the medium-term cycles.

[14] The transformed model is not trend-stationary because any changes in level of N have permanent effects on the trend.

[15] We use log-linearization around a deterministic balanced growth path and Dynare software package to obtain an approximation to the solution of the system of equations characterizing the equilibrium.

4.1 Baseline calibration

There are in total 20 parameters to calibrate. Our strategy is to calibrate a first set of nine parameters to the values provided by independent studies and the standard practice in quantitative macroeconomics literature. The final good production function parameter α is chosen to replicate the capital income share of $1/3$. The per-period utility function is assumed to have constant relative risk aversion form $u(c) = \left(c^{1-\sigma} - 1\right)/(1-\sigma)$, where we chose the value of 2 for the coefficient of relative risk aversion, which implies greater risk aversion that logarithmic utility.[16] The capital depreciation rate δ_k is set to 2.6 percent (approximately 10 percent per annum) in reference to the standard practice in the quantitative macroeconomics literature (see e.g. Chugh, 2016). Following Li and Hall (2020), the obsolescence rate of new technologies, δ_N, is set equal to 6.9 percent (~25% per annum). This value is also in line with the main conclusions from Pakes and Schankerman (1984) and Huang and Diewert (2011). The gross growth rate of the innovation possibilities frontier along the balanced growth path is given by $g_N = 1 - \delta_N + \chi\,(l_R)^{\iota}\,(a)^{1-\iota}$, where l_R is the number of workers in the R&D sector, and parameter ι denotes the elasticity of g_N relative to l_R. Following Goolsbee (1998), we fix the value for ι to 0.667. The technology coefficient associated with the innovation process is normalized to one ($\eta = 1$). Using the balanced growth path restrictions, the average of the TFP shocks process is set to one and the separation rate in the R&D sector is set to 0.027. Finally, we specify the parameter governing the worker's bargaining power $\xi = 0.45$, which is near the midpoint of the range of values in the macroeconomics-labor literature.[17]

The values of the remaining eleven parameters are set to allow the model to replicate selected empirical moments along the balanced growth path. All the moments we use as calibration targets are quarterly averages in the U.S. over the period 1951Q01–2006Q01. We choose the value for the discount factor β to match the annual long-term real interest rate of 5.0 percent in reference to the average annual return on the S&P500 index.[18] The elasticity of the production function with respect to intermediate inputs matches the investment-GDP ratio.[19] The curvature parameter of

[16] A value of 2 is also chosen in Walsh (2005). A plausible range of alternative values for σ varies between 0.5 and 2.5 (see Table 6.1 in DeJong and Dave (2005) for more details). Aghion and Howitt (1994), Mortensen (2005) among others show a value for σ greater than one guarantees the capitalization effect of growth.

[17] See, for instance, Shimer (2005), Hagedorn and Manovskii (2008), Hall and Milgrom (2008), Gertler and Trigari (2009), Gervais et al. (2015), Petrosky-Nadeau, Zhang, and Kuehn (2018), and Drautzburg, Fernández-Villaverde, and Guerrón-Quintana (2021).

[18] DeJong and Dave (2005), Hornstein, Krusell, and Violante (2005), among others, use the same value.

[19] Gomme and Rupert (2007) advocate the use of investment-GDP ratio rather than capital-output ratios. The periodic

Table 5: Baseline calibration. U.S. quarterly data: 1951–2006

1. External Calibration			
Parameter	Description	Parameter value	Source
α	Capital share in the output	0.333	National Income (BLS)
σ	Relative risk aversion	2.000	Mortensen (2005)
δ_k	Capital depreciation rate	0.026	$\sim$ 10% per annum (BEA)
δ_N	Technology depreciation rate	0.069	Li and Hall (2020)
ι	R&D labor elasticity	0.667	Goolsbee (1998)
η	Technology coefficient	1.000	Normalization
δ_R	Separation rate in R&D sector	0.027	BGP restriction
$\overline{\zeta}$	Average TFP process	1.000	BGP restriction
ξ	Worker's bargaining power	0.450	Midrange value in macro-labor

2. Internal Calibration				
Parameter	Target (U.S. data, 1951:01–2006:01)	Parameter value	Target data	Target model
β	Real interest rate	0.998	0.0127	0.0127
ψ	Investment-GDP ratio	0.692	0.2067	0.2159
ϕ	Average GDP growth	0.546	1.0054	1.0054
$\overline{c}_R$	R&D workers share (NSF, ν_l)	1.1e6	0.0058	0.0058
δ_l	Average job-finding rate	0.027	0.4529	0.4529
γ	Average labor market tightness	1.371	0.9746	0.9746
κ	Vacancy filling cost in % of wage	0.050	0.1400	0.1400
$\overline{b}$	Average unemployment rate	1.057	0.0566	0.0566
Downhill simplex minimization algorithm:				
ρ_ζ	Persistence TFP from Fernald (2014)	0.931	0.9296	0.9000
$\sigma_{\epsilon\zeta}$	Std. dev. TFP from Fernald (2014)	0.007	0.0126	0.0126
N_0	First observation of TFP level in log	3.203	3.1186	3.1186

the law of motion for the technology frontier ϕ is set so that the model reproduces the average output growth rate of 0.54 percent (approximately 2.16 percent per annum). Parameter $\overline{c}_R$ is set so that the model matches the aggregate share of workers employed in R&D sector. We use the data from the National Science Foundation (NSF) on employment in R&D. For calibrating our model we consider workers in R&D sector as the number of people employed by R&D-performing

revisions to the capital stock data are so large, and the conceptual questions about what should be included in the capital stock are so difficult to satisfactorily answer, that estimates of the capital-output ratios are too unreliable to use as calibration target.

companies who were engaged in scientific or engineering work at a level that required knowledge gained either formally or by experience. The separation rate δ_l in the final good sector ensures an average value for the job finding rate of 0.4529 as estimated from the Robert Shimer's database. We follow den Haan, Ramey, and Watson (2000) and choose the matching function of the form $m(u, v) = uv/(u^\gamma + u^\gamma)^{1/\gamma}$. This specification ensures that matching probabilities always lie in $[0, 1]$ while retaining the properties of monotonicity, concavity, and constant returns to scale. We calibrate the value of parameter γ to match the average value of vacancy–unemployment ratio of 0.975. We obtain this last value from den Haan and Kaltenbrunner (2009) who use JOLTS data and the Help Wanted index to estimate quarterly number of vacancies. As in Elsby and Michaels (2013), the vacancy posting cost κ is targeted to per worker hiring cost of 14 percent of quarterly worker compensation. We set the value of non-market activity for the workers $\overline{b}$ to match the average value for the unemployment rate of 5.66 percent. The targets for the calibration of the persistence (ρ_ζ) and volatility ($\sigma_{\epsilon\zeta}$) of the AR(1) process governing the productivity shocks are the first-order auto-correlation and volatility of the high-frequency component of Fernald's (2014) TFP series. Because the (value-added) TFP is endogenous in our model, we chose the values for these parameters that minimize the distance between the moments in the simulated data and their empirical counterpart. Finally, we set the initial mass of available intermediate goods in period zero (N_0) to match the TFP level in the first quarter of 1951 as measured by Fernald (2014).

There are two possibly less standard aspects in our calibration, which seem important for quantitative implications of the model. The first is the calibration of the innovation technology. We used available estimates in recent economic literature, the average GDP growth rate and the average share of workers employed in R&D over our sample period to assign values to the parameters governing R&D investment productivity and the incentives to become R&D-specialized worker. It is reassuring that the calibration implies the model is able to closely match the non-targeted average ratio of R&D investments to GDP along the balanced growth path, as can be seen in Table 6. Second, the calibration of the value of non-market activity for workers can have important influence on the ability of the model to amplify the effects of fluctuations on the labor market quantities. While not explicitly targeted, our calibration implies the value for the replacement ratio of 0.9. Reassuringly, this is in the ballpark of values used in the recent literature.[20]

[20]Although our replacement ratio is larger than the value of 0.75 in Pissarides (2009), it is relatively close to 0.85 in

Table 6: Non-targeted Moments

	Value	
Moments	Data/Literature	Model
R&D/GDP	0.0245	0.0274
Replacement ratio	0.75 – 0.97	0.899

4.2 Alternative calibration

Following Comin and Gertler (2006) we also consider an annual calibration. The moments used as calibration targets are now annual averages in the U.S. data (1953–2006). As summarized in Table 7, we use the same targets as in our baseline calibration for most parameters, just adequately adjusted for annual observations. The annual job finding and separation rates are obtained from the observed average quarterly job finding and separation rates provided in by Robert Shimer by following the transition probability tree over four quarterly sub-periods. This yields $f = 0.8547$ and $\delta_l = 0.052$ at the annual frequency. Differently from the quarterly calibration, the worker's bargaining power parameter ξ is now internally calibrated. We set its value to 0.0424 in order to obtain the same replacement ratio of 0.9 as implied by the quarterly calibration. The annual series for TFP used from which are calculated the target moments for calibration of ρ_ζ, $\sigma_{\epsilon\zeta}$, and N_0 is from BLS.

5 Results

We follow the RBC literature by examining how well a single shock to TFP, presumed to be the principal driving force of economic fluctuations, can explain the unconditional patterns in the data. The RBC literature focused on the ability of technology shocks to account for short-term fluctuations. We instead explore the ability of our model to account for both short-term and medium-term fluctuations.

Petrosky-Nadeau, Zhang, and Kuehn (2018) or to 0.88 as estimated by Christiano, Eichenbaum, and Trabandt (2016). Our value for the replacement ratio is, however, lower than the value used by Hagedorn and Manovskii (2008).

Table 7: Alternative calibration. U.S. annual data: 1953–2006

1. External Calibration			
Parameter	Description	Parameter value	Source
α	Capital share in the output	0.333	National Income (BLS)
σ	Relative risk aversion	2.000	Mortensen (2005)
δ_k	Capital depreciation rate	0.100	BEA
δ_N	Technology depreciation rate	0.250	Li and Hall (2020)
ι	R&D labor elasticity	0.667	Goolsbee (1998)
η	Technology coefficient	1.000	Normalization
δ_R	Separation rate in R&D sector	0.052	BGP restriction
$\overline{\zeta}$	Average TFP process	1.000	BGP restriction

2. Internal Calibration				
Parameter	Target (U.S. data, 1953–2006)	Parameter value	Target data	Target model
β	Real interest rate	0.994	0.0500	0.0500
ψ	Investment-GDP ratio	0.694	0.2068	0.2142
ϕ	Average gross output growth	0.261	1.0216	1.0216
$\overline{c}_R$	R&D workers share (NSF, ν_l)	9.8e4	0.0058	0.0058
δ_l	Average job-finding rate	0.052	0.8547	0.8547
γ	Average labor market tightness	0.890	0.9389	0.9389
ξ	Replacement ratio	0.042	0.9000	0.9000
κ	Vacancy filling cost in % of wage	0.824	0.4530	0.4530
$\overline{b}$	Average unemployment rate	0.350	0.0573	0.0573
Downhill simplex minimization algorithm:				
ρ_ζ	Persistence of TFP (BLS)	0.820	0.1055	-0.0492
$\sigma_{\epsilon\zeta}$	Std. dev. of TFP (BLS)	0.012	0.0116	0.0116
N_0	First observation of TFP level in log	4.036	3.9884	3.9884

5.1 Impulse response functions

We first analyze impulse response functions to gain some insight into the endogenous propagation mechanism of the model. In order to highlight the importance of the endogenous growth for the propagation and amplification of shocks we report two sets of impulse response functions in Figure 7. One generated from our benchmark model with endogenous growth (solid lines) and another generated from a model without R&D sector and with constant mass of one of the varieties of intermediate goods (dashed lines). In that version the long-run growth is generated by exogenous constant growth in the TFP of the final good production sector. This model thus corresponds to a standard RBC model with search and matching frictions in the labor market (RBC-SM). We re-calibrate the RBC-SM model to match the relevant subset of the same empirical moments as our benchmark model. The magnitude of the shock is of one percent deviation from the BGP and the impulse responses are depicted as percentage deviations from the models' balanced growth paths.

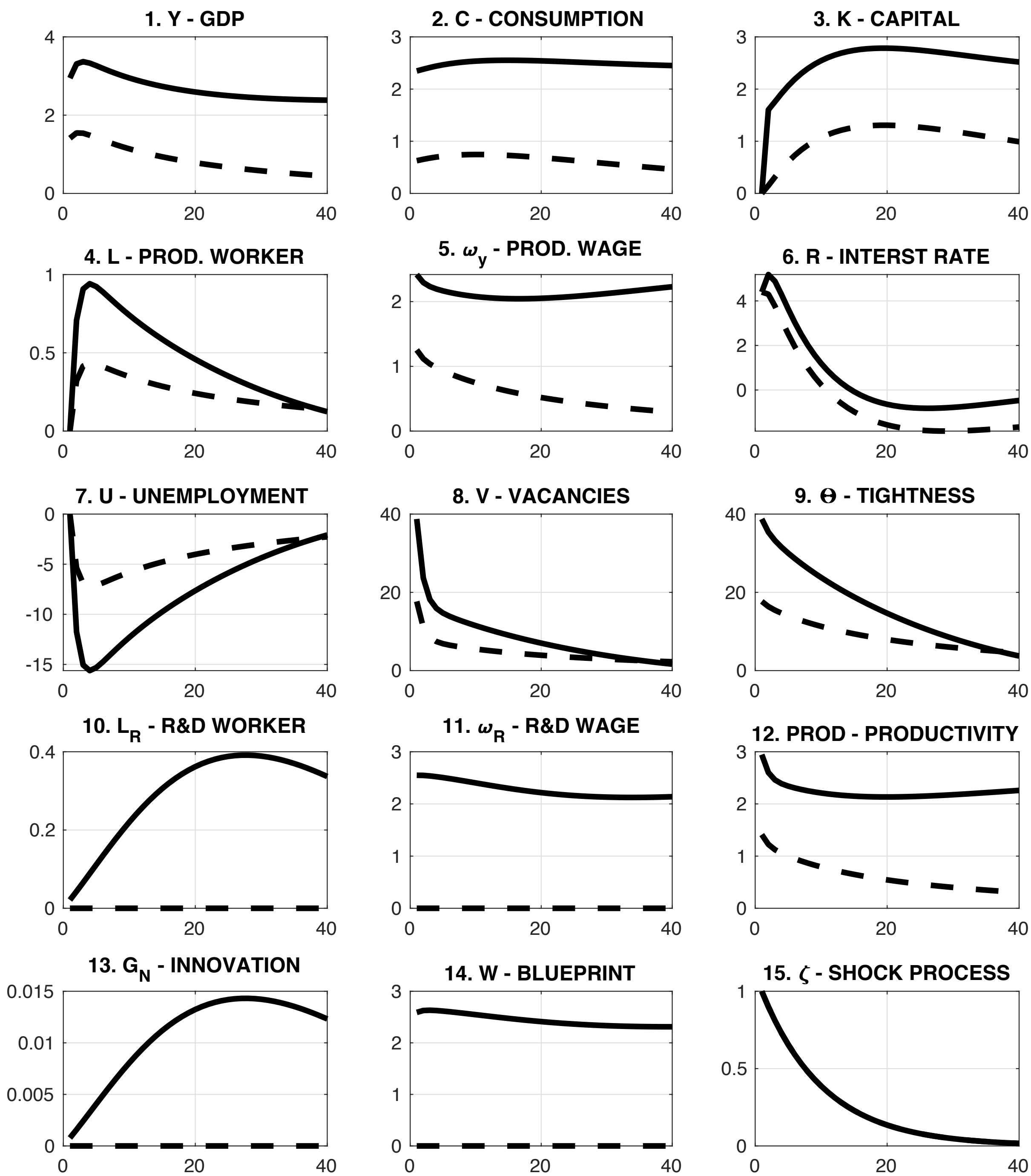
A positive TFP shock raises the productivity of intermediate inputs, capital, and labor in the final good production sector. As a result, for given prices, the demand for intermediate inputs and capital would increase. In equilibrium, this leads to a higher quantity of each existing variety of intermediate goods being used as input in production of the final good and to an increase in the price index of intermediate inputs. Notice however that on impact there is not an immediate increase in the number of varieties of intermediate goods since this mass is predetermined by previous period R&D efforts. Similarly, given that on impact the stock of capital is predetermined by previous period investment decisions (panel 3), the rise in demand for capital is entirely compensated by a sharp increase in the equilibrium rental rate of capital (panel 6). Because hires of workers by the final good firms are also predetermined, the quantity of labor input in the final good sector stays also unchanged on impact (panel 4). Nevertheless, the current jobs create a higher surplus to be shared between the firms and workers reflecting into a higher wage rate in the final good sector (panel 5).

Because the stochastic process governing TFP shocks is persistent, a high realization of the shock today creates expectations of higher productivity shocks also in the following periods. This increases the expected future productivity of intermediate inputs, capital, and labor, leading to a rise in the value of intermediate goods producer firms (panel 14), investment and future capital stock (panel 3), and the final good firms' incentives to post vacancies (panel 8). The higher value of intermediate goods firms spurs more innovation (panel 13) by increasing incentives to commit more resources to R&D (panel 10). In particular, the innovators demand more of specialized R&D labor. As a result, more workers become R&D specialists instead of searching for a job in the final good sector, the unemployment falls, and wages in the R&D sector rise (panel 11).[21]

Moreover, a TFP shock produces effects similar to those of news about future technologies.

[21]The value of L_R and U are modified already on impact but the magnitudes are small relative to the following periods rendering it difficult to clearly visualize in Figure 7.

Figure 7: Impulse Response Functions



Note: Figure 7 plots the model impulse responses to a TFP shock of one percent deviation from the BGP. The solid line in each panel depicts the response in our benchmark model with endogenous growth. The dashed line depicts the response in the RBC-SM model. The y-axis scale is in percentage deviations from the balanced growth path. The x-axis scale is the time horizon. The period length is one quarter.

The higher discovery of new varieties of intermediate inputs further enhances productivity of the final good sector in the subsequent periods, triggering a powerful feedback effect that leads to large and persistent responses of macroeconomic and labor variables to productivity fluctuations. This amplification mechanism generates responses of much larger magnitude in both the impact period and over the subsequent periods compared to those in the RBC-SM model in which the mechanism is absent.

Finally, notice that, in the benchmark model, GDP, consumption, capital stock and wages do not go back to their original balanced growth path levels in the long run. Instead, they stay permanently higher on a new balanced growth path levels. This is a consequence of a permanent increase in productivity due to the rise of the number of available intermediate input varieties. In contrast, in the RBC-SM model the variables simply revert to their original balanced growth path levels in the long run.

5.2 Medium-term cycles

We now quantitatively evaluate the capacity of the model to generate the medium-term fluctuations patterns observed in the U.S. data. We use simulation of the model to generate artificial time series of the same length as the available U.S. data. We generate 1,000 such simulations and for each simulation we compute counterparts to the empirical moments studied in Section 2. We then report average values of these statistics and compare them to the empirical evidence.[22] As when studying the impulse response functions, we also report the statistics generated by simulations of the exogenous growth model (RBC-SM) for comparison purposes.

Volatility Table 8 reports the percent standard deviations of the variables over the medium-term cycle, and its high-frequency component (the traditional business-cycle). Overall, the benchmark model does a reasonably good job in capturing the breakdown of the variation between the high- and medium-frequency components. Importantly, the benchmark model does particularly well in reproducing the volatility of labor market quantities at both medium-term and business-cycle

[22]We treat the artificial data exactly as we do the real data. In particular, in each simulation, we re-accumulate non-stationary time series by adding trend growth back. We then apply the same transformations and procedures to the artificial data as those described in Section 2 to isolate medium-term cycle and its high- and medium-frequency components.

frequencies and this is in stark contrast with the poor performance of the RBC-SM model along this dimension. The main amplification mechanism behind this success is the interaction between the effects of the TFP shock on expected future productivity in the final good sector and the demand for specialized R&D labor. As we have explained with the impulse response functions, a positive TFP shock indirectly raises innovators' demand for specialized labor. This creates new opportunities to which the unemployed workers can turn instead of searching for a job in the final good sector, and thus decreases unemployment. As a result, the elasticity of labor market tightness to productivity in our benchmark model is higher than in the RBC-SM model, which lacks this mechanism. The fact that the rise in demand for specialized R&D labor is pro-cyclical and persistent leads to amplified and persistent responses of labor market quantities to TFP shocks. It is worth to emphasize that this mechanism is different from those proposed in the most of macro-labor literature. In particular, the higher elasticity of labor market tightness in our model is not obtained through a reduction of what Ljungqvist and Sargent (2017) call the fundamental surplus of a job.

For other variables, the quarterly calibrated benchmark model generates volatility that is relatively close to that observed in the data over both the medium-term cycle and its high-frequency component with exception of R&D, which is less volatile in the model than in the data. The performance of the benchmark model is slightly better than that of the RBC-SM model, especially over the medium-term cycle. This difference is more apparent for the annually calibrated versions which also do a poorer job in reproducing the volatility of labor productivity and wages.

Persistence Table 9 shows that both the benchmark and the RBC-SM model are able to generate high first-order auto-correlation on the quarterly basis. However, examining more in detail the auto-correlograms for GDP and unemployment in Figure 8 reveals that the auto-correlation of the medium-term cycle decays faster in the RBC-SM model than in the benchmark model. This is also clearly visible in the bottom panel of Table 9 which shows that on annual basis, the RBC-SM model is not able to produce sufficient persistence in the medium-term cycle.

Cyclicality Figure 9 shows that the model captures quite well the cyclical co-movements of variables in the data at both the business cycle and medium-term cycle frequencies. However, in the medium-term cycle, labor productivity, vacancies, and wages tend to be more pro-cyclical in

Table 8: Data vs Model: Standard Deviations

	Medium-term cycle			High-frequency component		
	Data	Benchmark	RBC-SM	Data	Benchmark	RBC-SM
Quarterly Data						
GDP	3.30	3.53	2.49	1.49	1.52	1.28
Consumption	2.25	2.67	1.13	0.75	0.90	0.35
Investment	8.07	7.36	6.77	4.15	3.72	3.85
TFP	2.90	2.36	2.12	1.26	1.26	1.26
R&D	8.87	2.95	···	3.28	1.06	···
Productivity	3.27	2.45	2.28	1.25	1.13	1.20
Wage	2.95	2.26	2.20	1.03	1.00	1.15
Weighted wage[a]	···	2.25	···	···	0.98	···
Unemployment	21.99	20.91	3.91	12.16	8.44	1.81
Vacancies	23.77	27.24	5.29	13.45	15.48	3.33
Tightness	42.44	45.28	8.56	25.20	20.47	4.39
Annual Data						
GDP	3.50	3.00	1.30	1.38	1.12	1.07
Consumption	2.33	2.19	0.39	0.69	0.76	0.17
Investment	8.42	5.50	4.07	3.96	2.14	3.52
TFP	3.00	2.02	1.32	1.16	1.16	1.16
R&D	7.66	2.54	···	2.37	0.91	···
Productivity	3.31	1.98	1.29	1.12	1.15	1.10
Wage	3.12	0.77	0.62	0.98	0.32	0.50
Weighted wage	···	0.79	···	···	0.33	···
Unemployment	21.50	22.71	1.95	11.26	9.37	1.12
Vacancies	25.52	29.31	3.15	14.00	21.60	2.73
Tightness	43.37	41.77	3.66	24.30	18.70	2.26

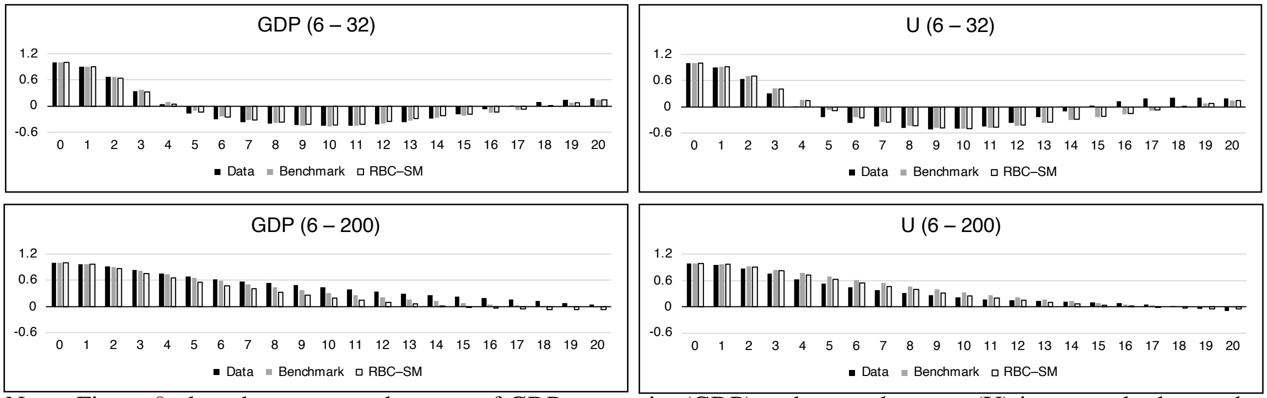
Note: The reported model standard deviations are average statistics over 1,000 simulations of a sample size corresponding to the data. [a]Weighted wage is the average wage across final good and R&D sectors weighted by the employment shares of each sector.

Table 9: Data vs Model: First-Order Autocorrelations

	Medium-term cycle			High-frequency component		
	Data	Benchmark	RBC-SM	Data	Benchmark	RBC-SM
Quarterly Data						
GDP	0.978	0.993	0.965	0.908	0.907	0.897
Consumption	0.989	0.994	0.985	0.922	0.902	0.910
Investment	0.976	0.991	0.959	0.927	0.913	0.897
TFP	0.983	0.991	0.956	0.930	0.900	0.894
R&D	0.984	0.994	...	0.913	0.902	...
Productivity	0.983	0.993	0.962	0.898	0.887	0.892
Wage	0.989	0.993	0.975	0.928	0.918	0.917
Weighted wage[a]	...	0.993	...	...	0.890	...
Unemployment	0.967	0.993	0.975	0.903	0.918	0.917
Vacancies	0.970	0.992	0.937	0.919	0.861	0.860
Tightness	0.965	0.993	0.964	0.913	0.895	0.894
Annual Data						
GDP	0.822	0.793	0.123	0.118	0.105	-0.221
Consumption	0.889	0.791	0.714	0.216	-0.022	-0.009
Investment	0.766	0.810	0.047	0.227	0.285	-0.226
TFP	0.836	0.577	0.014	0.105	-0.048	-0.239
R&D	0.849	0.782	...	0.049	-0.022	...
Productivity	0.862	0.521	0.060	0.071	-0.163	-0.241
Wage	0.894	0.727	0.519	0.268	-0.002	-0.127
Weighted wage	...	0.704	...	...	-0.072	...
Unemployment	0.726	0.747	0.548	0.121	0.007	-0.117
Vacancies	0.691	0.276	-0.036	0.173	-0.227	-0.333
Tightness	0.683	0.710	0.480	0.145	-0.041	-0.162

Note: The reported model auto-correlations are average statistics over 1,000 simulations of a sample size corresponding to the data. [a]Weighted wage is the average wage across final good and R&D sectors weighted by the employment shares of each sector.

Figure 8: Data vs Model: Auto-correlograms



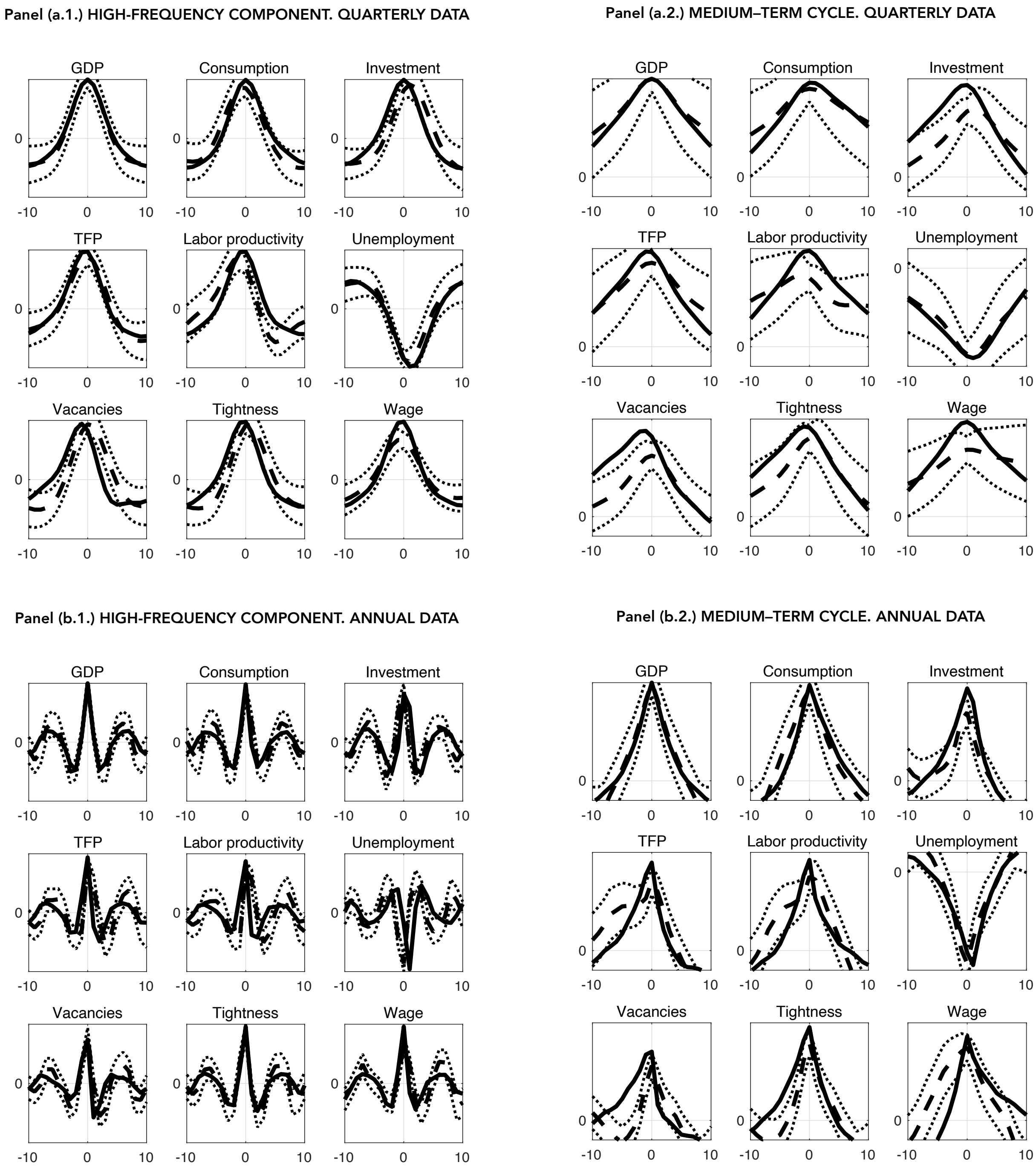
Note: Figure 8 plots the auto-correlograms of GDP per capita (GDP) and unemployment (U) in quarterly data and model simulated data for the high-frequency component (6-32) and the medium-term cycle (6-200). The reported model auto-correlations are average statistics over 1,000 simulations of a sample size corresponding to the data.

the model than in the data. The model also generates too much cross-correlation with past GDP per capita for investment, vacancies, and labor market tightness over the medium-term cycle.

Lead of R&D The R&D dynamics is important for amplification and propagation of shocks in our model. We now verify whether the co-movement of R&D with the key variables is also in line with the data. As we stressed in Section 2, taking in account the medium-term fluctuations in the annual data reveals a lead of R&D over several variables. In contrast, this lead is almost absent in the high-frequency component. Table 10 shows that the benchmark model is able to reproduce this pattern, to some extent, for GDP, consumption, TFP, labor productivity, and unemployment.

Figure 10 complements the evaluation by looking at detailed cross-correlograms of GDP per capita and unemployment rate with R&D at different leads in the quarterly real and simulated data. We can see that, in line with the data, the quarterly-calibrated benchmark model is able to generate a positive cross-correlation for up to 20 quarters' lead of R&D over GDP in the medium-term cycle and a negative cross-correlation for up to 20 quarters' lead of R&D over unemployment in the medium-term cycle. Moreover, these cross-correlations decay to zero by the lead of only 4 quarters in the high-frequency components. However, in the medium-term cycle, the model generates cross-correlations that are stronger than found in the real data, especially at closer leads.

Figure 9: Data vs Model: Cross Correlation with GDP per Capita



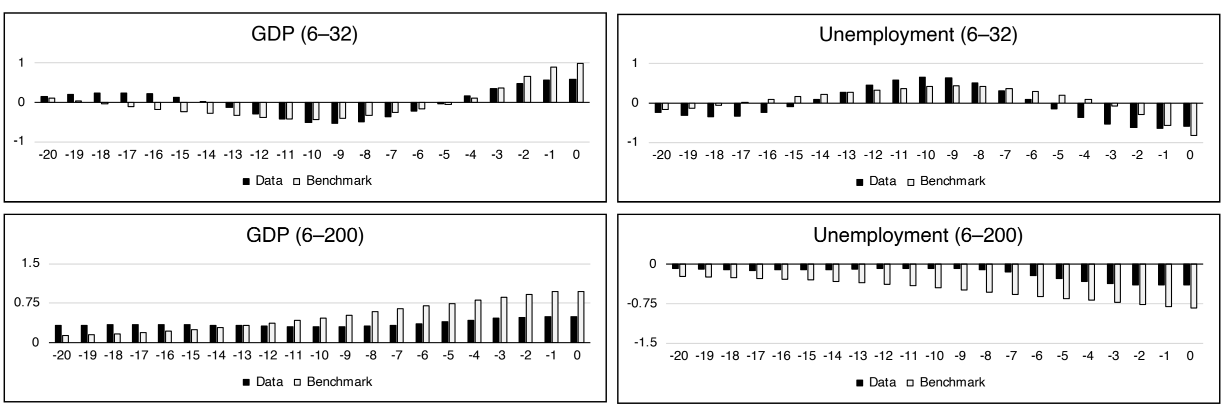
Note: Figure 9 plots the cross correlation with GDP per capita of the high-frequency component and the medium-term cycle at different leads and lags. The dashed lines depict the cross correlation in the data with 95–percent confidence bands (dotted lines). The solid lines depict the cross correlation in the benchmark model.

Table 10: Data vs Model: Cross-correlation with R&D at $t-5$

	Medium-term cycle		High-frequency component	
	Data	Benchmark	Data	Benchmark
Annual Data				
GDP	0.494	0.199	0.110	0.077
Consumption	0.590	0.115	0.055	0.074
Investment	0.074	0.260	0.202	0.106
TFP	0.518	0.258	-0.002	0.020
Labor productivity	0.616	0.168	-0.070	0.015
Wage	0.585	-0.089	-0.185	0.053
Weighted wage	···	-0.083	···	0.055
Unemployment	-0.308	-0.192	-0.087	-0.130
Vacancies	-0.115	0.283	0.027	-0.021
Labor market tightness	0.089	0.306	0.059	0.039

Note: The reported model cross-correlations are average statistics over 1,000 simulations of a sample size corresponding to the data. [a]Weighted wage is the average wage across final good and R&D sectors weighted by the employment shares of each sector.

Figure 10: Data vs Model: Cross-correlation of GDP and unemployment with R&D



Labor Market In order to evaluate how well the model is able to explain the key labor market relationships we summarize each of them by the slope of the regression line and the correlation coefficient implied by scatter plot of the relationship from the data. While the slope is informative about the elasticity of the relationship, the correlation coefficient is informative about its strength. Figure 11 shows the regression lines obtained from the real data (black) and the simulated data from the benchmark (red) and the RBC-SM (blue) models for the medium-term cycle (dashed) and its high-frequency component (solid). Table 11 then reports the associated slope and correlation coefficients.

In terms of the Beveridge curve (Panel (a) of Figure 11) both models seem to perform similarly, counter-factually generating a higher elasticity of vacancies to unemployment in the medium-term cycle than in its high-frequency component. The correlation between vacancies and unemployment in the simulated data is in line with the one observed in the real data in the medium-term cycle but it is much weaker for the high-frequency component. In terms of the Okun's law (Panel (b) of Figure 11) the benchmark model generates elasticity of unemployment to GDP that is close to that observed in the real data in the medium-term cycle. In contrast, this elasticity is too high in the RBC-SM model. In both models there is virtually no differential in the slope of the regression line for the medium-term cycle and for its high-frequency component. Looking at the correlation coefficients, we see that both models imply a strong relationship between the two variables, which is consistent with the real data, with a slightly better performance of the benchmark model. The relationship between labor productivity and labor market tightness (Panel (c) of Figure 11) in the benchmark model implies a higher elasticity of the tightness to productivity than in the data. However, the differential in the slope of the regression line between the medium-term cycle and its high-frequency component is quantitatively in line with the real data. In contrast, the RBC-SM model implies too low elasticity of tightness to productivity, which explains its failure to produce enough volatility in the labor market quantities. The RBC-SM model also fails to produce any significant differential in the value of this elasticity between the medium-term cycle and its high-frequency component. The correlation coefficients for this relationship implied by both models are too high compared to the data. This is typical in search and matching models models of the labor market with flexible wages. In the plain RBC-SM model there is a straightforward one to one relationship between labor productivity and labor market tightness. In the benchmark

Table 11: Data vs Model: Labor Market Relationships

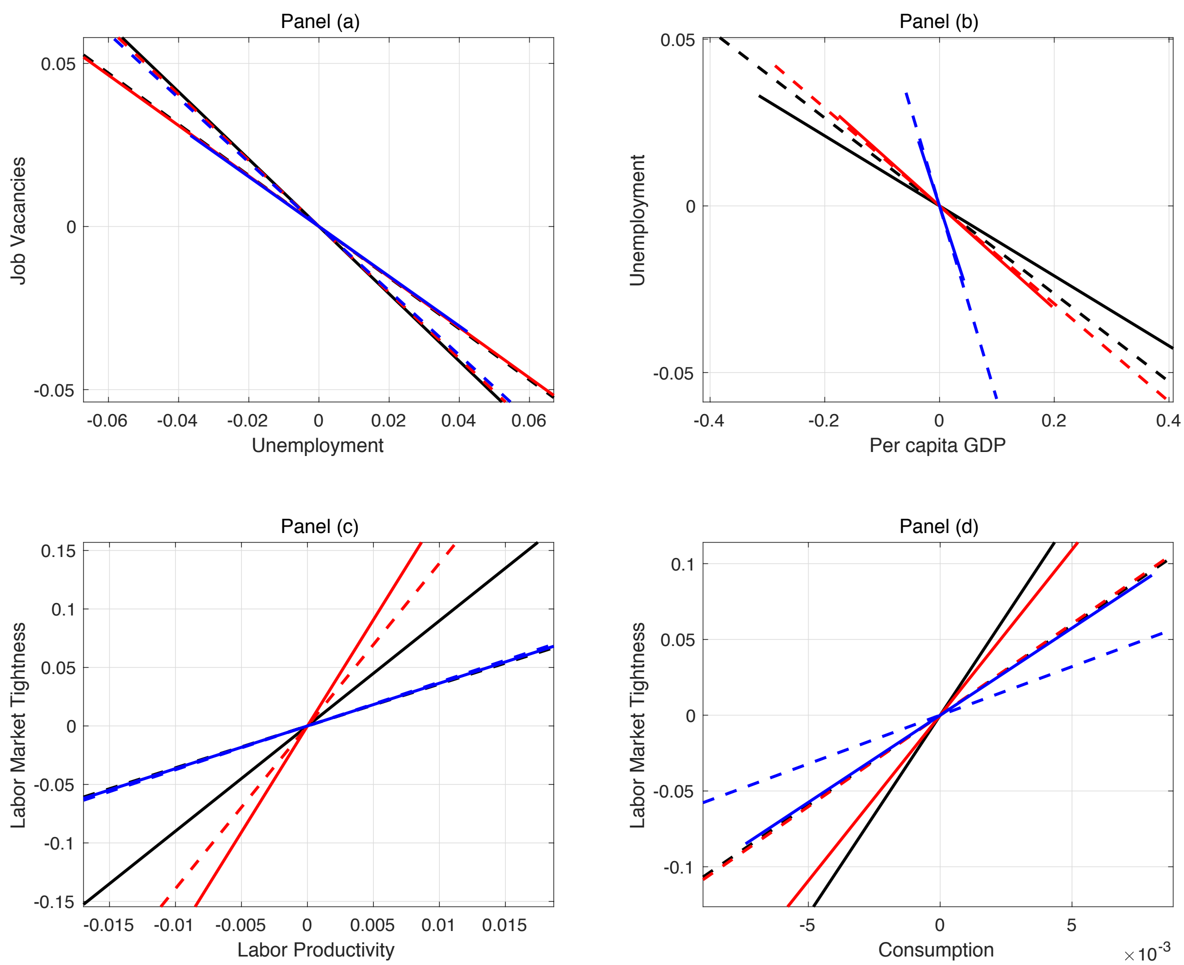
	Medium-term cycle			High-frequency component		
	Data	Benchmark	RBC-SM	Data	Benchmark	RBC-SM
Slope						
Unemp.–vacancies	-0.784	-1.012	-0.985	-1.032	-0.773	-0.763
GDP–unemployment	-0.132	-0.147	-0.581	-0.105	-0.154	-0.530
Productivity–thightness	3.581	13.911	3.745	8.993	18.161	3.650
Consumption–tightness	11.865	12.842	6.416	26.289	21.842	11.500
Correlation						
Unemp.–vacancies	-0.725	-0.754	-0.706	-0.933	-0.421	-0.411
GDP–unemployment	-0.878	-0.860	-0.900	-0.858	-0.851	-0.743
Productivity–thightness	0.276	0.770	0.999	0.445	0.997	1.000
Consumption–tightness	0.629	0.717	0.852	0.778	0.964	0.915

model, however, labor market tightness in the final goods sector is also affected by the number of workers who decide to specialize in R&D labor and the hiring incentives for the innovators. Because specializing in R&D labor involves quadratic adjustment costs, and past discoveries have permanent effects on the value of future innovation, the final goods labor market is affected by these effects more gradually which maps into a lower correlation between the labor productivity and the tightness in the medium-term cycle. Finally, for the relationship between labor market tightness and consumption (Panel (d) of Figure 11) the benchmark model implies elasticity between the two variables that are close to those in real data both in the medium-term cycle and its high-frequency component. In contrast, the elasticity implied by the RBC-SM model are is low. Both models imply slightly higher correlation coefficients than those observed in the real data for this relationship.

6 Conclusion

A growing body of literature stresses that in order to further improve our understanding of economic fluctuations, we need to consider the short-term behavior of the economy in connection to its longer-term dynamics. This paper provides an attempt to quantitatively evaluate the role of the R&D activity and innovation dynamics for explaining the short- and medium-term fluctuations of the

Figure 11: Data vs Model: Correlation Patterns for some Key Macro Variables.



Note: Figure 11 plots regression lines for correlation patterns for key labor market relationships. Solid lines correspond to the high-frequency component. Dashed lines correspond to medium-term cycle. Black lines correspond to the linear fit in the U.S. quarterly data. Red lines correspond to the linear fit in the data simulated with the benchmark model. Blue lines correspond to linear fit in the data simulated with the RBC-SM model.

labor market. Our results show that the expectations of the future evolution of productivity growth through innovation can significantly amplify and propagate the effects of current productivity shocks on the labor market variables. Many questions remain to be answered by future research, in particular concerning the implications for the conduct of labor market, innovation, and education policies.

To be completed...

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A Balanced growth path

The production function for the final good reads

$$y = A_{\#} N \zeta^{\frac{1}{\psi}} k^{\alpha} l^{1-\alpha}, \tag{45}$$

where $A_{\#} \equiv \left\{(1-\psi)^{1/\psi}\left[\mu/(1-\psi)\right]^{-1/\psi}\right\}^{1-\psi} > 0$. Next by taking Equation (45) in log, and then in first difference we get

$$(\ln y' - \ln y) = (\ln N' - \ln N) + \alpha(\ln k' - \ln k) + (1-\alpha)(\ln l' - \ln l), \tag{46}$$

where ζ is a stationary shock process. Along a BGP labor will grow at a rate $g_l = 1$. Capital will grow at the same rate as output because the real interest rate is constant. Using these facts, we find

$$(1-\alpha)(\ln y' - \ln y) = (\ln N' - \ln N). \tag{47}$$

Hence, along the BGP, output divided by the scaling factor $N^{1/(1-\alpha)}$ does not grow, i.e. it is stationary

$$y' N'^{-\frac{1}{1-\alpha}} = y N^{-\frac{1}{1-\alpha}}. \tag{48}$$

Thus we can induce stationarity in the production function by dividing both sides of the Equation (45) by the scaling factor $N^{1/(1-\alpha)}$:

$$\begin{aligned} \widetilde{y} &= A_{\#} \zeta^{\frac{1}{\psi}} N^{-\frac{\alpha}{1-\alpha}} \left[\frac{k}{N^{\frac{1}{1-\alpha}}}\right]^{\alpha} N^{\frac{\alpha}{1-\alpha}} l^{1-\alpha} \\ &= A_{\#} \zeta^{\frac{1}{\psi}} \widetilde{k}^{\alpha} l^{1-\alpha}, \end{aligned} \tag{49}$$

where $A_{\#} \equiv \left\{(1-\psi)^{1/\psi}\left[\mu/(1-\psi)\right]^{-1/\psi}\right\}^{1-\psi} > 0$. Equation (48) implies:

$$g_N = g_y^{1-\alpha}, \tag{50}$$

where g_y is the gross growth rate of output.