'You Will:' A Macroeconomic Analysis of Digital Advertising

Jeremy Greenwood, Yueyuan Ma, and Mehmet Yorukoglu[*]

Abstract

A model is developed where traditional and digital advertising finance the provision of free media goods and affect price competition. The economy is not efficient. Media goods are under provided. Additionally, there is too much advertising when ads cannot be perfectly directed toward potential buyers. The tax-cum-subsidy policy that overcomes these inefficiencies is characterized. The model is calibrated to the U.S. economy. The movement toward digital advertising increases consumer welfare significantly and is disproportionately financed by better-off consumers. The welfare gain from the optimal tax-cum-subsidy policy is much smaller than the one realized by the introduction of digital advertising.

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[*]Greenwood and Ma, University of Pennsylvania; Yorukoglu, Koç University. This is a report on research in progress. Comments are *welcome*! Address comments to Mehmet Yorukoglu at myorukoglu⊗ku.edu.tr

1 Introduction

1.1 The Question

Free media goods are everywhere. Think about Facebook, Google, Google Maps, Pandora, Twitter, Wikipedia, YouTube, and apps for dating, dieting, exercising, playing guitar, meditation, inter alia. Often these products are financed through advertising or the sale of marketing information for advertising purposes. Digital advertising has two benefits. First, the free media products financed by such advertising increase consumer welfare. Second, to the extent that digital advertising is more efficient than traditional advertising in spreading information about products, it might also spur competition among firms resulting in lower prices. Since media goods are not sold, they do not directly show up in the national income accounts. They don't show up indirectly either because advertising expenditure is deducted off of firms' profits and consequently does not show up in the national income accounts.

To address this question a variant of Butters' (1977) advertising model is used. Significant hot rodding has to be done to the vehicle to bring it up to speed for the task at hand. First, the framework is modified to allow for both digital and traditional advertising. Both types of advertising permit firms to convey information about products and prices to consumers, as in Butters (1977). Firms choose how much digital and traditional advertising to use. This decision depends on the relative cost effectiveness of these two information delivery mechanisms. Second, advertising is associated with the provision of free media goods. To incorporate the free provision of media goods, a full-fledged consumer sector is added. Free media goods are taken to complement leisure in utility, in the sense of Edgeworth and Pareto. Third, consumers differ by their income, while in Butters (1977) they are all the same. Distinct from Butters (1977), the maximum prices that consumers are willing to pay are endogenously determined as a function of the economic environment. A competitive equilibrium with digital and traditional advertising is characterized. As in Butters (1977), a distribution of prices emerges for a given product. The resulting equilibrium is not efficient, unlike Butters (1977), for two reasons. To start off with, free media goods are underprovided. Additionally, both digital and traditional adverts are sent to individuals who can't afford to buy the good at the advertised price. This wastes resources. A tax-cum-subsidy policy that overcomes these inefficiencies is presented. A version of the model is also considered where advertising can be directed toward only those customers who may buy the product. Once again, the free media goods distributed with advertising are underprovided.

Fourth, the developed model is calibrated using data on price markups, the ratio of advertising expenses to consumption expenditure, the ratio of spending on digital advertising relative to traditional advertising, the click-through rate for digital advertising, and the time spent on leisure by non-college- and college-educated individuals. This is something Butters (1977) could not have done at the time of his research. The welfare gain from the introduction of digital advertising is computed. The provision of free media goods boosts consumer welfare significantly. It also leads to more leisure, since media goods and leisure are complements in utility. The increase in leisure is more pronounced for the non-college educated vis à vis the college educated. The gain in utility from the rise in leisure is largely offset by a decline in regular consumption because people earn less now. The welfare benefit from the provision of free media goods is not captured in the GDP accounts for two reasons. To begin with, advertising is expensed or subtracted off of firms' profits and hence does not appear in GDP. And then, GDP and welfare are not the same thing; think about the welfare benefit of vaccines versus their cost. The analysis suggests that affluent consumers may finance a disproportionately large share of the cost of media goods because they purchase goods at higher prices. Yet, the move toward digital advertising may benefit affluent consumers more because it stimulates price competition at the higher price end of the goods market relative to the lower end.

1.2 Background

Advertising has been around for eons. Babylonian merchants employed barkers who advertised their wares by shouting out. The Romans used signage outside of stores to sell wares; a bush signified a wine shop. Painted notices on the walls of bathhouses in Pompeii told of upcoming

Figure 1: A 1919 toothpaste ad in the *Saturday Evening Post* magazine for S.S. White Dental Manufacturing Co. *Source*: Ad*Access, Duke Digital Repository.

exhibitions. Marshall (1920, p. 271) noted that "A single prominent position in a great thoroughfare promotes the sale of many various things." After the arrival of the printing press came newspapers and then magazines. Benjamin Franklin published advertisements in his newspaper, the *Pennsylvania Gazette*. He is credited with publishing in 1741 the first magazine ad in the United States in the short-lived *The General Magazine and Historical Chronicle, for all the British Plantations in America*.

Advertising became an industry in the 19th century. N.W. Ayer & Son was founded in Philadelphia in 1869. It sold complete advertising campaigns for businesses. It is credited with slogans such as "A diamond is forever" used by De Beers. A typical early 20th century magazine ad is displayed in Figure 1. Direct mail advertising started in 1872 with Aaron Montgomery Ward who launched a one page catalog, which was quickly followed by the Sear's Catalog.

Things changed rapidly in the 20th century with the advent of new technologies. Radio advertising started in the 1920s. In 1922 the first paid radio ad ran in New York City to promote the sale of apartments. It cost $50 for 50 minutes of airtime. The first paid television ad was for Bulova watches. It was broadcast in 1941 before a baseball game between the Brooklyn Dodgers and Philadelphia Phillies. Television advertising

Figure 2: The first clickable ad, part of AT&T's "You Will" campaign. *Source*: *The Atlantic*, 2017.

expanded with the introduction of cable tv in the 1950s. MTV introduced music videos that were really just commercials for music artists. Additionally, channels were started that were devoted to advertising, such as HSN and QVC.

The information age began in the 1970s. A descendent of direct mail advertising is email marketing. This started in 1978 with an ad sent by Digital Equipment Corporation via the Arpanet to 400 DEC computer users. It didn't really take off until the 1990s when many people started to use the internet through outlets such as Microsoft's Hotmail that offered free email starting in 1996. Last, online advertising started in the 1990s. The first clickable ad was on *Hotwired.com* in 1994, then the online version of *Wired* magazine–see Figure 2. It was part of AT&T's "You will" campaign that prognosticated about the future in the information age. The ad enjoyed a click-through rate of 44 percent and cost AT&T $30,000 for three months.

The composition of advertising spending changed as new vehicles for delivering ads cropped up, as Figure 3 shows. Ads in newspapers and magazines declined with the arrival of TV. Digital advertising rose with the advent of the information age. It's interesting to note that advertising's share of GDP has remained roughly constant in the postwar period at around 2 percent.

Online advertising is dominated by two giants, Facebook and Google. Google was founded in 1998 and Facebook in 2004. The ad revenue earned by these two companies (and Amazon) is shown in Figure 4 (right panel). Google's ad revenue shot up from around $70 million in 2001 to $135 billion in 2019. Likewise, Facebook's ascent is equally dramatic,

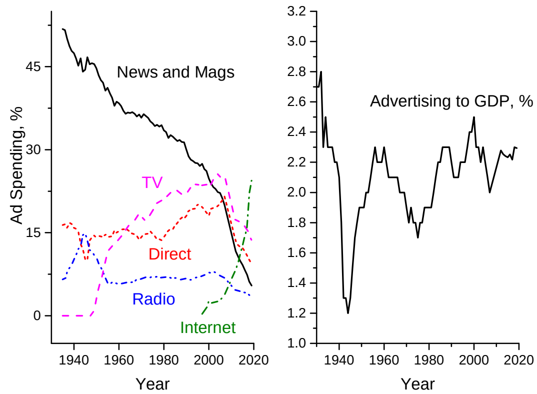

Figure 3: Advertising in the United States, 1935-2019. Advertising has consistently amounted to approximately 2 percent of GDP. Its composition has seen dramatic changes; however, as new mediums for communicating emerged. *Sources*: Douglas Galbi and AdAge.

rising from roughly \$2 to \$70 billion between 2010 and 2019. The first search engine was Archie, created in 1990. Alan Emtage, its creator, described an indexing technique that helped Archie catalogue "freely available or Public Domain documents, images, sounds and services on the network." Yahoo! Search was the first popular search engine, arriving in 1995. The next decade saw the rise of Google Search, which yielded better search results using an iterative algorithm that ranked web pages on the number of websites that linked to them and the ranking of these websites.

The first social media website is generally attributed to Six Degrees, founded in 1997. The name was based on the idea that people are linked to each other by six, or fewer, social connections. People could create profiles and "friend" each other. It had around 3.5 million users at its pinnacle. Things took off with the creation of MySpace in 2003. Between 2005 and 2008 it was the largest social media site in the world with over 100 million users per month. After 2008 Facebook dominated the social media world. Facebook had 2.5 billion monthly users in 2019.

A breakdown of online advertising revenue by format is also displayed in Figure 4 (left panel). Online search is the dominant vehicle for digital advertising, followed by social media. Google inserts online ads into its

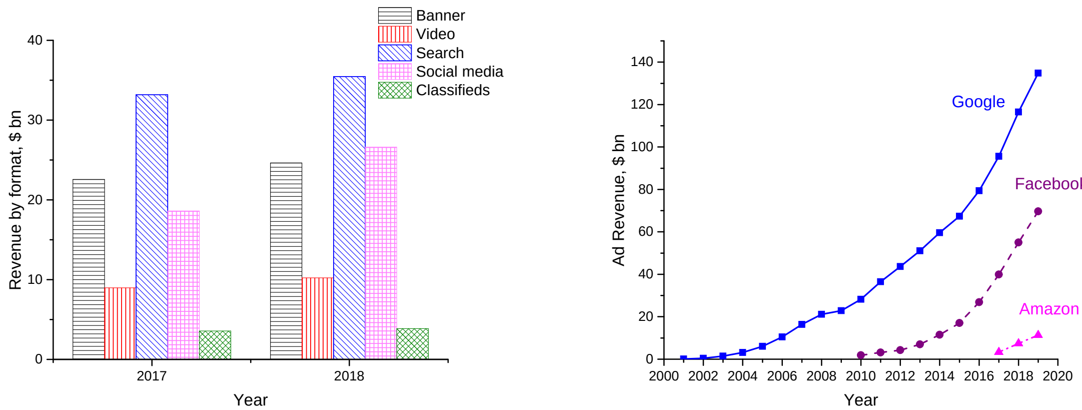

Figure 4: Right panel, ad revenue earned by Amazon, Facebook, and Google, 2001-2019. Left panel, distribution of U.S. advertising revenue by format, 2017 and 2018. *Source: statista*.

products, such as Google Search, using a pay-per-click pricing model. The search advertising cost per click was $0.69 in 2019. Google Search handled 5.4 billion search requests per day in 2019. Moving up from the third to the second position displayed by Google Search's results leads to a 31 percent increase in traffic. Advertisers pay for location. Apparently, only 0.78 percent of Google users make it to the second page of search results. The return on various mediums of advertising is presented in Figure 5, right panel. Digital search has the highest return in terms of sales per dollar spent on advertising. The left panel illustrates that spending by advertisers closely tracks the amount of time that consumers spend on the mediums.

A lot of digital content is provided for free via advertising. Think about the free goods just from Google: Chrome, Google Search, Google Maps, Gmail, Google Drive, YouTube, etc. Figure 6 shows the number of apps available in Google Play Store. In 2019 this was a whopping 2.8 million. Interestingly, consumers spend little for these products. Less than 14 percent of Google users spent more than $10 per digital media in the Google Play Store, as the left panel illustrates.

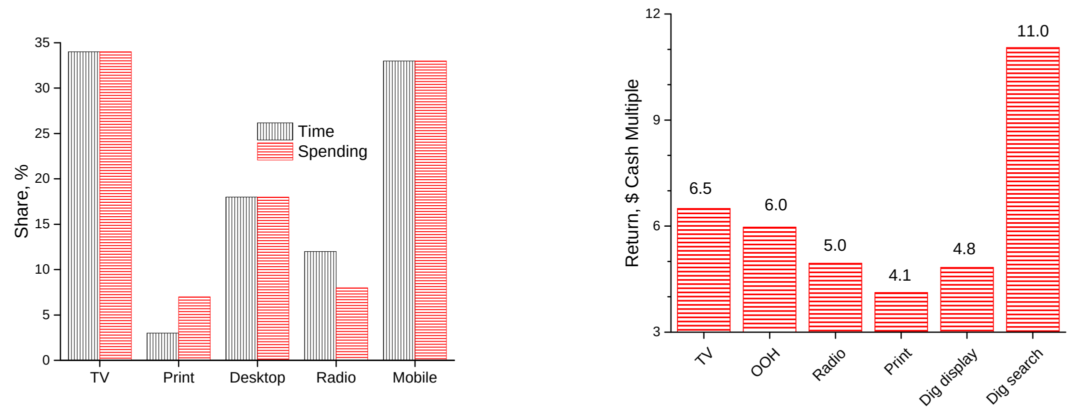

Figure 5: Right panel, the return per dollar of advertising by medium in the United States for 2017, measured as a cash multiple, 2001-2019. Left panel, U.S. advertising spending vs time spent by consumers by medium, 2018. *Source: statista.*

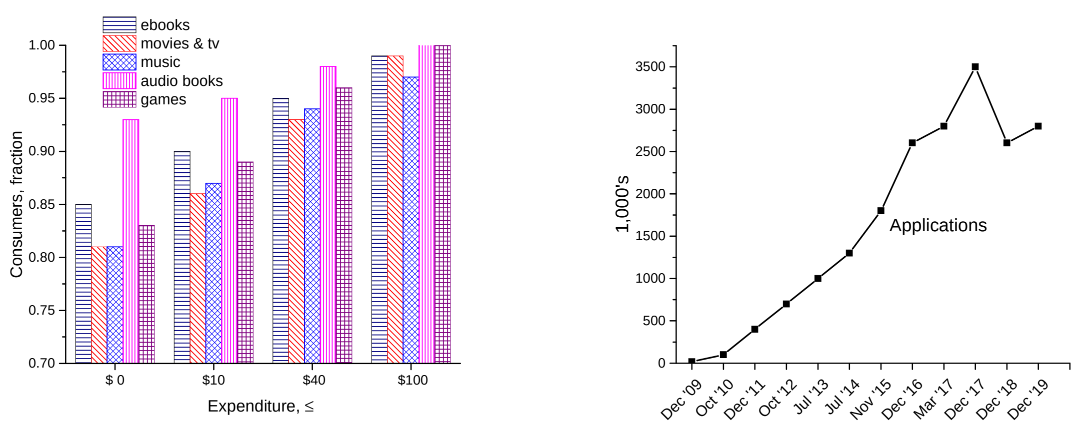

Figure 6: Right panel, applications in the Google Play Store, 2009-2019. Left panel, money spent by U.S. consumers on Google digital media products in 2017, presented in cumulative distribution form. *Source: statista.*

2 A Brief Review of the Advertising Literature

Advertising has been part and parcel of economic life for a long period of time, as Figure 3 suggests. Until the second part of the twentieth century, however, economists paid little attention to advertising. The roots of economic analysis of advertising can be traced back to insightful work of Marshall (1920). Economic analysis of advertising flourished since then.[1]

At a time when competitive equilibrium and full information were the fundamentals of economic thinking, economists were struggled with the question of why consumers would respond to advertising. Here two views emerged. The first one holds that advertising is persuasive, altering consumers' tastes and creating brand loyalty. Not surprisingly, according to the persuasive view, advertising has no real value to consumers, and can have important anti-competitive effects, resulting with increased economic concentration. Marshall (1920, p. 304 and 306) noted that "much of the modern expenditure on advertising is not constructive, but combative," and that "advertisements which are mainly combative generally involve social waste."

The second view holds that advertising is informative. According to this view, markets are characterized by imperfect consumer information that leads to market inefficiencies. Here, rather than being the problem, advertising emerges as a remedy offered by the market. Clearly, according to the informative view, advertising promotes competition. Marshall (1920, p. 305) also thought that advertising could be constructive by "the assistance, which they afford to customers by enabling them to satisfy their wants without inordinate fatigue or loss of time, would be appropriate, even if the business were not in strong rivalry with others." He noted that "exceptionally constructive are all those measures needed for explaining to people generally the claims of some new thing, which is capable of supplying a great but latent want."

In the approach taken here advertising is informative. The foundation of the informative view of advertising was laid by Ozga (1960) and Stigler (1961). They saw price dispersion as a reflection of consumer ig-

[1] Bagwell (2007) provides a detailed survey of the literature, so only a capsule summary is given here.

norance and advertising is a valuable source of information for consumers that results in a reduction in price dispersion. Telser (1964) significantly advanced the theoretical and empirical foundations for the informative view concluding that advertising is a sign of competition and is an important source of information for the consumers. Following these lines, Butters (1977) offered the first equilibrium analysis of advertising in a multi-firm model. He shows that advertising in equilibrium is efficient. Stegeman (1991) extends Butter's (1977) work with the assumption that consumers' valuations of products are heterogeneous. He demonstrates that informative advertising is then inefficient.[2]

Extending Butters' (1977) model to an economy where there is productivity heterogeneity across firms, Dinlersoz and Yorukoglu (2008) study how improvements in advertising technology affect industry equilibrium. In a related work, Dinlersoz and Yorukoglu (2012) analyzed how advertising technology affects firm dynamics. They show that entry, exit, and volatility in firm size and value, increase as advertising technology improves. Equilibrium in both models are efficient.

In more recent work, Perla (2019) builds a model where consumers learn about firms slowly through a network of connections between consumers and firms that endogenously evolves through the life cycle of an industry. Cavenaile and Roldan (2019) analyze the implications of advertising for firm dynamics and economic growth through its interaction with R&D investment at the firm level. They provide empirical evidence supporting substitution between R&D and advertising using exogenous changes in the tax treatment of R&D expenditures across U.S. states.

[2]Digital advertising was not around at the time of Stegeman's (1991) paper. Like Butters (1977) he does not have a fully fleshed out consumer sector, which isn't needed for their analyses. The latter is important for the current inquiry for two reasons. First, consumer behavior changes as the economy evolves due to technological progress in advertising and, second, tastes need to be specified for the welfare analysis. Additionally, Stegeman does not present the optimal tax-cum-subsidy policy that renders the advertising economy efficient. Last, he doesn't take the model to data; calibration was in its infancy at the time of his research.

3 Setup

Consider an economy with three types of goods; namely, generic consumption goods, media leisure goods, and leisure. At most a unit measure of varieties of regular consumption goods can be produced. There is free entry into the production of each variety of regular goods, $i \in [0, 1]$, subject to incurring a fixed cost of $\mathfrak{r}$. To sell its product a regular goods producer must advertise to potential customers, which is costly. Advertising can be done in two ways. The first way is through traditional advertising. The second way is via modern online advertising. A potential customer receives ads for a variety in a random manner. A producer of regular good-i is free to set the price, p_i, that it wants. This can differ across variety-i producers because consumers will vary in the advertised prices that they have in their information sets.

Ads are delivered via media goods, which are provided to consumers for free. There are $\mathfrak{m}$ media goods available. Media goods have a click-through rate that represents the number of ads that the good will deliver. The supply of media goods, $\mathfrak{m}$, is determined by the amount of advertising that firms want to do. The cost of providing these goods is absorbed as an advertising expense.

Turn now to the consumer/worker. Regular good-i must be consumed in the discrete quantity $c_i \in \{0, 1\}$. An individual might not consume the full spectrum of regular goods because either they didn't receive an ad for a good or because they couldn't afford them at the advertised price. Media leisure good-j is consumed in the discrete quantity $m_j \in \{0, 1\}$. Since media leisure goods are free the consumer will enjoy the full spectrum of what is currently available. There is a unit mass of people. Each person is indexed by a talent level $\tau \in \{\underline{\tau}, \overline{\tau}\}$, where $\underline{\tau} = 1 < \overline{\tau}$. The fractions of the population with $\underline{\tau}$ and $\overline{\tau}$ are denoted by $\mathfrak{t}$ and $1 - \mathfrak{t}$. A person with ability level $\tau = \underline{\tau}$ is unskilled and earns the wage rate 1. A skilled person, $\tau = \overline{\tau}$, earns the wage $\overline{\tau}$, but must incur a fixed education cost, $\mathfrak{e}$, in terms of time. In other words, the wage rate for unskilled labor is the numeraire, which implies that all goods prices are measured in terms of unskilled labor. An individual has one of time which they can split between working in the market, h, leisure, l, and education, $\mathfrak{e}$.

Preferences are given by

$$\theta \ln(\int_0^v c_i di) + \frac{(1-\theta)}{\rho} \ln[\kappa l^\rho + (1-\kappa)(\int_0^{\mathfrak{m}} m_j dj)^\rho], \text{ with } \rho < 0, \quad (1)$$

where v and $\mathfrak{m}$ demarcate the set of available regular and media goods. These preferences are well defined even when particular varieties of consumption goods are not consumed. Media goods can be mixed with leisure to generate utility; i.e., they are leisure goods. For example, you must spent time to enjoy an online game. The assumption that $\rho < 0$ implies that leisure, l, and leisure goods, the m_j's, are Edgeworth-Pareto complements in utility–in other words, the cross partial in utility is positive. The idea is that more leisure goods increase the marginal utility of leisure. Therefore, you will want more leisure at the margin. The notion of leisure complementing goods is in Greenwood and Vandenbrouke (2008) and Kopecky (2011). Kopecky (2008) suggests the decline in the price of leisure goods encouraged the elderly to spend a larger fraction of their life in retirement. Aguiar et al (2017) use this notion to argue that part of the recent decline in hours worked by young males is due to the advent of recreational computing. The individual's budget constraint is given by

$$\int_0^v p_i c_i di = \tau h(\tau) \equiv \begin{cases} \overline{\tau}(1 - l - \mathfrak{e}), & \text{skilled;} \\ 1 - l, & \text{unskilled,} \end{cases} \quad (2)$$

where, with some abuse of notation, in this context p_i represents the minimum price for good i that the consumer/worker has in his information set and $h(\tau)$ is the hours worked by a type-τ person.

4 Regular Goods Firms

Firms can freely enter into the production of any variety of regular goods subject to a fixed cost of $\mathfrak{r}$ (in units of unskilled labor). Suppose that there are v active varieties of goods with n firms producing each variety for a total of vn firms in the economy. The quantities v and n will be determined in equilibrium by the fact that firms must earn zero profits. Any variety of regular goods can be produced by a firm according the constant-returns-to-scale production

$$o = h/\gamma,$$

where o is the output of the good and h is the amount of labor employed. The unit cost of producing a good is γ.

To sell its product at time price p, a firm must reach out to customers, which involves advertising. Ads are delivered through media goods, which can be distributed through either a traditional or digital vehicle. Let a_t and a_d represent the number of traditional and digital ads that are sent out by the firm. To generate a_t traditional ads a firm must provide t media goods that each has a click through rate of ζ; i.e.,

$$a_t = \zeta t. \tag{3}$$

The cost (measured in terms of unskilled labor) for traditional advertising is

$$A(a_t) = \phi a_t^\alpha = \phi(\zeta t)^\alpha, \text{ with } \alpha > 1. \tag{4}$$

Likewise, digital ads are distributed via digital goods provision. A digital good has a click-through rate of $\psi < 1$. So, d digital goods will deliver a flow of ads, a_d, according to

$$a_d = \psi d. \tag{5}$$

The cost of producing a_d digital ads is

$$A(qa_d) = \phi(q\psi d)^\alpha, \tag{6}$$

where q is a technology factor reflecting the cost advantage of digital advertising.

Because consumers will differ in the ads that they have in their information sets, firms do not have to charge the same price. This information friction allows firms to charge a price higher than its marginal production cost, γ. Let $\underline{p}$ represent the lowest profitable price in equilibrium and likewise $\overline{p}$ denote the maximum profitable one. Now, a firm is free to charge any price p such that $\underline{p} \leq p \leq \overline{p}$. The higher the price, the less likely the firm will make a sale. The set of viable equilibrium prices, $\mathcal{P}$, is characterized later in Proposition 2. Since there is free entry into a variety, it must transpire that a firm will earn the same profit at any price, $p \in \mathcal{P}$.

4.1 Advertising

Let $S(p) = \Pr(\text{SALE}|p)$ be the probability that an ad at price p will generate a sale for the firm. This probability is exogenous for a firm and is unpacked later. The firm chooses its advertising strategy to maximize its profits at price p. So, its advertisements solve

$$\Pi(p;q) \equiv \max_{a_t,a_d}\{(p-\gamma)(a_t+a_d)S(p) - A(a_t) - A(qa_d)\}. \tag{7}$$

Here term $p-\gamma$ represents the firm's unit profits (excluding advertising costs) while $(a_t+a_d)S(p)$ is the firm's total sales. Its advertising costs are $A(a_t)+A(qa_d)$. The first-order conditions for a_t and a_d are

$$(p-\gamma)S(p) = \phi\alpha a_t^{\alpha-1} \text{ and } (p-\gamma)S(p) = \phi\alpha q^{\alpha} a_d^{\alpha-1}. \tag{8}$$

The common lefthand side of these expressions is the expected profit (or marginal benefit) from sending out an extra ad. The righthand sides represent the marginal costs of an extra traditional or digital ad. Clearly, the marginal cost of digital advertising increases with its cost factor, q.

Proposition 1 *(Advertising) All firms do the same amount of traditional, a_t, and digital advertising, a_d, even when charging different prices for their products, $p \in \mathcal{P}$. Digital advertising decreases with its cost factor, q.*

Proof. See the Appendix. ■

To understand the logic underlying the proposition, note that in equilibrium a firm is free to pick any price it desires. So, expected unit profits, $(p-\gamma)S(p)$, must be constant across equilibrium prices. Suppose not. Then firms with higher values for $(p-\gamma)S(p)$ would make more than firms with lower values because the former could always do the same amount of advertising as the latter.

If the marginal benefit is constant across prices, then from (8) so must be the marginal costs. This implies that a_t and a_d are invariant across prices, p. Finally, from the above first-order conditions for a_t and a_d, it is immediate that

$$a_t = [\frac{(p-\gamma)S(p)}{\alpha\phi}]^{1/(\alpha-1)} \text{ and } a_d = [\frac{(p-\gamma)S(p)}{q^{\alpha}\alpha\phi}]^{1/(\alpha-1)} = q^{\alpha/(1-\alpha)}a_t, \tag{9}$$

or equivalently

$$a_t = [\frac{(\underline{p} - \gamma)S(\underline{p})}{\alpha\phi}]^{1/(\alpha-1)} \text{ and } a_d = [\frac{(\underline{p} - \gamma)S(\underline{p})}{q^{\alpha}\alpha\phi}]^{1/(\alpha-1)}, \qquad (10)$$

where the second line follows from the proposition. If there are n firms producing each variety, then the total number of adverts per variety, a, is

$$a = n(a_t + a_d) = n(1 + q^{\alpha/(1-\alpha)})[\frac{(\underline{p} - \gamma)S(\underline{p})}{\alpha\phi}]^{1/(\alpha-1)}. \qquad (11)$$

5 Pricing

5.1 Advertised Price Distribution

Consumers receive ads randomly, without any targeting by firms–targeting is discussed in Section 12. Assume that there is a much larger mass of consumers vis à vis firms and that no consumer receives more than one ad from the same firm. Let a represent the number of ads for a variety per consumer in the economy. The number of ads, i, that a consumer receives will be distributed according to a Poisson distribution $e^{-a}a^i/i!$.[3] Now, let $P(p) = \Pr(\text{PRICE} \leq p)$ be the fraction of ads for a variety that have a price less than or equal to p. The function $P(p)$ is characterized later in Proposition 3.

Suppose a firm sends an ad to a consumer offering to sell the good at price p. The odds of a consumer with i *other* ads having no price lower than p are $[1-P(p)]^i$. Even when the firm's price p is the lowest one in the

[3] To see this, imagine an economy with a discrete number of consumers, $\mathfrak{c}$, who are flooded with $a\mathfrak{c}$ ads per variety. The probability that a consumer will receive i ads is distributed according to the binomial distribution

$$\binom{a\mathfrak{c}}{i}(\frac{1}{\mathfrak{c}})^i(1 - \frac{1}{\mathfrak{c}})^{a\mathfrak{c}-i},$$

where $1/\mathfrak{c}$ is the chance that a consumer gets an ad (success) and $1 - 1/\mathfrak{c}$ are the odds that they won't (failure). Out of a set of $a\mathfrak{c}$ ads there are $\binom{a\mathfrak{c}}{i}$ ways each event could happen. Finally,

$$\lim_{\mathfrak{c}\to\infty}\binom{a\mathfrak{c}}{i}(\frac{1}{\mathfrak{c}})^i(1 - \frac{1}{\mathfrak{c}})^{a\mathfrak{c}-i} = e^{-a}a^i/i!.$$

consumer's information set, the person may not buy the good because it is too expensive. Let $I(p;\tau)=1$ denote the situation when a type-τ consumer buys the firm's good at price p and $I(p;\tau)=0$ when not. It then follows that the probability that a consumer who has received an ad from the firm with price p will buy the good, given that they may have received $i=0,1,2,\cdots$ other ads, is given by[4]

$$\begin{aligned}S(p) &= \Pr(\text{SALE}|p) = e^{-a}\sum_{i=0}^{\infty}\frac{a^i}{i!}[1-P(p)]^i[\mathfrak{t}I(p;\underline{\tau})+(1-\mathfrak{t})I(p;\overline{\tau})]\\ &= e^{-aP(p)}[\mathfrak{t}I(p;\underline{\tau})+(1-\mathfrak{t})I(p;\overline{\tau})].\end{aligned}$$

Three prices play a central role in the analysis; namely, the minimum price in the economy, $\underline{p}$, the maximum price at which the unskilled will buy, $p(\underline{\tau})$, and the maximum price at which the skilled will purchase, $\overline{p}$. The minimum price is determined by technological considerations while the maximum prices also depend upon the outcome of the consumer problems for the unskilled and skilled, an important distinction from Butters (1977). The determination of $\underline{p}$ and $\overline{p}$ is discussed now with the specification of $p(\underline{\tau})$ following shortly after. Consider a firm that chooses to charge the minimum price, $\underline{p}$. All the ads that this firm sends out will result in purchases by consumers, implying $S(\underline{p})=1$. Since there is free entry into the production of any variety, this firm will earn zero profits. Hence,

$$\Pi(\underline{p};q)-\mathfrak{r}=0.$$

Solving this equation gives

$$\underline{p}=[\frac{\mathfrak{r}}{\Upsilon(q)}]^{(\alpha-1)/\alpha}+\gamma, \tag{12}$$

where

$$\begin{aligned}\Upsilon(q) &\equiv (1+q^{\alpha/(1-\alpha)})\phi^{1/(1-\alpha)}(\alpha^{1/(1-\alpha)}-\alpha^{\alpha/(1-\alpha)})\\ &= (1+q^{\alpha/(1-\alpha)})(\frac{1}{\phi})^{1/(\alpha-1)}(\frac{1}{\alpha})^{\alpha/(\alpha-1)}(\alpha-1)>0.\end{aligned}$$

[4] To go from the first to the second line, set $s=\sum_{i=0}^{\infty}(a^i/i!)x^i=[1+ax+(ax)^2/2!+(ax)^3/3!+\cdots]$, which implies that $ds/dx=[a+a^2(ax)+a^3(ax)+\cdots]=as$. Therefore, $(1/s)ds/dx=a$ so that $s=e^{ax}$. Now, let $x=1-P(p)$ to get $\sum_{i=0}^{\infty}(a^i/i!)[1-P(p)]^i=e^{a[1-P(p)]}$, from which the desired result follows.

(See the proof of Proposition 1 in the Appendix for guidance.) The minimum price, $\underline{p}$, is determined solely by technological factors. As a consequence so is the amount of traditional, a_t, and digital advertising, a_d, that each firm does, a fact that follows from (10) in conjunction with $S(\underline{p}) = 1$.

Since a firm is free to pick any price it must be the case that

$$\Pi(p'; q) = \Pi(p''; q), \text{ for any } p' \text{ and } p'' \in \mathcal{P}.$$

Proposition 1 states that all firms do the same amount of advertising. Therefore,

$$(p' - \gamma)S(p') = (p'' - \gamma)S(p''), \tag{13}$$

or equivalently

$$\begin{aligned}(p' - \gamma)e^{-aP(p')}[\mathfrak{t}I(p'; \underline{\tau}) + (1 - \mathfrak{t})I(p'; \overline{\tau})]\\ = (p'' - \gamma)e^{-aP(p'')}[\mathfrak{t}I(p''; \underline{\tau}) + (1 - \mathfrak{t})I(p''; \overline{\tau})].\end{aligned}$$

Turn to the firm that charges the highest price, $\overline{p}$. Only skilled consumers ($\tau = \overline{\tau}$) who have no other ads will buy the firm's product. Therefore, $S(\overline{p}) = e^{-a}(1 - \mathfrak{t})$, because $P(p) = 1$ (i.e., all ads have a price lower than $\overline{p}$). Therefore, evaluating the above expression at $p' = \overline{p}$ and $p'' = \underline{p}$ gives

$$e^{-a}(1 - \mathfrak{t})(\overline{p} - \gamma) = \underline{p} - \gamma,$$

so that the maximum price at which a skilled person buys a good is

$$\overline{p} = \frac{\underline{p} - \gamma}{e^{-a}(1 - \mathfrak{t})} + \gamma = \frac{[\mathfrak{r}/\Upsilon(q)]^{(\alpha-1)/\alpha}}{e^{-a}(1 - \mathfrak{t})} + \gamma. \tag{14}$$

Next, focus on the highest price that unskilled consumers can afford, $p(\underline{\tau})$. At any higher price there will be a discrete drop off in potential customers from 1 down to $1 - \mathfrak{t}$. To recover profits there must be discrete jump up in the lowest price above $p(\underline{\tau})$, denoted by $p_{\uparrow}(\underline{\tau})$. Since there are no prices in between $p(\underline{\tau})$ and $p_{\uparrow}(\underline{\tau})$ it transpires that $P(p(\underline{\tau})) = P(p_{\uparrow}(\underline{\tau}))$, which is formalized later in Proposition 3. The prices at the left and righthand sides of the jump must have equal profits, so that $(1 - \mathfrak{t})[p_{\uparrow}(\underline{\tau}) - \gamma] = p(\underline{\tau}) - \gamma$, which yields

$$p_{\uparrow}(\underline{\tau}) = \frac{p(\underline{\tau}) - \gamma}{1 - \mathfrak{t}} + \gamma. \tag{15}$$

Now, there must be firms charging every price, p, in the set $\mathcal{P} = [\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$. To understand why, suppose to the contrary that there is a hole in one of the intervals. Firms at the lower edge of the hole could increase profits by raising their price slightly, because this will not affect the number of customers they have. The proposition below describes the situation.

Proposition 2 *(Pricing) For any variety of regular goods there are firms charging every price, p, in the set $\mathcal{P} = [\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$. Take the aggregate amount of advertising per variety, a, as given. Then, both $\underline{p}$ and $\overline{p}$ are increasing in the entry cost, $\mathfrak{r}$, the marginal cost of production, γ, and the cost of digital advertising, q. Last, the maximum price, $\overline{p}$, is decreasing in the fraction of individuals, $1 - \mathfrak{t}$, who are skilled.*

Proof. See the Appendix. ■

It's probably obvious that an increase in the cost of doing business, as given by $\mathfrak{r}$, γ, and q, will lead to the pricing set $\mathcal{P} = [\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$ shifting rightward, because given the free-entry assumption firms must recover their costs. When there are more skilled consumers, $1 - \mathfrak{t}$, it becomes more profitable to charge the maximum price, $\overline{p}$, since the odds of an ad landing on a skilled person increase. But, again, perfect competition will drive the maximum price down so that firms earn zero profits.

Direct attention now to characterizing the distribution of prices in the set $\mathcal{P} = [\underline{p}, p(\underline{\tau})] \cup [p_{\uparrow}(\underline{\tau}), \overline{p}]$. Using the fact that $S(\underline{p}) = 1$ in equation (13) gives

$$(p - \gamma) e^{-aP(p)} [\mathfrak{t} I(p; \underline{\tau}) + (1 - \mathfrak{t}) I(p; \overline{\tau})] = \underline{p} - \gamma, \text{ for } p \in \mathcal{P}. \tag{16}$$

Since this equation must hold for all p in the pricing set, $\mathcal{P}$, it traces out the function $P(p)$.

Proposition 3 *(Advertised Price Distribution) The cumulative distribution for prices, $P(p)$, is given by*

$$P(p) = \Pr(\text{PRICE} \leq p) = \begin{cases} \ln\{(p - \gamma)/(\underline{p} - \gamma)\}/a, & \textit{for } p \in [\underline{p}, p(\underline{\tau})]; \\ \ln\{[p(\underline{\tau}) - \gamma]/(\underline{p} - \gamma)\}/a, & \textit{for } p \in [p(\underline{\tau}), p_{\uparrow}(\underline{\tau})]; \\ \ln\{(1 - \mathfrak{t})(p - \gamma)/(\underline{p} - \gamma)\}/a, & \textit{for } p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]. \end{cases} \tag{17}$$

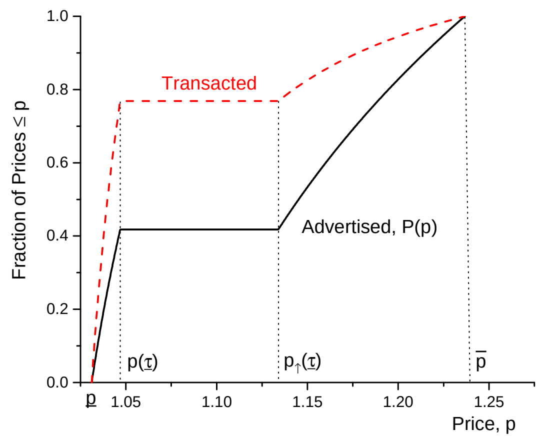

Figure 7: The cumulative distribution functions for both advertised prices, $P(p)$, and transacted prices that obtain in calibrated equilibrium for 2018–Section 10 discusses the model's calibration. It is not profitable for a firm to price in the open interval $\left(p(\underline{\tau}), p_{\uparrow}(\underline{\tau})\right)$. The distribution function for advertised prices stochastically dominates the one for transacted prices, because consumers buy at the lowest price in their information set.

The associated density function reads

$$P_1(p) = \begin{cases} 1/[a(p-\gamma)] > 0, for\ p \in [\underline{p}, p(\underline{\tau})]; \\ 0, \qquad\qquad\qquad\quad for\ p \in [p(\underline{\tau}), p_{\uparrow}(\underline{\tau})]; \\ 1/[a(p-\gamma)] > 0, for\ p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]. \end{cases} \tag{18}$$

Proof. See the Appendix. ■

Figure 7 illustrates the cumulative distribution for prices. For subsequent use note that $P_1(p)$ represents the fraction of ads offering to sell a variety at price p.

5.2 Number of Varieties

How many varieties, v, will be produced? Since there is free entry into the production of any variety of consumption goods all possible varieties will be sold. If this wasn't the case, a producer could move into a variety where no one else is producing and earn supra-normal profits because of the lack of competition in advertised prices. Individuals will not consume all varieties, though. People won't receive ads for some varieties and even when they do get ads some varieties may be too expensive for unskilled consumers.

Proposition 4 *(Number of Varieties) All consumption goods in the feasible set* $[0, 1]$ *will be produced; i.e.,* $v = 1$.

Proof. See the Appendix. ■

5.3 Maximum Price the Unskilled will Pay, $p(\underline{\tau})$

What is the maximum price, $p(\underline{\tau})$, at which an unskilled person will buy a good? To begin with, since $S(\underline{p}) = 1$, equation (13) also implies

$$S(p) = \frac{(\underline{p} - \gamma)}{(p - \gamma)}, \text{ for } p \in \mathcal{P}. \tag{19}$$

Let $B(p) = \Pr(\text{BUY})$ represent that the probability that a consumer will buy at price p. This is not quite the same as the probability that a firm will make a sale at price p, $S(p)$, because the latter averages over both types of consumers. The two probabilities are related as follows:

$$B(p) \equiv \Pr(\text{BUY}) = \begin{cases} S(p), & \text{for } p \in [\underline{p}, p(\underline{\tau})]; \\ S(p)/(1 - \mathfrak{t}), & \text{for } p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]. \end{cases}$$

For given variety, the odds of a purchase at price p by a consumer are $P_1(p)B(p)$. Since there is a unit mass of varieties, a type-τ person's budget constraint can be written as

$$a \int_{\underline{p}}^{p(\tau)} p P_1(p) B(p) dp = \tau h(\tau), \tag{20}$$

where $h(\tau)$ is hours worked and $p(\tau)$ denotes the time price of the most expensive good the person will buy; i.e., $p(\tau) = p(\underline{\tau})$, for $\tau = \underline{\tau}$, and $p(\tau) = \overline{p}$, for $\tau = \overline{\tau}$. Equation (20) pins down $p(\underline{\tau})$. To see this, set $\tau = \underline{\tau}$ in (20) and perform the required integration, while using (18) and (19), to obtain

$$a(\underline{p} - \gamma)\{\ln[\frac{p(\underline{\tau}) - \gamma}{\underline{p} - \gamma}] - \frac{\gamma}{p(\underline{\tau}) - \gamma} + \frac{\gamma}{\underline{p} - \gamma}\}/a = 1 - l(\underline{\tau}). \tag{21}$$

This equation determines $p(\underline{\tau})$.

6 Supply of Free Media Goods

By reference to (3) and (5), it is immediate that the quantity of media goods provided, $\mathfrak{m}$, is given by

$$\mathfrak{m} = n(\frac{a_t}{\zeta} + \frac{a_d}{\psi}) = n(\frac{\underline{p} - \gamma}{\alpha\phi})^{1/(\alpha-1)}[\frac{1}{\zeta} + \frac{q^{\alpha/(1-\alpha)}}{\psi}]. \tag{22}$$

7 The Consumer/Worker Problem

A consumer/worker's optimization problem is to maximize (1) subject to (2) by the choice of $\{c_i\}_i^v$ and l, for $\tau \in \{\underline{\tau}, \overline{\tau}\}$. Focus on a generic type-τ worker and index the regular goods from the lowest to the highest priced so that p_i is increasing in i. Let $c(\tau)$ signify the most expensive generic good consumed by the individual, which has the price $p(\tau)$. This also represents the person's overall consumption of generic goods because $c(\tau) = \int_0^{c(\tau)} c_i di$, as $c_i = 1$ for $i \in [0, c(\tau)]$. Now, from the budget constraint (2) it's clear that $c(\tau)$ can be written as a function of a person's productivity, τ, and hours worked, $h(\tau)$. So, write[5]

$$c(\tau) = C(h(\tau), \tau),$$

with

$$C_1(h(\tau), \tau) = \tau/p(\tau) > 0 \text{ and } C_2(h(\tau), \tau) = h(\tau)/p(\tau) > 0, \tag{23}$$

Using this fact, the consumer/worker's maximization problem can be reformulated as

$$W(\tau) = \max_{l(\tau)}\{\theta \ln[C(1 - l(\tau) - \mathfrak{e}; \tau, \mathfrak{e})] + \frac{(1-\theta)}{\rho} \ln[\kappa l(\tau)^\rho + (1-\kappa)\mathfrak{m}^\rho]\}. \tag{24}$$

The generic first-order condition for the leisure of a type-τ person, or $l(\tau)$, is:

$$\underbrace{\frac{\theta}{c(\tau)}\frac{\tau}{p(\tau)}}_{\text{MARGINAL COST OF LEISURE}} = \underbrace{(1-\theta)\frac{\kappa l(\tau)^{\rho-1}}{\kappa l(\tau)^\rho + (1-\kappa)\mathfrak{m}^\rho}}_{\text{MARGINAL BENEFIT OF LEISURE}}, \text{ for } \tau \in \{\underline{\tau}, \overline{\tau}\}. \tag{25}$$

[5]Note that $\int_0^{\mathfrak{c}} p_i c_i di = \int_0^{c(\tau)} p_i di = \tau h(\tau)$. To compute $C_1(h(\tau), \tau)$, take the total differential of the above equation while using Leibniz's rule to get $p_{c(\tau)} c_{c(\tau)} dc(\tau) = \tau dh(\tau)$ so that $dc(\tau)/dh(\tau) \equiv C_1(h(\tau), \tau) = \tau/p(\tau)$. A similar calculation gives the formula for $C_2(h(\tau), \tau)$.

The righthand side of this equation is the marginal benefit from an extra unit of leisure. It is increasing in the quantity of media leisure goods, $\mathfrak{m}$, since $\rho < 0$. The lefthand side is the marginal cost of leisure. An extra unit of leisure leads to a drop in income for a type-τ person. This causes a drop in regular consumption, $c(\tau)$, of $\tau/p(\tau)$, where $p(\tau)$ is the price of the last regular good consumed. This is multiplied by the marginal utility of regular goods, $\theta/c(\tau)$.

The upshot of this first-order condition is given by the proposition below.

Proposition 5 *(Consumption/Leisure) An individual's consumption and leisure satisfy the following properties:*

1. *Leisure, $l(\tau)$, is increasing in the number of media leisure goods, $\mathfrak{m}$;*

2. *Regular consumption, $c(\tau)$, is decreasing in the number of media leisure goods, $\mathfrak{m}$, and is increasing in the level of skill, τ;*

3. *Work effort, $h(\tau)$, rises with the cost of an education, $\mathfrak{e}$.*

Proof. See the Appendix. ■

The first point follows from the fact that an increase in the number of media goods, $\mathfrak{m}$, raises the marginal benefit of leisure, $l(\tau)$, because the two goods are complements in the utility function (i.e., $\rho < 0$). Next, the rise in leisure, $l(\tau)$, is connected with a drop in work effort, $h(\tau)$, that reduces regular consumption, $c(\tau)$. An increase in τ decreases the marginal cost of regular consumption in terms of forgone leisure. Hence, regular consumption rises. The third result transpires because an increase in $\mathfrak{e}$ raises the marginal cost of leisure for any given level of hours worked, $h(\tau)$, implying that regular consumption, $c(\tau)$, will be lower. This property is important because it implies that if an education is costly enough, then the skilled will work more than the unskilled. This allows the framework to explain the recent rise in the unskilled's leisure relative to the skilled's.

Last, the overall consumption of generic goods by a type-τ person, $c(\tau)$ for $\tau \in \{\underline{\tau}, \overline{\tau}\}$, is given by

$$c(\tau) = a \int_{\underline{p}}^{p(\tau)} B(p) P_1(p) dp \text{ [cf. (20)]}.$$

where again $P_1(p)B(p)$ represents the odds of a purchase at price p. Evaluating the integral at $\tau = \underline{\tau}$ gives consumption for an unskilled person,

$$\begin{aligned} c(\underline{\tau}) &= a \int_{\underline{p}}^{p(\underline{\tau})} \frac{\underline{p} - \gamma}{a(p-\gamma)^2} dp \text{ [using (18) and (19)]} \\ &= 1 - S(p(\underline{\tau})). \end{aligned} \tag{26}$$

The expression has an intuitive interpretation since $1 - S(p(\underline{\tau}))$ represents the odds for each variety of getting at least one advertised price less than or equal to $p(\underline{\tau})$. Alternatively, when $\tau = \overline{\tau}$ the formula yields a skilled person's consumption,

$$\begin{aligned} c(\overline{\tau}) &= a\left[\int_{\underline{p}}^{p(\underline{\tau})} \frac{\underline{p} - \gamma}{(p-\gamma)^2} dp + \frac{1}{1-\mathfrak{t}} \int_{p_{\uparrow}(\underline{\tau})}^{\overline{p}} \frac{\underline{p} - \gamma}{(p-\gamma)^2} dp\right] / a \\ &= (1 - e^{-a}). \end{aligned} \tag{27}$$

Here, $1 - e^{-a} = 1 - S(\overline{p})$ is the probability of receiving at least one ad per variety.

8 Equilibrium

In equilibrium the labor market must clear. The labor-market-clearing condition reads

$$\begin{aligned} &\gamma[\mathfrak{t}c(\underline{\tau}) + (1-\mathfrak{t})c(\overline{\tau})] + n[A(a_t) + A(qa_d) + \mathfrak{r}] \\ &\qquad = \mathfrak{t}[1 - l(\underline{\tau})] + (1-\mathfrak{t})\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}]. \end{aligned} \tag{28}$$

The lefthand side is the demand for labor. The first term, $\gamma[\mathfrak{t}c(\underline{\tau}) + (1-\mathfrak{t})c(\overline{\tau})]$, is the demand for labor originating from the consumption of regular goods. The second term represents the labor used in advertising and absorbed by the fixed costs associated with entry for the n regular firms, $n[A(a_t) + A(qa_d) + \mathfrak{r}]$. The righthand side is the supply of labor from unskilled and skilled workers. This condition can be thought of as tying down the number of entrants, n, into a variety of regular goods.

It's now time to take stock of things.

Definition of an Equilibrium *An equilibrium for the economy is defined by a solution for advertising, a_t, a_d, and a, overall consumption, $c(\underline{\tau})$, and $c(\overline{\tau})$, the quantity of media goods consumed, $\mathfrak{m}$, labor supply, $l(\underline{\tau})$ and $l(\overline{\tau})$, the number of firms producing a variety, n, and the prices of regular goods, $\underline{p}$, $\overline{p}$, $p(\underline{\tau})$,* and *$p_{\uparrow}(\underline{\tau})$, such that:*

1. *Advertising is done in accordance with (10) and (11), which determine a_t, a_d, and a, where $S(\underline{p}) = 1$. These solutions depend on the values for n and $\underline{p}$.*

2. *The minimum and maximum time prices for regular goods, $\underline{p}$ and $\overline{p}$, are regulated by (12) and (14), taking as given a.*

3. *The highest time price paid by an unskilled person, $p(\underline{\tau})$, is described by the pricing equation (21), assuming values for a, $l(\underline{\tau})$, and $\underline{p}$. The price for the skilled at the jump point, $p_{\uparrow}(\underline{\tau})$, is determined by (15) as a function of $p(\underline{\tau})$.*

4. *The quantity of media goods consumed, $\mathfrak{m}$, is given by (22), where the solution for $\mathfrak{m}$ is dependent on a_t, a_d, and n.*

5. *The solution to the consumer-worker's problem for $c(\tau)$ and $l(\tau)$ is governed by (25), (26), and (27) for $\tau \in \{\underline{\tau}, \overline{\tau}\}$, given $p(\tau)$ and $\mathfrak{m}$. These solutions take as given a, $\mathfrak{m}$, $\underline{p}$, $p(\underline{\tau})$, $p_{\uparrow}(\underline{\tau})$, and $p(\overline{\tau}) = \overline{p}$.*

6. *The labor market clears in accordance with (28), which gives the number of firms per variety, n, as a function of a_t, a_d, $c(\underline{\tau})$, $c(\overline{\tau})$, $l(\underline{\tau})$, and $l(\overline{\tau})$.*

9 Efficiency of the Equilibrium

The competitive equilibrium is not efficient. This transpires for two reasons why the equilibrium is not efficient. First, ads offering to sell goods at high prices are being sent to unskilled consumers that can never afford to buy them. This is a social waste of resources. Second, when engaging in advertising, firms do not take into account how the introduction of free media goods benefits the consumer. So, there is an underprovision of media goods.

The Pareto optima for the economy can be traced out by solving the following planning problem, where $\xi \in [0,1]$ is the relative planning weight that is being placed on unskilled individuals:

$$\max_{c(\underline{\tau}),c(\overline{\tau}),a_t,a_d,n,l(\underline{\tau}),l(\overline{\tau})} \Big(\xi\mathfrak{t}\theta \ln c(\underline{\tau}) + \frac{\xi\mathfrak{t}(1-\theta)}{\rho}\ln\{\kappa l(\underline{\tau})^{\rho} + (1-\kappa)[n(\frac{a_t}{\zeta}+\frac{a_d}{\psi})]^{\rho}\}$$
$$+ (1-\mathfrak{t})\theta \ln c(\overline{\tau}) + \frac{(1-\mathfrak{t})(1-\theta)}{\rho}\ln\{\kappa l(\overline{\tau})^{\rho} + (1-\kappa)[n(\frac{a_t}{\zeta}+\frac{a_d}{\psi})]^{\rho}\}\Big), \tag{29}$$

subject to

$$1 - e^{-(a_t+a_d)n} - c(\overline{\tau}) = 0, \tag{30}$$

and

$$\mathfrak{t}[1-l(\underline{\tau})] + (1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}] - \mathfrak{t}\gamma c(\underline{\tau}) - \gamma(1-\mathfrak{t})c(\overline{\tau}) - n[A(a_t)+A(qa_d)+\mathfrak{r}] = 0. \tag{31}$$

An interpretation of this problem is that the planner is giving unskilled and skilled people coupons in the amounts $c(\underline{\tau})$ and $c(\overline{\tau})$. Each coupon entitles a person to one good at the store they go to. The total amount of coupons handed out is constrained by the resource constraint (31). The advertisements give the locations of the stores that sell each variety. Without an ad the consumer will not know where to buy a variety. The odds of getting at least one ad for any particular variety are $1 - e^{-(a_t+a_d)n}$. So, equation (30) states that the consumption for the skilled is constrained by the ads they receive.

The allocations from the planning problem can be supported in a competitive equilibrium using a tax-cum-subsidy scheme. The excessive amount of advertising can be corrected by levying a fine on all advertising and providing a subsidy for consumers on all goods sold. Specifically, consumers require a proportional price reduction, r, in the amount

$$r = 1 - \gamma/p(\underline{\tau}), \tag{32}$$

and all advertising should be fined at the rate

$$f = r\overline{p}(1-\mathfrak{t})e^{-(a_t+a_d)n}. \tag{33}$$

The underprovision of media goods can be rectified by providing a subsidy per media good in the amount s, where

$$s = \mathfrak{t}\frac{(1-\kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\underline{\tau})})^{\rho-1} + (1-\mathfrak{t})\overline{\tau}\frac{(1-\kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\overline{\tau})})^{\rho-1}. \tag{34}$$

This is equivalent to subsidizing traditional and digital advertising at the rates s/ζ and s/ψ. The above policy should be financed by lump-sum taxation in line with

$$ra[\mathfrak{t}\int_{\underline{p}}^{p(\tau)} pP_1(p)B(p)dp + (1-\mathfrak{t})\int_{\underline{p}}^{\overline{p}} pP_1(p)B(p)dp] + sn(\frac{a_t}{\zeta} + \frac{a_d}{\psi})$$
$$= \mathfrak{t}t(\underline{\tau}) + (1-\mathfrak{t})t(\overline{\tau}) + fn(a_t + a_d), \tag{35}$$

where $t(\underline{\tau})$ and $t(\overline{\tau})$ are the lump-sum taxes levied on the unskilled and skilled. The way these taxes are raised affects the economy's income distribution.

Proposition 6 *(Efficiency) The solution to the planning problem (29) can be supported as a competitive equilibrium with the tax-cum-subsidy scheme specified by (32), (33), and (34) that is financed by lump-sum taxation in accordance with (35).*

Corollary 1 *(Single agent economy) Suppose there is only one type of consumer/worker. Then only a subsidy on media goods is required.*

Proof. See the Appendix. ■

The intuition for the above tax-cum-subsidy scheme is this. The skilled consume more varieties than the unskilled. A certain amount of advertising is required to effect this. There is no need to do any extra advertising to support the unskilled's consumption. So, the last variety sold to an unskilled person should be priced at its marginal production cost implying that $(1-r)p(\underline{\tau}) = \gamma$, where r is the required proportional price reduction. When determining how much advertising to do firms use the price p instead of the subsidized price $(1-r)p$, where the latter reflects the value of the good to a consumer. Since $p > (1-r)p$ there is propensity toward too much advertising. This is corrected by fining advertising in general at the rate f.

Last, firms neglect the fact that media goods are valuable to consumers. Therefore, they under provide them. This is rectified by subsidizing media goods. The subsidy, s, is just a weighted average of each group's marginal rate of substitution between leisure and media goods, as can be seen from (34). The marginal rates of substitution reflects how much an extra media good is worth to a person in terms of leisure. For a skilled person a unit of leisure is worth more than for an unskilled person, as reflected by $\overline{\tau}$. Last, the click-through rate specifies how efficient advertising is.

10 Calibration

In order to simulate the model, values have to be assigned to the following parameters: θ, κ, ρ, $\mathfrak{m}$, γ, α, ϕ, ζ, ψ, q, $\mathfrak{r}$, $\overline{\tau}$, $\mathfrak{e}$, and $\mathfrak{t}$. Most of the parameter values are unique to this study. The strategy is to pin down parameter values by using data on markups, the advertising-to-consumption ratio, the click-through rate, the hike in the ratio of spending on digital versus traditional advertising, and the step up in the time spent on leisure by non-college- and college-educated individuals.

Some parameter values can be set straightforwardly. The unskilled in the model are taken to be the non-college educated. They represent 65 percent of the population. The productivity of college graduates is set to match the income of this group relative to the non-college educated. At the observed levels of labor supplies this implies that college graduates are 2.35 times as productive as the non-college educated. Accordingly, $\mathfrak{t} = 0.65$ and $\overline{\tau} = 2.35$. The click-through rate on digital advertising is very low, roughly 2.5 percent. This dictates setting $\psi = 0.025$. The choice of some parameters are normalizations. On this, γ and ϕ control the units that output and advertising are measured in. Hence, $\gamma = \phi = 1$.

The rest of the parameter values are selected by targeting a set of stylized facts. The long and short of the calibration procedure is this–a detailed explanation is provided in Appendix 15. The model's calibration is divided into two parts; viz, the firm side that determines the advertising parameters and a consumer side that pins down the preference ones. These two parts are linked. On the firm side, an important parameter is the cost elasticity for advertising, α. To calibrate this parameter, a markup of 7 percent for the average transacted price over marginal production cost is chosen–this number is taken from Basu (2019).[6] As mentioned in the introduction, advertising has been roughly 2 percent of GDP for the last 100 years. This leads to the following two restrictions on the calibration exercise.

$$\begin{aligned}\text{MARKUP} &= 1.07 = E[p]/\gamma \\ &= [\frac{\int_{\underline{p}}^{p(\underline{\tau})} pB(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} pB(p)P_1(p)dp}{\int_{\underline{p}}^{p(\underline{\tau})} B(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} B(p)P_1(p)dp}]/\gamma,\end{aligned} \tag{36}$$

and

$$\text{A2C} = 0.02 = \frac{n[A(a_t) + A(qa_d)]}{\mathfrak{t}c(\underline{\tau}) + (1-\mathfrak{t})c(\overline{\tau})}. \tag{37}$$

The markup is calculated using the transacted price distribution, shown in Figure 7. The restrictions (36) and (37) are used to pin down a value for α, which governs the marginal cost of advertising. The backed-out value for α is similar to the one used by Dinlersoz and Yorukoglu (2012) to generate a reasonable equilibrium firm-size distribution. In fact, alternatively, if their value is used for α, then the model here would predict a markup of 7 percent. These restrictions also determine the fixed entry cost, $\mathfrak{r}$.

The ratio of spending on digital to traditional advertising rose from 0.02 percent 2003 to 0.28 in 2018. These numbers are used to calibrate the rise in the relative efficiency of digital advertising, or q, over this time period. Hence, the following condition is imposed on the calibration exercise:

$$\text{D2T} = \frac{A(qa_d)}{A(a_t)} = \begin{cases} 0.02, \text{for } 2003; \\ 0.07, \text{for } 2010; \\ 0.28, \text{for } 2018. \end{cases} \tag{38}$$

When calibrating the firm side of the model, the labor allocations from the consumer side are taken as given. Given these labor allocations, the firm-side calibration hits exactly the three data targets given by (36), (37), and (38).

[6] The size of price markups is controversial. The number used here is conservative: the larger is the price markup, the bigger will be welfare gain from digital advertising.

For the consumer side, the preference parameters θ, κ, and ρ, plus the parameter governing the cost of education, $\mathfrak{e}$, are chosen to match the levels of leisure for non-college and college educated for the years 2003, 2010, and 2018. Additionally, these observations are also used to infer a value for the click-through rate, ζ, on traditional advertising. Leisure is defined as all time spent on entertainment, social activities, relaxing, active recreation, sleeping, eating, and personal care; this definition corresponds with Aguiar and Hurst (2007, Table III, measure 2). The trend in leisure is charted in Figure 8. Leisure for the non-college educate rose from 64.1 percent of time not working in 2003 to 65.4 in 2018. The increase for college graduates was from 60.7 to 61.6. In each year college graduates enjoyed less leisure than the non-college educated. Galbi (2001) has noted that, historically speaking, increases in discretionary time use are closely related to the waxing in time spent on media. So, the figure also tracks the gain in leisure since 2003 accounted for by the time consumed on media; namely, TV, radio, reading, movies, computers, and games. The model will be calibrated to the gains in leisures linked with the increased time spent on media.

Since media goods are free their quantity is not recorded in the national income accounts. The data on time spent not working, both over time and between the non-college and college educated, is used to infer the quantity of media goods. Since media goods and leisure are Edgeworth-Pareto complements, an increase in the supply of the media goods should lead to more time spent not working, ceteris paribus; recall Proposition 5. This type of strategy was introduced in Goolsbee and Klenow (2006) and followed by Brynjolfsson and Oh (2012). Things are more complicated here, though. The advent of digital advertising also affects the prices of consumer goods, which will have an impact on leisure as well.

Let $\text{LEISURE}_t(\tau)$ represent the leisure target for a type-τ person in year t. Then, formally speaking, the parameter values in question solve

$$\min_{\theta,\kappa,\rho,\mathfrak{e},\zeta} \sum_{\tau=\underline{\tau},\overline{\tau}} \sum_{t=03,10,18} [l_t(\tau) - \text{LEISURE}_t(\tau)]^2, \tag{39}$$

subject to (36), (37), and (38). The constraints take into account how the choice of the preference parameters interacts with the firm-side cal-

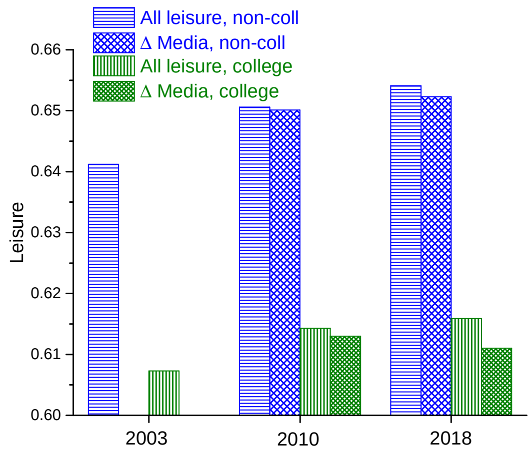

Figure 8: Leisure for the non-college and college educated. Also shown is the increase in leisure since 2003 comprised by shifts in time spent on media (Δ Media). The bars for 2003 and the cross-hatched ones for 2010 and 2018 are used in the model's calibration. *Source*: American Time Use Survey.

ibration.

The upshot from the calibration procedure is displayed in Table 2.

11 Welfare

A person's welfare, $W(\tau)$, reads

$$W(\tau) = \theta \ln(c(\tau)) + \frac{(1-\theta)}{\rho} \ln[\kappa l(\tau)^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}], \text{ for } \tau \in \{\underline{\tau}, \overline{\tau}\},$$

where $c(\tau)$, $l(\tau)$, and $\mathfrak{m}$ represent the allocations for consumption, leisure, and media goods under some particular scenario. From this it is clear that any change in welfare can be broken down into changes in $c(\tau)$, $l(\tau)$, and $\mathfrak{m}$. Now consider two different scenarios, A and B. In order to move to regime A a type-τ person living in regime B would have to be compensated by boosting his regime-A consumption by the factor

$$\text{EV}(\tau) = e^{[W_B(\tau) - W_A(\tau)]/\theta} - 1.$$

Calibrated Parameter Values

Parameter Values	*Description*	*Identification*
Consumers		
$\theta = 0.3499$	Consumption weight	Data, Eq (39)
$\kappa = 0.0060$	Weight on leisure, CES	Data, Eq (39)
$\rho = -5.0896$	Elasticity of substitution	Data, Eq (39)
$\mathfrak{t} = 0.65$	Low-type fraction	Imposed
$\overline{\tau} = 2.3433$	High-type productivity	Data
$\mathfrak{e} = 0.0952$	Cost of skill	Data, Eq (39)
Firms		
$\gamma = 1$	Marginal production cost	Normalization
$\mathfrak{r} = 0.0028$	Entry fixed cost	Data, Eqs (36) and (37)
Advertising		
$\alpha = 3.0148$	Cost elasticity	Data, Eqs (36) and (37)
$\phi = 1$		Normalization
$q_{03} = 12.0922, q_{10} = 5.9132$	Efficiency of digital adv.	Data, Eq (38)
$q_{18} = 2.3302$		
$\psi = 0.025$	Click-through rate, digital	Data
$\zeta = 0.4281$	Click-through rate, traditional	Data, Eq (39)

Table 1: The parameter values that result from the calibration procedure.

Data Targets

Description	*U.S. Data*	*Model*
Markup, 2018	1.07	1.07
Advertising/consumption, 1919-2019	0.022	0.022
Digital/traditional advertising		
2018	0.282	0.282
2010	0.070	0.070
2003	0.024	0.024
Leisure		
Non-college, 2018	0.6523	0.6520
College, 2018	0.6110	0.6115
Non-college, 2010	0.6501	0.6505
College, 2010	0.6130	0.6124
Non-college, 2003	0.6412	0.6411
College, 2003	0.6073	0.6074

Table 2: The data targets used in the calibration exercise and the corresponding numbers for the model. The calibration procedure hits the firm side numbers exactly while maximizing the model's fit for leisure.

That is, $\text{EV}(\tau)$ measures a type-τ person's equivalent variation.[7]

11.1 The Change in Welfare from 2003 to 2018

Between 2003 and 2018, advertising became more efficient. This had three effects. First, consumers benefited from the introduction of new media goods.[8] Second, leisure rose. Third, the reduction in hours was associated with a decline in consumption. By how much did welfare improve overall?

Table 3 shows the results. Welfare increased for the non-college and college educated by 2.5 and 2.6 percent, in terms of consumption. For both groups of individuals, there is a significant increase in welfare due to the expansion of free media goods connected with digital advertising. The non-college educated realize a significant gain in welfare from their rise in leisure. This occurs because media goods and leisure are complements in utility; recall Proposition 5. The welfare gain from the increase in leisure is mostly offset by a decline in non-college educated consumption. The college educated enjoyed a smaller improvement in welfare from the rise in leisure. Their decline in consumption is negligible. The reduced work effort by the college-educated is counteracted by a reduction in prices stimulated by increased competition. The large boost in welfare generated by the free provision of media goods is not reflected in GDP for two reasons. First, advertising spending is deducted from firm's profits in the GDP accounts. Corrado, Hulten, and Sichel (2009) recommend counting (a portion of) advertising as an intangible investment in the GDP accounts–McGrattan and Prescott (2010) express a similar view. This would increase GDP by a maximum of 2 percent. Second, GDP is not the same as economic welfare. For example, elec-

[7] In otherwords, $\text{EV}(\tau)$ solves the equation

$$W_B(\tau) = \theta \ln[(1+\text{EV}(\tau))c_A(\tau)] + \frac{(1-\theta)}{\rho}\ln[\kappa l_A(\tau)^\rho + (1-\kappa)(\mathfrak{m}_A)^\rho].$$

[8] Marshall (1920, p. 307) notes "the dependence of newspapers and magazines on receipts from advertisements. They are thereby enabled to provide a larger amount of reading matter than would otherwise be possible ..."

The Increase in Welfare from 2003 to 2018

	EV	Consumption	Media Goods	Leisure
Non-college	2.5%	-2.43%	1.81%	3.01%
College	2.6%	-0.02%	1.39%	1.21%

Table 3: The welfare gains from the expansion of free media goods arising from the advent of digital advertising. These welfare gains are decomposed into the effects that digital advertising had on regular consumption, media goods provision, and leisure.

tricity constitutes around 2 percent of expenditure yet Greenwood and Kopecky (2013) estimate it has a compensating variation of 92 percent with there existing no equivalent variation; i.e., it isn't possible to give a person today enough income to compensate them for living without electricity.

The above estimate of the improvement in welfare is not out of line with other work. Goolsbee and Klenow (2006) estimate that in the internet was worth somewhere between 2 to 3 percent of income to the average consumer in 2005, but this could be as high as 27 percent depending on the preferred specification. Greenwood and Kopecky (2013) place the welfare gain from the introduction of personal computers at somewhere between 2 to 3 percent of GDP in 2004. Brynjolfsson and Oh (2012) calculate, using the Greenwood and Kopecky (2013) method, that the introduction of free media goods was worth about 5 percent of consumption in 2011.

It's dangerous to prognosticate about the future, but suppose, as a thought experiment, that technological advance in digital advertising continues until 2040 at the same rate as between 2003 and 2018. From 2018 to 2030 the non-college educated would see their welfare climb by an additional 1.7 percent, while the college educated would enjoy a benefit of 4.1 percent. By 2040 the respective numbers would be 3.0 and 7.8 percent. The cumulative welfare gains from 2003 on are shown in Figure 9. These welfare gains can be broken down. The free provision of media goods see strong diminishing returns kick in after 2018. The extra supply of free media goods increases welfare for the non-college- and college-educated population by 0.04 and 0.03 percent for the 2018-2040 period. This is trivial compared with the gain between 2003 and 2018.

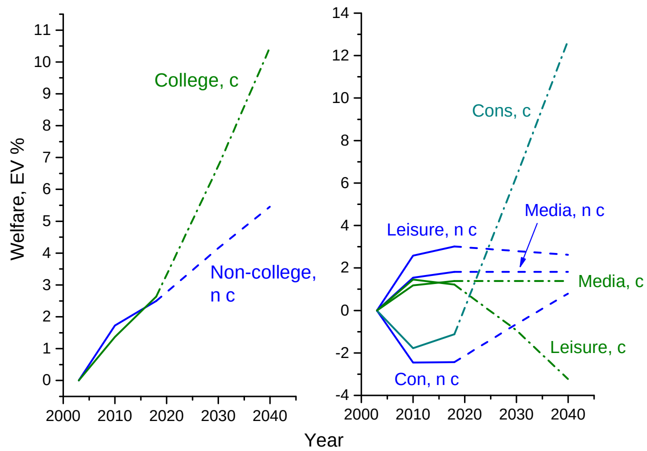

Figure 9: Cumulative welfare changes for the non-college and college educated population. The dashed and dot-dashed portions of the lines show the extrapolations from 2018 to 2040. Over this future period the college-educated gain a lot in welfare from generic consumption due to increased competition at the upper end of the price distribution. This is partially offset by a decline in welfare because of a reduction in leisure motivated by the rise in the return on working for the college educated.

Most of the hike in welfare over this period derives from more generic consumption resulting from more intense price competition; for the two parties, the numbers are 3.2 and 12.3 percent. Interestingly, leisure drops for both parties, which contributes welfare losses of 0.3 and 4.4 percent. The more precipitous loss for the college educated occurs because they realize a significant boost in their effective real wage because of a drop in prices at the upper end of the price distribution.

11.2 Who pays?

Who is implicitly paying for the provision of free media goods? Specifically, does consumption by the upper end of the population help the lower end by stimulating the supply of free media goods? To begin with, the non-college share of advertising expenditure is given by

$$\frac{\mathfrak{t}[1 - l(\underline{\tau}) - \gamma c(\underline{\tau})] - \mathfrak{t}S(p(\underline{\tau}))n\mathfrak{r}}{n[A(a_t) + A(qa_d)]},$$

where $\mathfrak{t}S(p(\underline{\tau}))$ is the share of advertised prices below $p(\underline{\tau})$ that are sent to the non-college educated. The numerator is non-college income less the cost of their consumption, $\mathfrak{t}[1 - l(\underline{\tau}) - \gamma c(\underline{\tau})]$, minus their prorated share of the fixed cost of production, $\mathfrak{t}S(p(\underline{\tau}))n\mathfrak{r}$. By this metric the

Share of Digital Advertising Costs

	Undirected		*Directed*	
	Share	Share/(Pop Share)	Share	Share/(Pop Share)
Non-college	27.15%	42%	39.62%	61%
College	72.85%	208%	60.38%	173%

Table 4: The fraction of the cost of free good provision paid for by the non-college- and college-educated populations. The last two columns refer to the directed advertising model introduced in Section 12.

Raising the College Premium

	EV	Consumption	Media Goods	Leisure
Non-college	0.003%	-0.01%	0.003%	0.01%
College	17.02%	14.56%	0.002%	2.39%

Table 5: The impact on welfare from increasing college-educated productivity by 20 percent in the 2018 benchmark equilibrium. The welfare gains are broken down into those arising from shifts in regular consumption, media goods provision, and leisure.

non-college educated pay 27 percent of the cost of advertising–see Table 4. Note that the non-college educated represent $100 \times \mathfrak{t} = 65$ percent of the population, so the percentage share per person is only 42 percent. The college-educated pay more than their share because they buy goods at higher prices where the markups are larger.

To gain further insight into this question, the labor-market productivity of the college educated, $\overline{\tau}$, is increased by 20 percent. Welfare for the non-college educated is boosted by a paltry 0.003 percent–see Table 5. This is smaller than Lucas' (1987) estimate of the welfare gains from eliminating business cycles. The gain comes from a 24.8 percent increase in the provision of free media goods. This stimulates leisure slightly, because media goods and leisure are complements in utility. Consumption drops because of the decrease in hours worked. Not surprisingly, the college-educated enjoy a big improvement in welfare, which derives mostly from their increase in consumption due to a higher labor income.

Implementing the Efficient Equilibrium

EV, Non-college	EV, College	r	f	s	$\frac{t(\underline{\tau})}{1-l(\underline{\tau})}$	$\frac{t(\overline{\tau})}{1-l(\overline{\tau})-\mathfrak{e}}$
0.02%	0.03%	5.9%	1.0%	0.00%	1.69%	2.21%

Table 6: The tax-cum-subsidy policy needed to make the competitive equilibrium efficient and the welfare gains from doing so.

11.3 Public Policy

By how much would welfare improve if the tax-cum-subsidy scheme proposed in Section 9 was implemented? The upshot is presented in Table 6. Moving to the efficient equilibrium has a small welfare gain, worth about 0.02 percent for the non-college educated and about 0.03 percent for the college educated. These are smaller than some of the magnitudes calculated in traditional welfare analyses, such as Rees's (1963) estimate of the welfare cost of labor unions, which he found to be 0.13 percent of GDP. Implementing the efficient equilibrium would require a fairly large intervention in the economy. The purchase of consumption goods would have to be subsidized at 6 percent in order to align the marginal price paid by the non-college educated to its marginal production cost. Advertising in general would face a small fine of 1.0 percent. Media goods provision would have to be subsidized at an insignificant rate to compensate for the underprovision of media goods. Last, the lump-sum taxes required to implement the program would amount to 1.7 percent of labor income for the non-college educated and 2.2 percent for the college educated. While in the rarefied confines of the model such a policy is desirable, this is unlikely to be the case in the real world especially given the small welfare gain. The advertising equilibrium modeled is surprisingly close to being efficient.

12 Directed Advertising

Advertisers now collect vast amounts of information on consumers. Suppose instead that advertising can be directed only toward those consumers who will potentially buy the product, but that anyone can use the free media goods used to disseminate the ad. In such a setting there is no point sending an ad with a very high price to a consumer who can't afford to purchase the good at this price. The framework essentially bifurcates into two spheres of economic activity, one for each consumer type. A firm can decide which group of consumers to sell to and at what price. These two spheres are only linked via the free-entry condition and the provision of free media goods. A capsule summary of the revised setup is now presented.

First, firms separate into two groups that are mutually exclusive, those that sell to low- and high-type consumers. The number of firms per variety in each group is different, denoted by n_τ, for $\tau \in \{\underline{\tau}, \overline{\tau}\}$. Within each group firms solve an advertising problem of the form (7). Since firms' profits must be the same across groups and prices, all firms in the economy will do the same amount of traditional and digital advertising, a_t and a_d. Denote the total amount of adverts within any variety for a group by $a_\tau = n_\tau(a_t + a_d)$, for $\tau \in \{\underline{\tau}, \overline{\tau}\}$.

Second, each group of consumers faces their own advertised price distribution, $P_\tau(p)$ for $\tau \in \{\underline{\tau}, \overline{\tau}\}$. This occurs because they are targeted separately. As before, let the maximum prices for each group be represented by $p(\underline{\tau})$ and $\overline{p}$. These prices solve

$$[p(\underline{\tau}) - \gamma]e^{-a_{\underline{\tau}}/\mathrm{t}} = \underline{p} - \gamma \text{ and } (\overline{p} - \gamma)e^{-a_{\overline{\tau}}/(1-\mathrm{t})} = \underline{p} - \gamma.$$

The minimum price, $\underline{p}$, is the same as in the equilibrium with undirected advertising because, as was mentioned, this price depends only on technological considerations. As a consequence, the amount of traditional, a_t, and digital advertising, a_d, that each firm does is identical in the two equilibriums. The two advertised price distributions are

$$P_\tau(p) = \Pr(\text{PRICE} \leq p) = \begin{cases} \ln\{(p-\gamma)/(\underline{p}-\gamma)\}\mathbf{t}/a_{\underline{\tau}}, & \text{for } \tau = \underline{\tau}; \\ \ln\{p-\gamma]/(\underline{p}-\gamma)\}(1-\mathbf{t})/a_{\overline{\tau}}, & \text{for } \tau = \overline{\tau}. \end{cases}$$

Neither price distribution exhibits a flat portion associated with a jump in prices.

Third, there are separate resource constraints for each of the two spheres:

$$\gamma c(\underline{\tau}) + n_{\underline{\tau}}[A(a_t) + A(qa_d) + \mathfrak{r}]/\mathbf{t} = 1 - l(\underline{\tau}),$$

and

$$\gamma c(\overline{\tau}) + n_{\overline{\tau}}[A(a_t) + A(qa_d) + \mathfrak{r}]/(1-\mathbf{t}) = \overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}].$$

Last, the consumption of media goods, $\mathfrak{m}$, for both groups of individuals is given by

$$\mathfrak{m} = (n_{\underline{\tau}} + n_{\overline{\tau}})(a_t/\zeta + a_d/\psi).$$

How would a move from a world where advertising is undirected to a world where it is directed affect welfare? To conduct this experiment, the parameter values from the benchmark are retained so as to make things comparable. The results are somewhat surprising–see Table 7. Welfare drops ever so slightly for the non-college educated by 0.02 percent

MOVE TOWARD DIRECTED ADVERTISING

	EV	Consumption	Media Goods	Leisure
Non-college	-0.02%	0.05%	-0.03%	-0.05%
College	5.03%	9.88%	-0.02%	-4.44%

Table 7: The welfare gains from a move toward directed advertising These welfare gains are decomposed into the effects that digital advertising has on prices, media goods provision, and leisure.

but moves up for the college educated by 5.03 percent, in terms of consumption. Media goods consumption falls insignificantly for both groups because now there is marginally less advertising overall. This leads to a loss in welfare, ceterus paribus. The non-college educated reduce their leisure, because leisure and media goods are Edgeworth-Pareto complements in utility. The college educated realize a large gain in welfare from increased consumption because price competition is stimulated at the upper end of the price distribution. For the college educated the average price that they pay for goods drops by 4.6 percent, while there is no impact on the average price paid by the non-college educated. The maximum price paid by the college educated falls by 11.8 percent. As can be seen from the first-order condition (25) for the college educated, this amounts to an increase in the college-educated real wage, $\overline{\tau}/\overline{p}$, that stimulates work effort and discourages leisure. Figure 10 shows the shift in the transacted price distribution for the college educated. Last, note that the share of directed advertising paid for by the college educated drops–see Table 4.

12.1 Public Policy

The directed advertising economy is virtually efficient. A move to the efficient equilibrium leads to infinitesimal welfare gains of 0.003 and 0.001 percent for the non-college and college educated–see Table 8.

13 Conclusions

A model is developed where firms must advertise to sell goods. There are two modes of advertising; namely, traditional and digital. Advertising is executed via the provision of free media goods. These media goods complement leisure in utility. Since there is randomness in the ads that

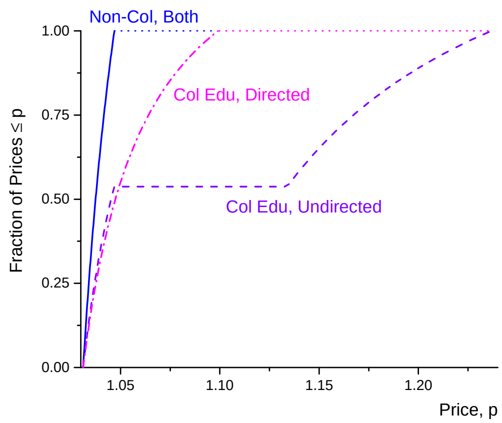

Figure 10: The cumulative distribution functions for transacted prices under both directed and undirected advertising. The college-educated purchase from a much better price distribution when advertising is directed. For the non-college educated the two prices distributions are virtually identical.

THE EFFICIENT EQUILIBRIUM WITH DIRECTED ADVERTISING

EV, Non-college	EV, College	r	f	s	$\frac{t(\underline{\tau})}{1-l(\underline{\tau})}$	$\frac{t(\overline{\tau})}{1-l(\overline{\tau})-\mathfrak{e}}$
0.003%	0.001%	0%	0%	0.007%	0.01%	0.13%

Table 8: The tax-cum-subsidy policy needed to make the competitive equilibrium with directed advertising efficient and the welfare gains from doing so.

consumers receive, firms set different prices for the exact same product. Hence, an equilibrium distribution of prices emerges. The advertising equilibrium is not efficient. First, free media goods are underprovided. Second, some advertising is wasteful in the sense that ads are sent to consumers who can't afford to purchase the good at the posted price. A tax-cum-subsidy policy that overcomes these inefficiencies is developed. Part of this policy involves subsidizing media goods provision and taxing advertising.

The developed model is matched up with some stylized facts from the U.S. data; in particular, the average price markup, the ratio of advertising expenses to consumption expenditure, the click-through rate for digital advertising, the growth in the ratio of spending on digital advertising relative to traditional advertising, and the rise in the time spent on leisure that was connected with media for both non-college- and college-educated people. Interestingly, the framework is consistent with the recent decrease in hours worked for the non-college educated relative to the college educated. The provision of free media goods via advertising is connected with a large increase in welfare. This gain in welfare is not incorporated into GDP because advertising is subtracted off from firms' profits. Additionally, GDP is not a good measure of welfare when new goods are introduced into an economy. College-educated consumers pay a disproportionately large share of the cost of these media goods because they purchase the higher-priced goods. They may benefit from the introduction of digital advertising, however, due to the expansion of price competition at the upper end of the goods market relative to the lower end. The tax-cum-subsidy policy that overcomes these inefficiencies associated with advertising has a small impact on welfare, which is swamped by the welfare gain from the free provision of media goods.

The competitive equilibrium with undirected advertising is compared with one where advertising is directed toward consumers that might actually buy the product. There is a slightly smaller supply of free media goods in the world with directed advertising because there is less advertising. This (negligibly) hurts those consumers who wouldn't have bought the product in the economy with undirected advertising. It benefits those consumers who would have bought the product in the economy with undirected advertising because now there is more price competition, which results in increased consumption.

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14 Appendix: Proofs

14.1 Competitive Equilibrium with Undirected Advertising

Proof of Proposition 1 (Advertising). Plugging these solutions for a_t and a_d, given by the first line of (9), into the objective function (7) gives

$$\Pi(p;q) = [(p-\gamma)S(p)]^{\alpha/(\alpha-1)}\Upsilon(q),$$

where

$$\begin{aligned}\Upsilon(q) &\equiv (1+q^{\alpha/(1-\alpha)})\phi^{1/(1-\alpha)}(\alpha^{1/(1-\alpha)}-\alpha^{\alpha/(1-\alpha)}) \\ &= (1+q^{\alpha/(1-\alpha)})(\frac{1}{\phi})^{1/(\alpha-1)}(\frac{1}{\alpha})^{\alpha/(\alpha-1)}(\alpha-1) > 0.\end{aligned}$$

Now, consider two firms charging two different prices, p' and p'', in the set $\mathcal{P}$. It must transpire that $\Pi(p'; q) = \Pi(p''; q)$, which can only be true if $[(p' - \gamma)S(p')]^{\alpha/(\alpha-1)} = [(p'' - \gamma)S(p'')]^{\alpha/(\alpha-1)}$. But then, from (9), the solutions for a_t and a_d must be the same. ■

Proof of Proposition 2 (Pricing). It's trivial to see from (12) and (14) that $\underline{p}$ and $\overline{p}$ are increasing in $\mathfrak{r}, \gamma$, and q. Last, $\overline{p}$ falls with $(1 - \mathfrak{t})$, as is immediate from (14). ■

Proof of Proposition 3 (Price Distribution). Equation (16) implies that

$$P(p) = \frac{1}{a} \ln\{\frac{(p - \tau)[\mathfrak{t}I(p; \underline{\tau}) + (1 - \mathfrak{t})I(p; \overline{\tau})]}{\underline{p} - \gamma}\}.$$

The result follows by noting that $\mathfrak{t}I(p; \underline{\tau}) + (1 - \mathfrak{t})I(p; \overline{\tau}) = 1$, when $p \in [\underline{p}, p(\underline{\tau})]$, and $\mathfrak{t}I(p; \underline{\tau}) + (1 - \mathfrak{t})I(p; \overline{\tau}) = 1 - \mathfrak{t}$, when $p \in [p_{\uparrow}(\underline{\tau}), \overline{p}]$. Last, since there are no firms that price in the range $[p(\underline{\tau}), p_{\uparrow}(\underline{\tau})]$ the distribution function is flat over this interval. ■

Proof of Proposition 4 (Number of Varieties). Suppose that some consumption good i is not produced. A producer could enter the variety, charging the maximum price, $\overline{p}$, while advertising in the amounts a_t *and* a_d. All high-type consumers receiving an ad would buy this good. The resulting level of supra-normal profits is

$$\begin{aligned}(\overline{p} - \gamma)(a_t + a_d)(1 - \mathfrak{t}) - A(a_t) - A(qa_d) - \mathfrak{r} > \\ (\overline{p} - \gamma)(a_t + a_d)e^{-a}(1 - \mathfrak{t}) - A(a_t) - A(qa_d) - \mathfrak{r} = 0.\end{aligned}$$

These positive profits violate the zero-profit condition. ■

Proof of Proposition 5 (Consumption/Leisure). To conserve on notation let $c = c(\tau)$, $l = l(\tau)$, and $p_c = p(\tau)$. Focus on the first-order condition (25), which can be rewritten as[9]

$$\underbrace{\frac{\theta}{c}\frac{\tau}{p_c}}_{\text{MC}} = \underbrace{(1 - \theta)\frac{\kappa}{\kappa l + (1 - \kappa)\mathfrak{m}^{\rho} l^{1-\rho}}}_{\text{MB}}. \tag{40}$$

The above first-order condition can be represented diagrammatically, as shown in Figure 11. The lefthand side of (40) represents the marginal

[9] The righthand side is the marginal utility of leisure, l. Note that the marginal utility of leisure goods, $\mathfrak{m}$, has the symmetric form

$$(1 - \alpha)\frac{(1 - \kappa)\mathfrak{m}^{\rho-1}}{\kappa l^{\rho} + (1 - \kappa)\mathfrak{m}^{\rho}} > 0.$$

Taking the derivative of this with respect to l also gives the cross partial given in (41). That is, if leisure is an Edgeworth-Pareto complement with leisure goods, then leisure goods are an Edgeworth-Pareto complement with leisure.

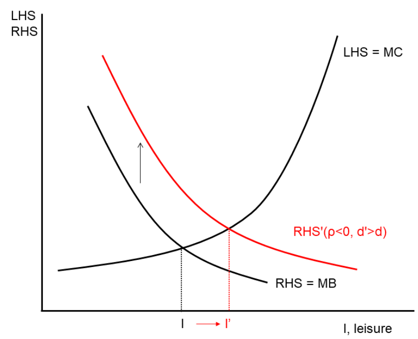

Figure 11: Leisure, l, is determined at the point where the marginal benefit and marginal cost curves intersect. The diagram shows what happens when the number of leisure goods increase from d to d'.

cost of leisure (MC). This is increasing in l, so the marginal cost curve is upward sloping. On this, note that both c and p_c are decreasing in l by (23). The righthand side is the marginal benefit of leisure (MB). The righthand side is decreasing in l so the marginal benefit curve is downward sloping.

1. To demonstrate the first point that leisure, l, will increase with the number of media goods, $\mathfrak{m}$, note that

$$\frac{d\text{MB}}{d\mathfrak{m}} = -\rho(1-\theta)\frac{\kappa(1-\kappa)l^{1-\rho}}{[\kappa l + (1-\kappa)\mathfrak{m}^{\rho}l^{1-\rho}]^2}\mathfrak{m}^{\rho-1} > 0, \text{ as } \rho < 0. \quad (41)$$

The marginal cost curve will stay in position, because it is not a function of $\mathfrak{m}$.

2. If leisure, l, increases with the free provision of media goods, then work effort, $h(\tau)$, and income, $\tau h(\tau)$, must fall. This leads to a drop in regular consumption, c. To show that regular consumption, c, is increasing in the level of skill, τ, convert the first-order condition (40) for l into one for c by using the budget constraint (2). For a skilled person this will read

$$\frac{\theta}{c}\frac{\overline{\tau}}{p_c} = (1-\theta)\frac{\kappa}{\kappa[(1-\mathfrak{e})-(1/\overline{\tau})\int_0^c p_i di] + (1-\kappa)\mathfrak{m}^{\rho}[(1-\mathfrak{e})-(1/\overline{\tau})\int_0^c p_i di]^{1-\rho}}. \quad (42)$$

(For the unskilled person just set $\overline{\tau} = \mathfrak{e} = 0$.) The lefthand side is the marginal benefit of regular consumption, c, while the righthand side is its marginal cost. The marginal cost curve rises in c while the marginal benefit curve declines in c. Here an increase in τ decreases the marginal cost of consumption, while is raises the marginal benefit. Hence, c will increase.

3. Last, to establish that work effort for the skilled, $h(\overline{\tau}) = 1 - l - \mathfrak{e}$, is increasing in the cost of education, $\mathfrak{e}$, return to equation (40). Note that the marginal cost of leisure rises with $\mathfrak{e}$ because $c = C(h(\overline{\tau}), \overline{\tau})$ will be smaller at any given level of l by (23). The righthand side is unaffected by $\mathfrak{e}$.

■

14.2 Efficiency of the Undirected Advertising Equilibrium

To conserve on notation, let the subscript 1 denote an allocation for the unskilled person and 2 the skilled one. The planning problem (29) then rewrites as

$$\max_{c_1,c_2,a_t,a_d,n,l_1,l_2} \Big(\xi\mathfrak{t}\theta \ln c_1 + \frac{\xi\mathfrak{t}(1-\theta)}{\rho} \ln[\kappa l_1^{\rho} + (1-\kappa)[n(\frac{a_t}{\zeta} + \frac{a_d}{\psi})]^{\rho}]$$
$$+ (1-\mathfrak{t})\theta \ln c_2 + \frac{(1-\mathfrak{t})(1-\theta)}{\rho} \ln[\kappa l_2^{\rho} + (1-\kappa)\underbrace{[n(\frac{a_t}{\zeta} + \frac{a_d}{\psi})]}_{=\mathfrak{m}}^{\rho}]\Big),$$

subject to

$$(1-\mathfrak{t})(1 - e^{-(a_t+a_d)n}) - (1-\mathfrak{t})c_2 = 0,$$

and

$$\mathfrak{t}(1-l_1) + (1-\mathfrak{t})\overline{\tau}(1 - l_2 - \mathfrak{e}) - \mathfrak{t}\gamma c_1 - \gamma(1-\mathfrak{t})c_2 - n[A(a_t) + A(qa_d) + \mathfrak{r}] = 0.$$

Attach the Lagrange multiplier ω to the first constraint and the one λ to the second.

The first-order conditions are:

$$\xi\theta\frac{1}{c_1} = \lambda\gamma, \tag{43}$$

$$\theta\frac{1}{c_2} = \omega + \lambda\gamma = \lambda(\omega/\lambda + \gamma), \tag{44}$$

$$\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\zeta}{\kappa l_1^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}} + (1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\zeta}{\kappa l_2^{\rho} + (1-\kappa)\mathfrak{m}^{\rho}}$$
$$+ \omega(1-\mathfrak{t})ne^{-(a_t+a_d)n} = \lambda n A_1(a_t),$$

$$(45)$$

$$\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\psi}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}+(1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}n/\psi}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho} + (1-\mathfrak{t})\omega ne^{-(a_t+a_d)n}=\lambda nqA_1(qa_d), \tag{46}$$

$$\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}(a_t/\zeta+a_d/\psi)}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}+(1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}(a_t/\zeta+a_d/\psi)}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho} + (1-\mathfrak{t})\omega(a_t+a_d)e^{-(a_t+a_d)n}=\lambda[A(a_t)+A(qa_d)+\mathfrak{r}],$$

$$(47)$$

$$\xi(1-\theta)\frac{\kappa l_1^{\rho-1}}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}=\lambda, \tag{48}$$

and

$$(1-\theta)\frac{\kappa l_2^{\rho-1}}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho}=\lambda\overline{\tau}. \tag{49}$$

Following Negishi (1960), the question asked is whether or not there is a competitive equilibrium with the set of taxes and subsidies specified by (32), (33), (34), and (35) that shares the planning problem's allocations for $c_1, c_2, a_t, a_d, n, l_1$, and l_2. If so, then the competitive equilibrium with the proposed tax-cum-subsidy scheme is Pareto optimal.

Before proceeding to proving that the competitive equilibrium with the proposed tax-cum-subsidy scheme is Pareto optimal, motivated by (46), conjecture that the subsidy on each media goods, s, is

$$s=[\xi\mathfrak{t}(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa l_1^\rho+(1-\kappa)\mathfrak{m}^\rho}+(1-\mathfrak{t})(1-\theta)\frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa l_2^\rho+(1-\kappa)\mathfrak{m}^\rho}]/\lambda. \tag{50}$$

Using (48) and (49) it can be seen that the righthand side of this expression collapses so that

$$\begin{aligned} s=&\frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa}[\mathfrak{t}\frac{1}{l_1^{\rho-1}}+(1-\mathfrak{t})\overline{\tau}\frac{1}{l_2^{\rho-1}}]\\ =&\mathfrak{t}\frac{(1-\kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\underline{\tau})})^{\rho-1}+(1-\mathfrak{t})\overline{\tau}\frac{(1-\kappa)}{\kappa}(\frac{\mathfrak{m}}{l(\overline{\tau})})^{\rho-1} \text{ [by (22)]}.\end{aligned}$$

This subsidy per media good is equivalent to subsidizing traditional and digital advertising at the rates s/ζ and s/ψ. The proportional price reduction on generic goods, r, implies that

$$(1-r)p(\underline{\tau})=\gamma \text{ [cf (32)]}. \tag{51}$$

Proof of Proposition 6 (Efficiency). To start with focus on the consumption/leisure allocations, while assuming that the solutions for advertising and the number of firms agree in both situations.To show that the planning problem with the specified planning weight ξ can be supported as a competitive equilibrium with the proposed subsidy-cum-tax policy, let

$$\frac{\omega}{\lambda} + \gamma = (1-r)\overline{p}.$$

Using this together with equations (44) and (49) gives the skilled consumer's first-order condition in the competitive equilibrium. Under both regimes $c_2 = (1 - e^{-(a_t+a_d)n})$. This, along with the consumption/leisure first-order condition, implies that the solution for the skilled person's leisure, l_2, will be the same in both scenarios. Analogously, using (51) in conjunction with equations (43) and (48) gives the unskilled consumer's first-order condition. Then, the labor-market-clearing condition (28) implies the unskilled person's consumption, c_1, is the same. So, the allocations for c_1, c_2, l_1, and l_2 from the planning problem can be supported as a competitive equilibrium with the proposed subsidy-cum-tax policy.

Now turn to advertising. In the competitive equilibrium, $\underline{p} - \gamma = A_1(a_t) + f - s/\zeta$ and $\underline{p} - \gamma = A_1(qa_d) + f - s/\psi$. Rewriting equation (45) while using the formula for s and adding f to both sides yields

$$s/\zeta + (1-\mathfrak{t})(\omega/\lambda)e^{-(a_t+a_d)n} + f = A_1(a_t) + f.$$

Using formula (33) for the fine on advertising, f, then gives

$$s/\zeta + (1-\mathfrak{t})(\omega/\lambda + r\overline{p})e^{-(a_t+a_d)n} = A_1(a_t) + f.$$

Noting that $\omega/\lambda = (1-r)\overline{p} - \gamma$ leads to

$$s/\zeta + (1-\mathfrak{t})(\overline{p} - \gamma)e^{-(a_t+a_d)n} = A_1(a_t) + f$$

or

$$\underline{p} - \gamma = A_1(a_t) + f - s/\zeta, \tag{52}$$

because $(1-\mathfrak{t})\overline{p}(1-\gamma)e^{-(a_t+a_d)n} = \underline{p}(1-\gamma)$. This is the first-order condition for a_t in a competitive equilibrium. Similarly, from equation (46) it can be seen that

$$s/\psi + (1-\mathfrak{t})(\omega/\lambda)e^{-(a_t+a_d)n} + f = qA_1(qa_d) + f,$$

implying

$$\underline{p} - \gamma = qA_1(qa_d) + f - s/\psi. \tag{53}$$

This is the efficiency condition for digital advertising, a_d, that arises in the competitive equilibrium. Therefore, the competitive solutions for a_t

and a_d, under the proposed subsidy-cum-tax policy, satisfy the planning problem.

Last, move on to the number of firms. Multiply (45) by a_t/λ and (46) by a_d/λ and then sum the resulting equations to get

$$s(a_t/\zeta+a_d/\psi)n+(1-\mathfrak{t})(\omega/\lambda)n(a_t+a_d)e^{-(a_t+a_d)n} = na_tA_1(a_t)+nqa_dA_1(qa_d),$$

where formula (50) for s has been used. Similarly, multiply (47) by n/λ and subtract the result from the above equation to obtain

$$a_tA_1(a_t) + qa_dA_1(qa_d) = A(a_t) + A(qa_d) + \mathfrak{r}. \tag{54}$$

Finally, multiplying the efficiency conditions (52) and (53) for traditional and digital advertising by a_t and a_d, respectively, and then summing the two equations while making use of (54) gives

$$\begin{aligned}(\underline{p} - \gamma)(a_t + a_d) &= a_tA_1(a_t) + qa_dA_1(qa_d) + (a_t + a_d)f - s(a_t/\zeta + a_d/\psi)\\ &= A(a_t) + A(qa_d) + \mathfrak{r} + (a_t + a_d)f - s(a_t/\zeta + a_d/\psi).\end{aligned}$$

This is the zero-profit condition for a firm when there is a subsidy for media goods provision. This implies that the solution for n from the planning problem will be shared by the competitive economy with the proposed subsidy-cum-tax policy. ■

Suppose now that there is only one type of consumer/worker. Without loss in generality, let this be the high type. For this specialized case just a subsidy on media goods is required in the amount

$$s = \frac{(1-\kappa)\mathfrak{m}^{\rho-1}}{\kappa}\overline{\tau}\frac{1}{l_2^{\rho-1}}.$$

Proof of Corollary 1 (Single agent economy). The proof is a straightforward modification of the previous proof. When there is only the high-type person, the first-order conditions (43) and (48) no longer appear, so disregard them. Now, set $\omega/\lambda + \gamma = \overline{p}$. Using this together with equations (44) and (49) gives the consumer's first-order condition in a competitive equilibrium. To complete things, set $f = r = \mathfrak{t} = 0$. Then parrot the remaining steps in the above proof (ignoring the ones for the unskilled person) while using the revised formula for s. ■

14.3 Recovering r, f, s, $t(\underline{\tau})$, and $t(\overline{\tau})$ from the Planning Problem

The tax-cum-subsidy scheme that renders the competitive equilibrium efficient can be recovered from the solution to the planning problem.

First, the subsidy on digital advertising, s, can be calculated from (34) using the planning problem allocations for a_t, a_d, $l(\underline{\tau})$, $l(\overline{\tau})$, and n.

Second, the proportional price reduction, r, and the fine on advertising, f, are immediate from (32) and (33), if the prices $p(\underline{\tau})$ and $\overline{p}$ are known. To recover these two prices, from the competitive equilibrium it transpires that

$$[1 - S(p(\underline{\tau}))] = [1 - \frac{\underline{p} - \gamma}{p(\underline{\tau}) - \gamma}] = c_1 \text{ [equations (19) and (26)]},$$

which implies

$$p(\underline{\tau}) = \frac{\underline{p} - \gamma}{1 - c_1} + \gamma = \frac{e^{-a}(1 - \mathfrak{t})(\overline{p} - \gamma)}{1 - c_1} + \gamma,$$

where the term on the far right follows from substituting out for $\underline{p} - \gamma$ using (14). Next, from the two consumer's problems, in the competitive equilibrium with the proposed tax-cum-subsidy scheme, it transpires that

$$p(\underline{\tau}) = \Xi\overline{p},$$

where

$$\Xi \equiv (\frac{c_2}{c_1})\frac{\kappa l(\underline{\tau}) + (1 - \kappa)\mathfrak{m}^{\rho} l(\underline{\tau})^{1-\rho}}{\overline{\tau}[\kappa l(\overline{\tau}) + (1 - \kappa)\mathfrak{m}^{\rho} l(\overline{\tau})^{1-\rho}]}.$$

Therefore,

$$\overline{p} = \frac{\gamma(1 - \Delta)}{\Xi - \Delta} \text{ and } p(\underline{\tau}) = \Xi\frac{\gamma(1 - \Delta)}{\Xi - \Delta},$$

where

$$\Delta \equiv \frac{e^{-a}(1 - \mathfrak{t})}{1 - c_1}.$$

Since a, c_1, $\mathfrak{m}$, $l(\underline{\tau})$, and $l(\overline{\tau})$, are known from the planning problem so are Ξ and Δ.

Finally, by modifying (26) and (59), the lump-sum taxes levied on the unskilled and skilled, $t(\underline{\tau})$ and $t(\overline{\tau})$, read as

$$t(\underline{\tau}) = 1 - l(\underline{\tau}) - (1 - r)a(\underline{p} - \gamma)\{\ln[\frac{p(\underline{\tau}) - \gamma}{\underline{p} - \gamma}] - \frac{\gamma}{p(\underline{\tau}) - \gamma} + \frac{\gamma}{\underline{p} - \gamma}\}/a$$

and

$$\begin{aligned} t(\overline{\tau}) = {} & \overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}] \\ & -(1 - r)a(\underline{p} - \gamma)\{\ln[\frac{p(\underline{\tau}) - \gamma}{\underline{p} - \gamma}] - \frac{\gamma}{p(\underline{\tau}) - \gamma} + \frac{\gamma}{\underline{p} - \gamma}\}/a \\ & -\frac{(1 - r)a(\underline{p} - \gamma)}{1 - \mathfrak{t}}\{\ln[\frac{\overline{p} - \gamma}{p_{\uparrow}(\underline{\tau}) - \gamma}] - \frac{\gamma}{(\overline{p} - \gamma)} + \frac{\gamma}{p_{\uparrow}(\underline{\tau}) - \gamma)}\}/a, \end{aligned}$$

where

$$p_{\uparrow}(\underline{\tau}) - \gamma = \frac{p(\underline{\tau}) - \gamma}{1 - \mathfrak{t}}.$$

15 Appendix: Calibration

To solve the model values for the following parameter values are needed: θ, κ, ρ, γ, $\mathfrak{r}$, α, ϕ, ζ, ψ, q, $\mathfrak{e}$, $\mathfrak{t}$, and $\overline{\tau}$. The idea is to pin down values for $\mathfrak{r}$, α, q, ζ, ψ, θ, κ, ρ, and $\mathfrak{e}$ by using data on markups, the advertising-to-consumption ratio, the click-through rate on digital advertising, the increase in the ratio of spending on digital versus traditional advertising, and the rise in time spent on leisure by non-college educated and college educated persons. Out of the remaining parameters, $\mathfrak{t}$ and $\overline{\tau}$ can be assigned values directly from the data. The last two parameters, γ and ϕ, are normalized to 1. As will be seen, at the calibration point the calibration procedure will determine a value for n. The steps in the procedure are as follows:

1. *Calibrating* α. Two facts are used to do this, namely the average markup, MARKUP, and advertising's share of consumption, A2C. These facts are taken to apply for the whole period in question, and therefore for the year 2018.

   (a) A formula for α. In the model all firms have the same advertising expenses, zero profits, and hence revenue net of production costs. Hence, focus on the firms charging the lowest price, $\underline{p}$. To start with, equation (8) implies

   $$(\underline{p} - \gamma)a_t = \phi\alpha a_t^{\alpha} \text{ and } (\underline{p} - \gamma)a_d = \phi\alpha q^{\alpha} a_d^{\alpha}.$$

   This gives

   $$(\underline{p} - \gamma)n(a_t + a_d) = \alpha n[A(a_t) + A(qa_d)].$$

   Dividing through by total sales, $\mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]$, then results in

   $$\frac{(\underline{p} - \gamma)n(a_t + a_d)}{\mathfrak{t}[1 - l(\underline{\tau})] + (1 - \mathfrak{t})\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}]} = \alpha \frac{n[A(a_t) + A(qa_d)]}{\mathfrak{t}[1 - l(\underline{\tau})] + (1 - \mathfrak{t})\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}]}$$
   $$= \alpha \times \text{A2Cs},$$

   where it should noted that sales equals consumption expenditure in the model. Therefore,

   $$\alpha = \frac{1}{\text{A2C}} \times \frac{(\underline{p} - \gamma)a}{\mathfrak{t}[1 - l(\underline{\tau})] + (1 - \mathfrak{t})\overline{\tau}[1 - l(\overline{\tau}) - \mathfrak{e}]}.$$

To use this formula, values are needed for a, $\underline{p}-\gamma$, $1-l(\underline{\tau})$, and $1-l(\overline{\tau})-\mathfrak{e}$. The latter two quantities come from the consumer side of the calibration; that is, the model's predictions for $1-l(\underline{\tau})$ and $1-l(\overline{\tau})-\mathfrak{e}$ at the 2018 calibration point, as shown in Table 2. Information on the average price markup, MARKUP, is used to solve for a and $\underline{p}-\gamma$.

(b) Using the MARKUP to determining a and $\underline{p}-\gamma$. In the model, the average price markup, is given by

$$\text{MARKUP} = \frac{E[p]}{\gamma} = \frac{\int_{\underline{p}}^{p(\underline{\tau})} pB(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} pB(p)P_1(p)dp}{\int_{\underline{p}}^{p(\underline{\tau})} B(p)P_1(p)dp + (1-\mathfrak{t})\int_{p(\underline{\tau})}^{\overline{p}} B(p)P_1(p)dp}$$
$$= \frac{\mathfrak{t}[1-l(\underline{\tau})] + (1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]}{[1-\mathfrak{t}[(\underline{p}-\gamma)/(p(\underline{\tau})-\gamma)]-(1-\mathfrak{t})e^{-a}]}. \tag{55}$$

The numerator follows from (28) since this is proportional to aggregate spending. The denominator is proportional to aggregate consumption and follows from (26) and (27). Next, the labor-market-clearing condition (28) implies that

$$\gamma[\mathfrak{t}c(\underline{\tau})+(1-\mathfrak{t})c(\overline{\tau})]+a(\underline{p}-\gamma) = \mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}],$$

since $n[A(a_t) + A(qa_d) + \mathfrak{r}] = a(\underline{p}-\gamma)$ by the zero-profit condition. Solving out for $c(\underline{\tau})$ and $c(\overline{\tau})$ using (26) and (27), while noting that $B(p(\underline{\tau})) = (\underline{p}-\gamma)/[p(\underline{\tau})-\gamma]$, then yields

$$\gamma[1-\mathfrak{t}(\frac{\underline{p}-\gamma}{p(\underline{\tau})-\gamma})-(1-\mathfrak{t})e^{-a}]+a(\underline{p}-\gamma) = \mathfrak{t}[1-l(\underline{\tau})]+(1-\mathfrak{t})\overline{\tau}[1-l(\overline{\tau})-\mathfrak{e}]. \tag{56}$$

Last, $p(\underline{\tau})$ must solve

$$a(\underline{p}-\gamma)\{\ln[\frac{p(\underline{\tau})-\gamma}{\underline{p}-\gamma}]-\frac{\gamma}{p(\underline{\tau})-\gamma}+\frac{\gamma}{\underline{p}-\gamma}\}/a = 1-l(\underline{\tau}). \tag{57}$$

Equations (55), (56), and (57) represent a system of three equations in three unknowns, which can be used to find a solution a, $\underline{p}-\gamma$, and $p(\underline{\tau})$ predicated upon the observed markup, MARKUP, and the labor allocations for 2018 at the calibration point that are reported in Table 2.

2. *Calibrating the fixed entry cost,* $\mathfrak{r}$. Since all firms earn zero profits,

$$\mathfrak{r} = (\underline{p}-\gamma)^{\alpha/(\alpha-1)}(1+q^{\alpha/(1-\alpha)})\phi^{1/(1-\alpha)}(\alpha^{1/(1-\alpha)}-\alpha^{\alpha/(1-\alpha)}) > 0,$$

where $\underline{p}$, γ, q, and α have all been previously determined.

3. *Calibrating the cost of advantage of digital advertising, q, for the years 2003, 2010, and 2018.* These can be recovered from the observed ratio of digital ad spending to traditional ad spending, D2T, for the years 2003 and 2018. For the year 2010 and interpolated value is for D2T. To see this, from (4), (6), and (10) it is apparent that

$$\frac{A(qa_d)}{A(a_t)} = (\frac{qa_d}{a_t})^{\alpha} = (qq^{\alpha/(1-\alpha)})^{\alpha} = q^{\alpha/(1-\alpha)} = \text{D2T}.$$

   Therefore,

$$q = (\text{D2T})^{(1-\alpha)/\alpha},$$

   where α is known from the first step.

Calibrating the preference parameters, θ, κ, ρ, the cost of an education, $\mathfrak{e}$, and the click-through rate on traditional advertising, ζ. This is done by solving problem (39) which tries to match the prediction up the model's predictions for leisure versus 6 observations on leisure from U.S. data for non-college- and college-educated people for the years 2003, 2010, and 2018. Central to this data matching problem is the first-order condition

$$\frac{\theta}{c(\tau)}\frac{\tau}{p(\tau)} = (1-\theta)\frac{\kappa}{\kappa l(\tau) + (1-\kappa)\mathfrak{m}^{\rho} l(\tau)^{1-\rho}}, \text{ for } \tau \in \{\underline{\tau}, \overline{\tau}\}.$$

The quantity of digital media goods consumed, na_d/ψ, and the price of the last good consumed, $p(\tau)$, are quantities that can be recovered from the information produced in Steps 1 to 3, conditional upon values for $1 - l(\underline{\tau})$ and $1 - l(\overline{\tau}) - \mathfrak{e}$. The quantity of traditional media goods consumed, na_t/ζ, depends upon the click-through rate, ζ, for which there is no information available. So, ζ must be calibrated. The model's leisure quantities, $l_t(\underline{\tau})$ and $l_t(\overline{\tau})$, come from calibrating θ, κ, ρ, $\mathfrak{e}$, and ζ so as to match up, as close as possible, the model's predictions for leisure with the stylized facts from the data, LEISURE$_t(\tau)$ for $t = 2003, 2010$, and 2018, and $\tau = \{\underline{\tau} = \text{non-college}, \overline{\tau} = \text{college}\}$.