Taxing the Rich [*]

(PRELIMINARY AND INCOMPLETE)

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Abstract

Recently it has been argued that a progressive wealth tax may have large beneficial effects on the distribution of welfare in society and effectively no adverse effects on real economic activity. This paper quantitatively evaluates the merits of this view within a dynamic general equilibrium model in which wealth taxes distort the effort that managers expend and tilt their choice of projects towards less risky but also less innovative ventures. Our preliminary simulations show that even a simple version of the model accounts well for the increase in wealth inequality in the United States over the past 30 years. Such a model implies a substantial aggregate output loss from wealth taxes of the magnitude currently debated.

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1 Introduction

Recently, a number of authors have argued that a progressive wealth tax may have large beneficial effects on the distribution of welfare in a society and effectively no adverse effects on real economic activity (see Saez and Zucman 2019 and the references therein). This paper quantitatively evaluates the merits of this view in a model in which the accumulation of wealth plays a critical role in aligning the private incentives of a firm's top executives or entrepreneurs, referred to as *managers*, with those of the ultimate owners of a firm, a dimension that the optimal taxation literature has mainly ignored.

In our quantitative model, we find that highly progressive wealth taxes distort the incentives of top managers in a firm to build up expertise in managing the firm and, in contrast to the presumption of the advocates of such taxes, do indeed have large distortionary effects on the economy. Such taxes also discourage managers from undertaking *right-tailed projects*, that is, projects with a low probability of a very high return that would otherwise be chosen in the absence of such taxes. Our main conclusion is that the advocates of very progressive wealth taxes reach their conclusions in part because they abstract from two key forces. First, when managers' incentives need to be aligned to those of a firm by rewarding successful managers with increments to wealth that move closely with the fortunes of the firm, then progressive taxes can greatly distort these incentives. Second, when managers have the discretion to guide firms' investments towards different types of projects, taxes which effectively truncate the very high but low probability rewards—those that make projects with right-tailed payoffs attractive in the first place—can greatly distort the real allocations in an economy. We note that both of these forces can be quantitatively important even in a model in which a panel regression of paid worked hours on income taxes rates shows only very low elasticities of hours to tax rates.

The obvious starting point of a study of wealth taxes is an economy with heterogeneous consumers that endogenously generates the observed patterns of earnings and wealth. We proceed in what follows by first doing so and then evaluating the equilibrium affects of sharply altering the U.S. tax code from its current settings so as to incorporate the wealth taxes currently debated.

In our first step, we build on the existing quantitatively literature on income and wealth distributions with heterogeneous consumers has largely emphasized the role of earnings inequality in generating wealth inequality. Indeed, the most popular approach to generating heterogeneity in wealth is to embed an idiosyncratic process for earnings into a model in which consumers can save only through uncontingent bonds. These Aiyagari-Bewley economies generate wealth inequality from the precautionary savings motives of consumers who face exogenous stochastic processes for their income.

As Benhabib, Bisin, and Luo (2017) and Benhabib and Bisin (2018) have emphasized, however, such economies typically do not reproduce well a key feature of the data: the right tail of the wealth distribution is substantially thicker than that of the income distribution. That is, the fraction of economy-wide wealth held by a top share of the wealth distribution, say, the top 5%, 1%, 0.1%, is much larger than the corresponding fraction of economy-wide income held by the same top share of the income distribution. One branch of the literature deals with this issue by adding an *awesome* state in which an consumer's income randomly jumps up temporarily by a large amount so that these temporarily very rich consumers save vast amounts as a precaution for the time when normal states return (see Castaneda, Diaz-Gimenez, and Rios-Rull 2003 and the somewhat critical discussion in Benhabib and Bisin 2018).

A second branch of this literature on the wealth distribution focuses on the important role played by entrepreneurs in accounting for the top percentiles of the wealth distribution. In this work, entrepreneurs are modeled as sole owners of non-public firms who also manage them. In these setting, a firm's capital is entirely financed by owners using their personal wealth as collateral for borrowing. The models, while insightful, do not attempt to capture that a sizable fraction of the income earned by consumers at the top of the wealth distribution is labor income, more attributable to top executives and managers in publicly-owned firms financed by equity and debt rather than to sole owners (see Quadrini 2000 and Cagetti and De Nardi 2006).

A third branch of this literature emphasizes that the endogenous response of human capital accumulation to higher income tax rates dampens the optimal progressivity of optimal income taxes. This point was emphasized by Badel, Huggett, and Luo (2020).

Our model expands on several themes in this literature in accounting for the observed income and wealth distributions, namely the important roles of managers in firms and endogenous human capital accumulation but, critically, formalizes the distortions to managers' incentives and project choices arising from progressive wealth and income taxes. We think of consumers in a managerial position as having their compensation tied to the idiosyncratic fluctuations in their firm's fortunes for *agency*, that is, incentive, reasons. These consumers could be owner-entrepreneurs, in which case their income might be classified as business income, but they could also be top managers who are not necessarily owners of their firms and whose earnings might be classified as labor income.

We allow consumers to choose whether to be managers or workers in a firm in each period as in the span-of-control model of Lucas (1978). Consumers or *agents* begin with some initial endowments of human capital as a worker and human capital as a manager. In each period, their unobserved effort choice affects the probability distribution over their human capital and, hence, their efficiency units of labor. Managers' efficiency units of labor are not observed and their compensation is governed by incentive-compatible Markovian labor contracts that link their current earnings and continuation wealth to the output of the firm they manage. Workers' efficiency units of labor supplied are observed and they are paid a given wage per unit of the observable amount of their efficiency units of labor. The motivation for these assumptions is that it is much easier to monitor the effective output of workers performing rather routine tasks then it is to monitor that of top managers. Instead, the best way to provide a manager with incentives to exert the desired amount of effort is to connect their compensation to the output of the group of workers that this manager leads.

Since it is optimal to tie a manager's compensation to the output of a firm, we endogenize the feature that the effective return on wealth for a manager has an idiosyncratic component based on the particular division or firm that the manager leads. In this sense, we derive as part of the optimal contract what Angeletos (2007) directly assumed.

Our preliminary simulations show that even a simple version of the model accounts well for the increase in wealth inequality in the United States over the past 30 years. Such a model implies a substantial aggregate output loss from wealth taxes of the magnitude currently debated.

2 Economy

Here we sketch out a simple precursor to our full model.

2.1 Technologies, Timing, and States

Consider a consumer at the end of period $t-1$. This consumer has a wealth level W_{t-1} and a human capital vector $h_{t-1} = (h_{mt-1}, h_{mt-1})$ which denotes the current managerial and worker human capital of the consumer. At this time, the consumer chooses to work in period t as either a manager or a worker and we denote this occupation choice by $\kappa_{t-1} \in \{m, w\}$. A consumer's human capital in period t is given by

$$h_{mt} = z_{mt} h_{mt-1} \text{ and } h_{wt} = z_{wt} h_{wt-1}.$$

The effect of effort by a consumer is to change the probability distribution over human capital $h_t = (h_{mt}, h_{wt})$ that is used in production in at date t. This effort is the private information of the consumer. In particular, if the consumer chooses to work as a manager in period t, the effort e_t determines the density $d\pi^m(z_t|e, z_{t-1})$ over $z_t = (z_{mt}, z_{wt})$ whereas if the consumer chooses to work as a worker in period t, the effort e_t determines the density $d\pi^w(z_t|e, z_{t-1})$ over z_t. The presence of z_{t-1} in the conditional densities π^m and π^w allows for serial correlation in idiosyncratic productivity of consumers that manifests itself as serially correlated effective units of human capital supplied to firms h_{mt} and h_{wt}.

The output from a firm managed by manager with human capital h_{mt}, capital k_t, and (total effective) labor input ℓ_t is given by the constant returns to scale production function $F(h_{mt}, k_t, \ell_t)$.

At the end of period $t-1$, the *state of a consumer* is $s_{t-1} = (h_{t-1}, z_{t-1}, W_{t-1}, \kappa_{t-1})$. The aggregate state is the measure of consumers over s_t, namely $\mu(s_t)$. We often suppress this state in what follows and we let t on the decision rules indicate the dependence on the state.

2.2 Intermediary

Consider a representative intermediary that handles contracting, capital accumulation, and hiring workers. The intermediary that enters period $t-1$ with capital k_{t-1} and chooses investment x_{kt-1} at the end of period $t-1$, enters period t with capital

$$k_t = (1-\delta)k_{t-1} + x_{kt-1}$$

For any consumer, at the end of period $t-1$, the intermediary observes that consumer's state $s_{t-1} = (h_{t-1}, z_{t-1}, W_{t-1}, \kappa_{t-1})$. For a consumer with state s_{t-1} who chooses to work as a worker, the intermediary pays w_t for each unit of human capital $h_{wt}(s_{t-1}, z_t)$ supplied by the worker which the intermediary observes in period t. For a consumer with state s_{t-1} who chooses to work as a manager, the intermediary enters into a incentive compatible contract which consists of recommended effort level $e_t(s_{t-1})$ and for each realized vector z_t, a wage payment $w_{mt}(s_{t-1}, z_t)$, a wealth level $W_t(s_{t-1}, z_t)$, and an occupation $\kappa_t(s_{t-1}, z_t)$ so that a contract is a vector

$$x_t(s_{t-1}) = (e_t(s_{t-1}), w_{mt}(z_t, s_{t-1}), W_t(z_t, s_{t-1}))$$

for each s_{t-1}. In terms of information, the intermediary does not observe the effort level e_t but observes the realized vector of human capital z_t in t.

Here the key difference between the manager and the worker is that the manager's wage $w_{mt}(z_t, s_{t-1})$ is determined by a nonlinear contract, and part of the contract is the manager's end of period wealth, $W_t(z_t, s_{t-1})$, which implicitly means that the intermediary can observe and effectively restrict any other financial investments the manager makes. In contrast, the workers have simple linear contracts for wages per effective unit of human capital supplied, and the intermediary has no ability to control the end of period wealth.

This intermediary takes as given the intertemporal prices $\{Q_t\}$ and the wage rates $\{w_t\}$ for effective units of labor from workers and accepts contracts (x_t, s_{t-1}) that lead to nonnegative expected profits. Let $\tilde{\mu}(x_t, s_{t-1})$ be the measure of such contracts that it accepts, the intermediary chooses the capital $k_t(z_t; x_t, s_{t-1})$ and labor $\ell_t(z_t; x_t)$ that is allocated to that managers where z_t is the vector of realized productivities to maximize

$$\sum_t Q_t \left[\int_{x_t} \int_{z_t} [F(z_m h_{mt-1}, k_t(z_t; x_t, s_{t-1}), \ell_t(z_t; x_t, s_{t-1})) - w_t \ell_t(z_t; x_t, s_{t-1}) - w_{mt}(z_t; x_t, s_{t-1})] \right.$$
$$\left. d\pi^m(z_t|e, z_{t-1}) d\tilde{\mu}_m(x_t, s_{t-1}) \right] - \sum_t Q_t \left[k_t - (1-\delta) k_{t-1} \right]$$

subject to

$$\lambda_t : k_t \leq \int_{x_t} \int_{z_t} k_t(z_t; x_t, s_{t-1}) d\pi^m(z_t|e, z_{t-1}) d\tilde{\mu}_m(x_t, s_{t-1})$$

$$h_t(z_{mt}; x_t) : h_{mt} \leq z_{mt} h_{mt-1}.$$

Here k_t is the aggregate capital of the intermediary in period t and $(k_t(z; x_t, s_{t-1}), \ell_t(z; x_t, s_{t-1}))$ are the capital and labor assigned to a manager with state s_{t-1}, contract x_t, and period t human capital $z = (z_m, z_w)$. We then have:

Lemma 1 *The intermediary will accept any incentive compatible contract $w_{mt}(x_t), e_t(x_t)$ that satisfies*

$$\int_z \left[F(h_{mt}, a_k(w,R) h_{mt}, a_\ell(w,R) h_{mt}) - w_t a_\ell(w,R) h_{mt} - R a_k(w,R) h_{mt} \right] d\pi^m(z|e, z_{t-1}) - w_{mt} \geq 0,$$

where $h_{mt} = z_m h_{mt-1}$ and $z = (z_m, z_w)$

$$\int_z \Pi(w,R) z h_{mt-1} d\pi^m(z|e, z_{t-1}) - w_{mt} \geq 0 \tag{1}$$

where $\Pi(w,R) \equiv F(1, a_k(w,R), a_\ell(w,R)) - w a_\ell(w,R) - R a_k(w,R)$.

Proof: Note that the first order conditions of the intermediary are

$$k_t(z_{mt}; x_t) : Q_t F_k(z_{mt}; x_t) = \lambda_t \tag{2}$$

$$\ell_t(z_{mt};x_t) : F_\ell(z_{mt};x_t) = w_t \tag{3}$$

$$k_t : Q_{t-1} - (1-\delta)Q_t = \lambda_t \tag{4}$$

so substituting out λ_t from (2) into (4) and defining $1/R_t = Q_t/Q_{t-1}$ gives

$$F_{kt}(h,k,\ell) + (1-\delta) = \frac{1}{Q_{t-1,t}} \equiv R_t$$

which together with (3) implies that the optimal k/h and ℓ/h ratios are determined by

$$F_{kt}(1,\frac{k}{h},\frac{\ell}{h}) + 1 - \delta = \frac{1}{Q_{t-1,t}} \equiv R_t \tag{5}$$

$$F_\ell(1,\frac{k}{h},\frac{\ell}{h}) = w_t \tag{6}$$

Using (5) and (6), the optimal k and ℓ satisfy

$$k = a_k(w_t,R_t)h_m, \text{and } \ell = a_\ell(w_t,R_t)h_m.$$

2.3 Manager's Contracting Problem

Consider the contracting problem of a manager with state $s_{t-1} = (h_{t-1}, z_{t-1}, W_{t-1}, m)$ at the end of period $t-1$. This manager choose the contract $x_t = (e_t, w_{mt}(z_t), W_t(z_t), \kappa_t(z_t))$ that specifies effort e_t, contigent labor earnings, $w_{mt}(z_t)$, contingent wealth, $W_t(z_t)$, and the contingent occupation $\kappa_t(z_t)$ as well as current contingent consumption $c_t(z_t)$ to solve

$$V_{mt}(s_{t-1}) = \max_{x_t,c_t(z)} \int_z [u(c_t(z),e_t) + \beta \max\{V_{mt+1}(h_t,z_t,W_t,m), V_{\ell t+1}(\gamma_m h_{mt}, h_{wt}; z_t, W_t, \ell)\}]\, d\pi^m(z|e,z_{t-1})$$

where $\gamma_m < 1$ subject to the budget constraint

$$c_t(z) + \frac{1}{R_{t+1}} W_t(z) = w_{mt}(z) + W_{t-1}$$

the law of motion for human capital,

$$h_{mt} = z_{mt}h_{mt-1} \text{ and } h_{wt} = z_{wt}h_{wt-1} \text{ where } h_t = (h_{mt}, h_{wt})$$

where, given $c_t(z_t)$ and the contract x_t, the recommended effort is incentive compatible in that it solves

$$\max_{e_t} \int_z [u(c_t(z),e_t) + \beta \max\{V_m(s_t), V_w(s_t)\}]\, d\pi^m(z|e,z_{t-1}) \tag{7}$$

and the zero profit condition, where z_m is the first element of $z = (z_m, z_w)$

$$\int [\Pi(w,R) z_m h_{mt-1} - w_{mt}(z)]\, d\pi^m(z|e,z_{t-1}) \geq 0.$$

Note that if the consumer continues working as a manager in period $t+1$, then the state at the end of period t is $s_t = (h_t, z_t, W_t, \kappa_t)$. Instead, if the consumer switches to working as a worker in period $t+1$ then this consumer loses part of the managerial human capital in that $s_t = (\gamma_m h_{mt}, h_{wt}; z_t, W_t, \ell)$ with $\gamma_m < 1$. In what follows, we will adopt the first order condition approach and replace the global incentive constraint, (7), with its local form, written as a first-order condition,

$$\int_z [u(c_t(z), e_t) + \beta \max \{V_m(s_{t+1}), V_w(s_{t+1})\}] \frac{\partial \pi^m(z|e, z_{t-1})}{\partial e} + \int u_e(c(z_k), e) d\pi^m(z_k|e, z_{t-1}) = 0. \quad (8)$$

2.3.1 Worker's Problem

Consider next a consumer that at the end of period $t-1$ chooses to work as a worker in period t. This consumer takes a given the wage w_t and the gross interest rate R_{t+1} and solves

$$V_\ell(s_{t-1}) = \max_{e_t, c_t(z), W_t(z), \kappa_t(z)} \int [u(c_t(z), e_t) + \beta \max\{V_m(h_{mt}, \gamma_\ell h_{wt}; z_t, W_t, m), V_\ell(h_{mt}, h_{wt}; z_t, W_t, \ell)\}] d\pi^w(z|e, z_{t-1})$$

subject to

$$c_t(z) + \frac{1}{R_{t+1}} W_t(z) \leq w_t h_{wt} + W_{t-1}$$

$$h_{mt} = z_{mt} h_{mt-1} \text{ and } h_{wt} = z_{wt} h_{wt-1}.$$

Here we are explicitly imposing that these contracts are linear in human capital which will not be optimal because the workers would prefer risk-sharing contracts.

2.4 Measures and Market Clearing

An equilibrium consists of a measure $\mu_t(s_{t-1})$ on $s_{t-1} = (h_{t-1}, z_{t-1}, W_{t-1}, \kappa_{t-1})$, which imply $\mu_{mt}(s_{t-1})$ and $\mu_{\ell t}(s_{t-1})$, aggregate capital stock K_t, the value functions $\{V_{mt}, V_{\ell t}\}$, the contracts $x_t(s_{t-1})$, the effort, consumption, wealth, and occupation decision rules for managers and workers, $e_t(s_{t-1})$, $c_t(z, s_{t-1})$, $W_t(z_t, s_{t-1})$, $\kappa_t(z_t, s_{t-1}))$ where we note that the decision rules for the manager are implied by the contract, $x_t(s_{t-1})$, the intermediary decision rules $k_t(x_t, z_t)$ and $\ell_t(x_t, z)$, the prices $w_t, Q_{t,t+1}$ and $R_{t,t+1} = 1/Q_{t,t+1}$, that satisfy:

i) optimality workers and managers; ii) optimality for intermediaries; iii) market clearing for goods, using $s_{t-1} = (h_{t-1}, z_{t-1}, W_{t-1}, \kappa_{t-1})$

$$\begin{aligned} & \int_{s_{t-1}} \int_z c(x_t, z_t) d\pi^{\kappa_{t-1}}(z|e(s_{t-1}), z_{t-1}) d\mu_t(s_{t-1}) + k_{t+1} \\ = & \int_{s_{t-1}} \int_z F(h_t(z_t; s_{t-1}), k_t(z_t; s_{t-1}), \ell_t(z_t; s_{t-1})) d\pi^m(z|e(s_{t-1}), z_{t-1}) d\mu_{mt}(s_{t-1}) + (1-\delta) k_t; \end{aligned}$$

iv) market clearing for labor

$$\int_{s_{t-1}} \int_z \ell_t(z_t; s_{t-1}) d\pi^m(z|e(s_{t-1}), z_{t-1}) d\mu_m(s_{t-1}) \leq \int_{s_{t-1}} \int_z z_{wt} h_{wt-1} d\pi^w(z|e(s_{t-1}), z_{t-1}) d\mu_\ell(s_{t-1});$$

v) Market clearing for savings, using $s_{t-1} = (h_{t-1}, z_{t-1}, W_{t-1}, \kappa_{t-1})$

$$\begin{aligned} K_t &= \int_{s_{t-1}} \int_z W_t(z_t, s_{t-1}) d\pi^\kappa(z|e(s_{t-1}), z_{t-1}) d\mu(s_{t-1}) \\ K_t &= \int_{s_{t-1}} \left[\int_z W_t(z_t, s_{t-1}) d\pi^m(z|e(s_{t-1}), z_{t-1}) d\mu_\ell(s_{t-1}) + \int_z W_t(z_t, s_{t-1}) d\pi^\ell(z|e(s_{t-1}), z_{t-1}) d\mu_\ell(s_{t-1}) \right]; \end{aligned}$$

vi) market-clearing contracts

$$\tilde{\mu}_{mt}(x_t(s_{t-1}), s_{t-1}) = \mu_{mt}(s_{t-1});$$

v) the law of motion for μ_t

$$\mu_{t+1}(B) = \int_{G(B)} \int_z d\pi^{\kappa}(z|e(s_{t-1}), z_{t-1}) d\mu_t,$$

where

$$G(B) = \{s_{t-1}|(h_t, z_t, W_t(z_t, s_{t-1}), \kappa_t(z_t, s_{t-1}) \in B\}.$$

3 A Simple Example

In this section we show that our simple framework can generate substantial income inequality at the top. We now consider a simple partial equilibrium version of the model in which we abstract from the career choice and model only managers. Also we take wages and interest rates as given. We are interested in seeing if the model can generate a thick right tail of wealth for the managers.

Here we only explicitly model managers who now have a single type of human capital denoted h. The workers and capital are summarized by the profit function

$$\Pi(w, R) \equiv F(1, a_k(w, R), a_\ell(w, R)) - wa_\ell(w, R) - Ra_k(w, R)$$

which is profits of the intermediary after substracting the wage payments to workers and the rental payments on capital, but not yet subtracting out the payments to managers.

For this simplified model, consider the value function of a manager in which the state consists of a scalar h which this manager's human capital as a manager and this manager's current wealth W. For simplicity conisider the i.i.d. case in which the density $\pi^m(z_{mt}, z_{wt}|e_t, z_{mt-1}, z_{wt-1})$ reduces to $\pi(z_t|e_t)$. Then the manager's problem can be written recursively as

$$V(h, W) \max_{e, \{w(z), W(z)\}_z} \int_z \pi(z|e) \left[\frac{c(z)^{1-\sigma}}{1-\sigma} g(e) + \beta V(zh, W(z)) \right] \tag{9}$$

subject to the budget constraint

$$c(z) + \frac{W(z)}{R} = W + w(z)$$

the local incentive constraint

$$\int_z \frac{\partial \pi(z|e)}{\partial e} \left[\frac{c(z)^{1-\sigma}}{1-\sigma} g(e) + \beta \right] + \int_z \frac{\partial \pi(z|e)}{\partial e} \frac{c(z)^{1-\sigma}}{1-\sigma} g'(e) = 0 \tag{10}$$

and the zero profit condition for the intermediary.

$$\int \pi(z|e) \left[\Pi(w, R) zh - w(z) \right] = 0 \tag{11}$$

3.1 Reducing the State to One Dimension

We can establish this result:

Lemma 2 *The value function (9) has the form*

$$V(h,W)=\frac{W^{1-\sigma}}{1-\sigma}\phi\left(\frac{h}{W}\right). \tag{12}$$

Proof: Substitute in the budget constraint and the form for the continuation value from (12) to obtain

$$V(h,W)=\max_{e,\{w(z),W(z)\}_z}\int_z \pi(z|e)\left\{\frac{\left[W+w(z)-\frac{W(z)}{R}\right]^{1-\sigma}}{1-\sigma}g(e)+\beta\frac{W(z)^{1-\sigma}}{1-\sigma}\phi(\frac{zh}{W(z)}))\right\} \tag{13}$$

the local incentive constraint

$$\int_z \frac{\partial\pi(z|e)}{\partial e}\frac{\left[W+w(z)-\frac{W(z)}{R}\right]^{1-\sigma}}{1-\sigma}g(e)+\int_z \frac{\partial\pi(z|e)}{\partial e}\frac{\left[W+w(z)-\frac{W(z)}{R}\right]^{1-\sigma}}{1-\sigma}g'(e)=0 \tag{14}$$

and the zero profit condition for the intermediary.

$$\int \pi(z|e)\left[\Pi(w,R)zh-w(z)\right]=0 \tag{15}$$

Now consider a change of variables by letting $x(z)=w(z)/W$ and $y(z)=W(z)/W$, then factoring out $W^{1-\sigma}$ in the objective and the first constraint and dividing the zero profit condition by W, using

$$\frac{zh}{W(z)}=z\frac{W}{W(z)}\frac{h}{W}=\frac{z}{y(z)}\frac{h}{W},$$

we can rewrite this problem as

$$V(h,W)=\max_{e,\{x(z),y(z)\}_z}W^{1-\sigma}\int_z\left\{\frac{\left[1+x(z)-\frac{y(z)}{R}\right]^{1-\sigma}}{1-\sigma}g(e)+\beta\frac{y(z)^{1-\sigma}}{1-\sigma}\phi(\frac{z}{y(z)}\frac{h}{W})\right\}d\pi(z|e), \tag{16}$$

the local incentive constraint

$$\int_z \frac{\partial\pi(z|e)}{\partial e}\frac{\left[1+x(z)-\frac{y(z)}{R}\right]^{1-\sigma}}{1-\sigma}g(e)+\int_z \frac{\partial\pi(z|e)}{\partial e}\frac{\left[1+x(z)-\frac{y(z)}{R}\right]^{1-\sigma}}{1-\sigma}g'(e)=0, \tag{17}$$

and the zero profit condition for the intermediary

$$\int \pi(z|e)\left[\Pi(w,R)z\frac{h}{W}-x(z)\right]=0, \tag{18}$$

where in the last term of (16) we rewrote the argument of $\phi(\cdot)$ using

$$\frac{zh}{W(z)}=z\frac{W}{W(z)}\frac{h}{W}=\frac{z}{y(z)}\frac{h}{W}.$$

Now, from inspection, the only place the state variables (h,W) enter are in the last term of (16) and in the bracketed term in (18) and in both places these states enter only as the ratio h/W. Hence, the maximized value in (16) has the form

$$V(h,W)=W^{1-\sigma}\phi(\frac{h}{W}),$$

which establishes the result.

3.2 The Quantitative Example

Here we consider the simplified problem with the transformed state, which here we denote by $\tilde{h} = h/W$ and the transformed controls $x(z)$ and $y(z)$. We suppose there is a measure 1 of managers each of who survive with probability δ. In each period the measure $(1-\delta)$ of managers who die are replaced by an equal measure of new managers who have an initial ratio of human capital to wealth $\tilde{h}$ equal to 1.

We assume that the manager's productivity $z \in [0,\infty]$ has a density $d\pi(z|e)$ given by $f(\mu(e), \sigma)$. Here effort e increases the mean of the distribution over z via the function $\mu(e)$. We also assume that the per-period utility function is

$$u(c,e) = \log(c)g(e).$$

We assume that the disutility of effort e, given by

$$g(e) = \psi \frac{(1-e)^{1-\eta}}{1-\eta}$$

with $\psi > 0$, and $\eta \in (0,1)$. In this way, we restrict effort to line in the unit interval. Hence, the problem of the manager in (16) can be written as

$$\phi(\tilde{h}) = \max_{e,x(z),y(z)} \int \left[\log\left(1 + x(z) - \frac{1}{R}y(z)\right) g(e) + \delta\beta \log(y(z))\, \phi\left(\frac{z}{y(z)}\tilde{h}\right)\right] f(z|e)\, dz, \tag{19}$$

subject to

$$\begin{aligned}\int \left[\log\left(1 + x(z) - \frac{1}{R}y(z)\right) g(e) + \delta\beta \log(y(z))\, \phi\left(\frac{z}{y(z)}\tilde{h}\right)\right] \frac{\partial(z|e)}{\partial e} dz \\ + \int \log\left(1 + x(z) - \frac{1}{R}y(z)\right) g'(e) f(z|e)\, dz = 0,\end{aligned}$$

$$\int \left[\Pi z \tilde{h} - x(z)\right] f(z|e)\, dz = 0.$$

Solving the model implies finding policy functions for $\{e, x(z), y(z)\}$ and a value function $\phi(\tilde{h})$. As for the productivity process, we assume z is drawn from a log-normal distribution with mean $\mu(e)$ and constant variance; The mean of the distribution depends positively on managers effort and is given by the $\mu(e) = \alpha + \beta e$.

Most of the parameter take standard values. We consider a discount factor, β, equal to 0.9, a value of $R = 1.05$, and a annual probability of dying of 0.10. We set $\psi = 1$ and $\eta = 0.75$. The value of Π in this case is a scale parameter which we set to 1. Finally, we assume that $\alpha = -1$ and $\beta = 1.5$ so that an individual that exerts an effort level of 2/3, draws productivity from a distribution with average productivity equal to 1.

Preliminary Results. The solution of the model provides insights about the distribution of wealth and its evolution over time. For instance, Figure 1 shows that this simple model can generate a distribution of wealth with a Pareto tail which is quite similar to what is found in the data. To construct this figure, we simulate our model using a large sample managers (10,000) over a long period of time (1,200 periods). We then take the stationary distribution and we calculate the ratio of the average wealth above a certain threshold, that is, the ratio of the average wealth of those individuals with net wealth above 10 millions over 10 millions. We do this for different threshold levels ranging from 5 million to 80 million. Importantly, if the distribution is Pareto, this ratio should converge to a constant as we increase the threshold. Panel A of Figure 1 shows the results from our model simulation whereas Panel B shows the same calculations but derived from the US Survey of Consumer Finance for two years, 1998 and 2007. The similarities between the model simulation and the data are quite remarkable.

A second exercise pertains to the dynamics of wealth concentration. In the United States, wealth inequality has increased over the last 30 years accompanied by an increase in the share of wealth accrued by the top 1% of households. Our model can reproduce this increase in wealth concentration along the transition towards the steady state. To see this, we consider a 30 years period from our simulated data. The results are show in Figure 2 for the model (Panel A) from the World Inequality Database. We present the share of the top 1% of wealth normalized to its value in 1980. Again, the model does a good job in replicating the increase in wealth concentration at the top. Hence, we conclude that this simple model, can generate the patterns of wealth accumulation observed in the data.

The Impact of Linear Taxes. Finally, we evaluate the impact of taxes in our environment using a simple linear tax system. This serves a first approach to the taxation analysis we will perform using our fully fledged model. We consider three different linear taxes: one that reduces wages in a τ_y-fraction, second that reduces total wealth in a τ_W-fraction, and a third that reduces the returns in a τ_R-fraction. Following the notation in Equation (19), we can write the budget constraint as

$$\frac{c}{W} \quad = \quad (1-\tau_W) + (1-\tau_y)\, x\,(z) - \frac{y\,(z)}{1-(1-\tau_R)\, r},$$

where c/W is the consumption-to-wealth ratio.

We vary these taxes, one at the time, and evaluate what is the overall impact on aggregate effort and output. Intuitively, as taxes increase, they distort manager's effort decisions, reducing the aggregate effort and output. Figure 3 shows the aggregate output (panel A) and aggregate effort (panel B) for different values of taxes. An increase in taxes has the expected effect on aggregate output and effort, with higher taxes implying lower aggregate output and lower effort from the manager. Among the different linear taxes, the tax on wealth seems to have the stronger effect, reducing aggregate output almost one-to-one. Interestedly, effort does not change much—it declines less than 1%—indicating that a small change in aggregate effort can generate significant changes in aggregate economic activity. These results are most likely a lower bound for the over all impact of taxes: Since a non-linear system that has a larger effect on the wages and savings rates of high productivity managers, one would expect a large effect on aggregate effort and output.

References

Badel, Alejandro, Mark Huggett, and Wenlan Luo. 2020. Taxing top earners: a human capital perspective. *Economic Journal* 130 (629), 1200-1225.

Benhabib, Jess, and Alberto Bisin. 2018. Skewed wealth distributions: Theory and empirics. *Journal* *of Economic Literature* 56 (4), 1261-91.

Benhabib, Jess, Alberto Bisin, and Mi Luo. 2017. Earnings inequality and other determinants of wealth inequality. *American Economic Review* 107 (5), 593-97.

Cagetti, Marco, and Mariacristina De Nardi. 2006. Entrepreneurship, frictions, and wealth. *Journal of Political Economy* 114 (5), 835-870.

Castaneda, Ana, Javier Diaz-Gimenez, and Jose-Victor Rios-Rull. 2003. Accounting for the US earnings and wealth inequality. *Journal of Political Economy* 111 (4), 818-857.

Lucas Jr, Robert E. 1978. On the size distribution of business firms. *Bell Journal of Economics*, 508-523.

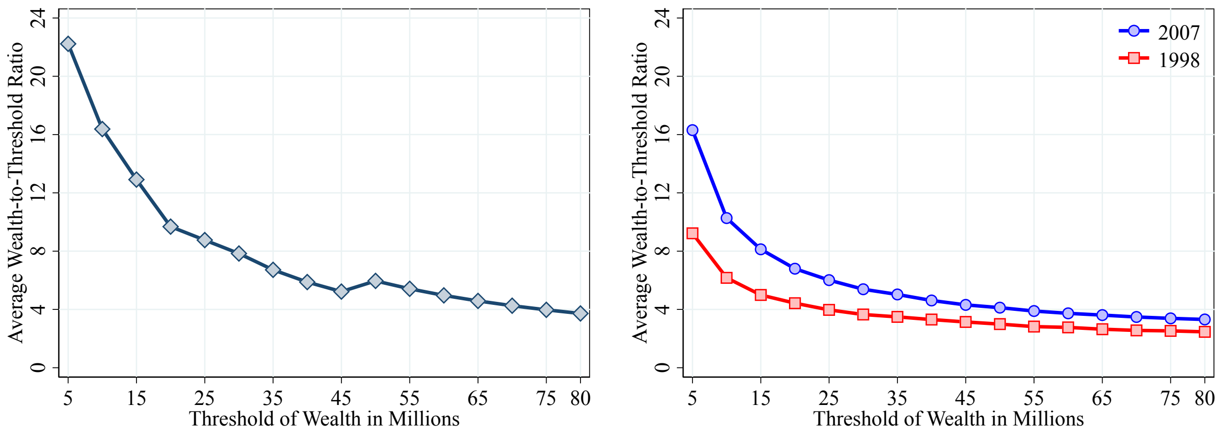
Quadrini, Vincenzo. 2000. Entrepreneurship, saving, and social mobility. *Review of Economic Dynamic*s 3 (1), 1-40.

Saez, Emmanuel, and Gabriel Zucman. 2019. How would a progressive wealth tax work? Evidence from the economics literature. Mimeo University of California Berkeley.

Figure 1 – Ratio of Average Wealth Above a Fixed Threshold

(a) Model Simulation

(b) US Data: Survey of Consumer Finance

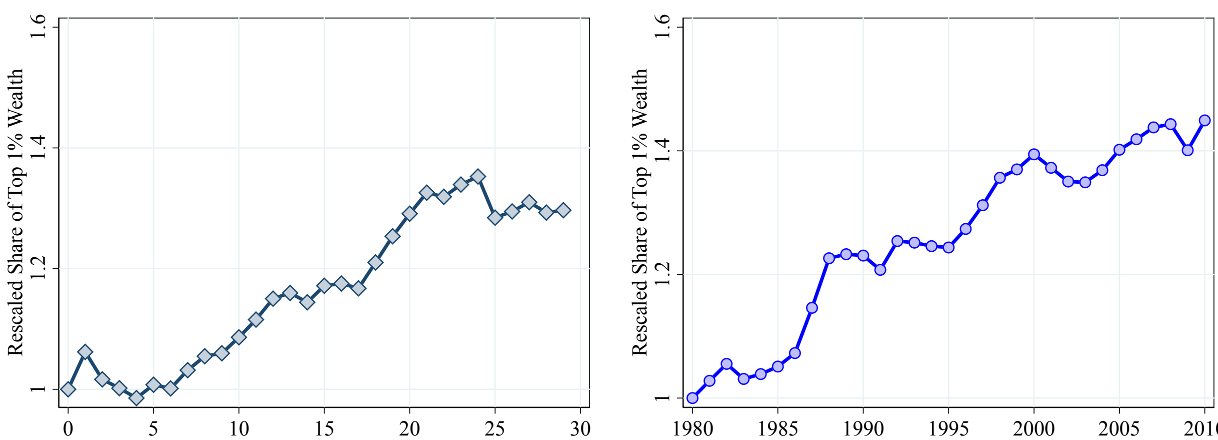


Notes: Figure 1 shows ratio between the average wealth of individuals over a certain threshold and the corresponding threshold using data from the simulated model—Panel A—and calculated from the Survey of Consumer Finance—Panel B. We calculate the ratio among all households with positive in a particular year.

Figure 2 – Share of Top 1% of Wealth Re-scaled to Initial Period

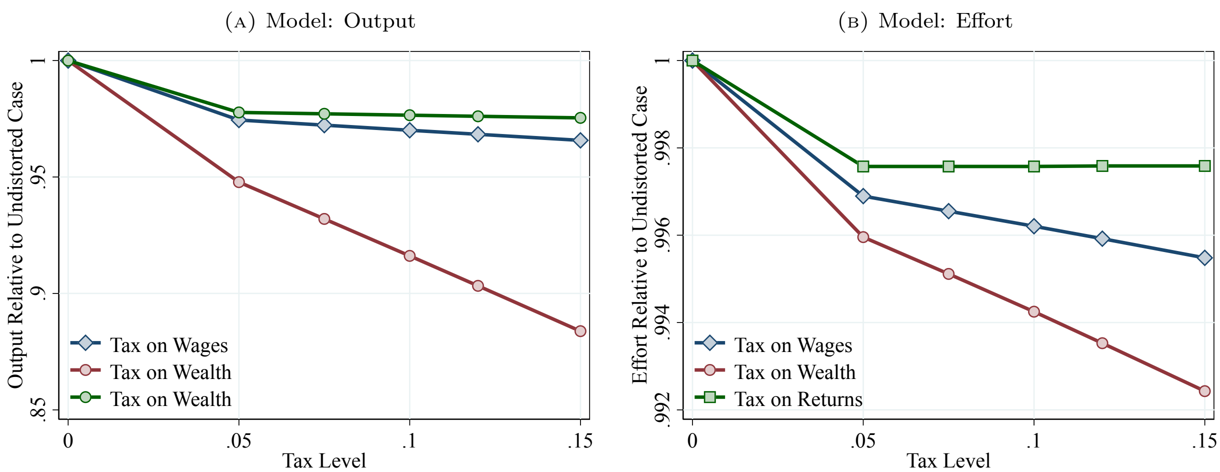
(a) Model Simulation

(b) US Data: World Inequality Database



Notes: Figure 2 shows the share of the top 1% of wealth obtained from the model simulation—Panel A—for the US data provided by the World Inequality Database—Panel B.

Figure 3 – Effect of Linear Tax on Aggregate Output and Aggregate Effort



Notes: Figure 3 shows the aggregate output and aggregate effort implied by the model re scaled by the corresponding value in the un distorted economy (all taxes equal to 0).