Money, Credit and Imperfect Competition Among Banks

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Abstract

This paper develops a micro-founded monetary framework where bank market power in lending is endogenous and responds to policy. We study its equilibrium consequences for financial intermediation, allocations and welfare. Calibrated to match the empirical money demand relationship the model also matches the co-movement between the average bank markup and the policy interest rate in macro data. We show that noisy consumer loan search generates in equilibrium a non-degenerate distribution of loan interest rates, the characteristics of which rationalize the empirically observed relationship between the dispersion and average level of bank lending markups at both the U.S. national and state levels. Further, we find that financial intermediation need not be welfare-improving if inflation is sufficiently low, a result which speaks to concerns regarding market power in the banking sectors of low-inflation countries. Lastly, there is a role in equilibrium for demand stabilization via the central bank's liquidity-management policy given a long-run inflation target. Efficiency gains arise from the central bank's ability to reduce the ability of banks exploit their market power in loan pricing.

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1 Introduction

In this paper, we study bank lending in a micro-founded monetary model in which banks' market power is endogenous. We begin by developing a theory of dispersion in loan interest rates that assumes neither monopolistic competition nor idiosyncratic shocks to either banks' demand or cost conditions. We calibrate the model to aspects of both aggregate money demand and the average banking markup and show that it is consistent with observations on the relationship between loan rate dispersion and average market power. We then consider the welfare effects of financial intermediation when banks' market power is endogenous. Finally, we consider the optimal banking-sector stabilization policy implied by our environment.

It is well documented that there is substantial concentration in the banking industry in all developed countries. For example, post-2007, the market share of the top-three banks in Germany is about 78%; in the U.K. this is roughly 58%; in Japan, this is 44%; and, in the U.S., the corresponding share is 35%.[1] Considering the U.S. as an example, there are also high profit margins in the banking sector, with markups of around 90%, and evidence of imperfect interest-rate pass-through, with a Rosse-Panzar H-statistic of 50%.[2]

In our analysis of micro-level data in Section 2, we find dispersion in loan rates for identical loan products in United States, even with controls for geography and other characteristics.[3] Martín-Oliver, Salas-Fumás and Saurina (2007) and Martín-Oliver, Salas-Fumás and Saurina (2009) also find price dispersion in loan rates for identical loan products in the case of Spanish banks. We compare our empirical observations to our model's predictions section 6.3).

In theoretical work where banking can be shown to be essential (*i.e.* welfare improving) it is often assumed that banks are perfectly competitive. For example, in Berentsen, Camera and Waller (2007), banks intermediate between anonymous consumers with liquidity needs and others who face the risk of holding idle money. In the presence of inflation (*i.e.* away from the Friedman Rule) banks raise welfare by

[1] Averaged across annual 2007-2019 time series data (available from Bankscope, 2020).

[2] See Corbae and D'Erasmo (2015, 2018). The Rosse-Panzar H-statistic measures the degree of competition in the banking market. It measures the elasticity of banks' revenues relative to input prices. Under perfect competition, the H-statistic equals 1 since an increase in input prices raise marginal cost and total revenue by the same amount. Under a monopoly, an increase in input prices raises marginal costs, lowers output and revenues, so that the H-statistic is less than or equal to 0. When the H-statistic is between 0 and 1, one usually presumes a monopolistically competitive industry.

[3] We use micro data from Ratewatch. More details can be found in Appendix ??.

compensating for the risk of idle funds by paying deposit interest.[4]

We begin with this basic environment and add an endogenous form of *market power* in bank lending. Specifically, we consider noisy consumer search for credit lines with banks along the lines of Burdett and Judd (1983). This induces an equilibrium where there is (intra-temporal) heterogeneity in lending rates for homogeneous loans. The distribution (or dispersion) of loan rates in equilibrium depends on both the wealth of agents and on monetary and fiscal (*i.e* taxation) policies. Banks face an equilibrium trade-off between wanting to raise their loan rate markups to increase profit per customer (viz. an intensive margin), and, wanting to lower their loan rates to increase the number of borrowers to whom they successfully lend (viz. an opposing extensive margin). The model nests a case with competitively pricing banks—approaching the case of Berentsen, Camera and Waller (2007)—as one limit, and, another case with a monopolistic bank as the other extreme.

At empirically plausible low inflation rates, dispersion in loan rates diminishes and banks tend to exploit their intensive-margin markups more. In this way they effectively extract goods trade surplus from consumers. As their need for insurance is low (with a low inflation tax) welfare in an equilibrium with banks can be actually lower than in an equilibrium without them. A result that speaks to why policymakers in many low-inflation countries may be particularly concerned with market power in the banking sector. As inflation rises, the insurance provided by banks becomes more important than they can be welfare improving. In addition, if demand is low in the sense that not many consumers seek loans, then competition among banks along the extensive margin can lower the markup sufficiently to render banking welfare improving even at low inflation.

Since our model contains Bertrand-pricing as a parametric limit, we can replicate the competitive banking equilibrium of Berentsen, Camera and Waller (2007) as a special case. We decompose the money demand condition for an agent *ex ante* into several parts. On part is identical to that arising in the model of Berentsen, Camera and Waller (2007). We can, however, isolate further new marginal benefit-versus-cost terms that capture the effect of equilibrium market power (and its attendant loan-interest risk) on agents' decisions to accumulate real balances.

A novel insight arises from these new terms: We use them to show how redistributive

[4] Berentsen, Camera and Waller (2007) is set in a Lagos and Wright (2005) environment in which contractual frictions render promises to repay not incentive compatible and thus money facilitates welfare-improving exchange (Kocherlakota, 1998). Inflation, however, erodes the value of money and reduces welfare. Competitive banks serve to insure agents against the risk of holding idle money.

taxation policies matter by affecting agents' marginal benefits of holding additional liquidity (self insurance). In Berentsen, Camera and Waller (2007), these redistributive tax instruments are irrelevant to agents' money demand *ex-ante*. We close by studying optimal interest-rate and tax policies designed to alleviate the effects of fluctuations in aggregate demand (and the attendant demand for bank loans). This exercise is similar in spirit to those considered by Berentsen and Waller (2011) and Boel and Waller (2019), with the exception that in our case, redistributive tax policies matter. That is, our optimal banking-stabilization policy problem has at least one additional policy instrument on top of the monetary interest-rate instrument in Berentsen and Waller (2011).

The remainder of the paper is organized as follows. In Section 2, we provide micro-data evidence on the relation between consumer loan-rate markups and markup dispersion. In Section 3, we lay out the details of the model and its component decision problems of households, firms, government and banks. In Section 4, we describe the stationary monetary equilibrium of the model economy. Here we also discuss what is new in this model. In Section 6, we study the insights from the model quantitatively, by first calibrating it to U.S. data. Using numerical results, we illustrate the insights on equilibrium market power in the banking sector. Here we also provide an empirically testable prediction on loan-interest dispersion and markups. In Section 7 we study an optimal (steady-state) policy design of interest-rate (or inflation) policy alongside redistribution taxation as a function of banking-sector loan-demand uncertainty. We conclude in Section 8.

2 Empirical Evidence

In this section, we examine the empirical relationship between the dispersion and average level of markups in micro-level lending rate data from bank branches in US. Specifically we study two measures of lending rate markups: (1) the raw markup of lending rates over the federal funds rate; and (2) a markup orthogonalized using a set of control variables. First, at the national level we find a positive relationship between standard deviation and average level of markups at monthly frequency. Second, we find *negative* relationships at the national level between the coefficient of variation of markups and their average. For both markup measures, these empirical results are consistent with the theoretical predictions that come below..

2.1 Data

Interest Rate data. To assess the relationship between markup dispersion among banks and the average markup level, we obtain interest rate data from RateWatch, which provides monthly interest rate data at the branch level for several types of consumer lending products. Our baseline analysis focuses on unsecured consumers lending products to be consistent with our model settings. Specifically, for personal loans, we choose the most commonly used product: Personal Unsecured Loan for Tier 1 borrowers.[5] Our primary sample includes 496,942 branch-month observations from January 2003 to December 2017, involving 11,855 branches. To calculate each branch's markup against federal funds rate, we collect daily effective federal funds data from Federal Reserve H15 report.

Bank and county data. We obtain commercial bank's information from their call reports. Specifically, we collect information on each commercial bank's reliance on deposit financing, leverage ratio, credit risk and bank size.

The Federal Deposit Insurance Corporation (FDIC) provides information on branch level deposits holdings for all FDIC-insured institutions in their summary of deposits (SOD) dataset. We use this data set to approximate each branch's local market competition and the impact of its commercial bank branch network. To capture the state of the local market competition environment, we calculate each branch's deposit share in its county, the Herfindahl-Hirschman Index (HHI) of county's deposit holdings and number of branch counts in the county. To measure one branch's parent commercial bank's branch network, we calculate one branch's deposit share in its commercial bank, the Herfindahl-Hirschman Index (HHI) of commercial bank's deposit holdings and number of branch counts in the commercial bank.

We also collect county-level socioeconomic information; including median income, the poverty rate, population and the average house price from census data. We also collect county-level unemployment data and number of business establishments from Bureau of Labor Statistics, county-level real GDP and GDP growth from the Bureau of Economic Analysis to control for local economic activities.

[5] As a robustness check, we also use mortgage rates as alternative variable to calculate markups. Specifically, we choose 30-Year Fixed Mortgage rate with an (origination?) size of $175,000. Our key results still hold when we use mortgage rates. Our results continue to hold also when using rates on personal loans with different borrower quality.

2.2 Markup

Raw Markup. As the baseline, we calculate each branch's markup relative to the federal fund rate. Specifically, the branch level raw markup is calculated as

$$Markup_{b,i,c,s,t} = (Rate_{b,i,c,s,t} - FF_t)/(1 + FF_t) \tag{2.1}$$

Here b stands for the branch, i for the commercial bank to which the branch belongs, c for the county in which branch is located, s for the state and t for the date for which RateWatch reports the branch rate information.

Orthogonalized Markup. Branch level loan rate pricing could be related to local social economic factors, deposit market competition, bank branch networks and bank's characteristics. Consequently, we orthogonalize the branch level markup to those potential factors and repeat our previous analysis on the re-scaled residual. First, we use the following OLS regression to obtain the residual $\epsilon_{b,i,c,s,t}$

$$Markup_{b,i,c,s,t} = \alpha_0 + \alpha_1 X_{b,i,c,s,t} + \alpha_2 X_{i,t} + \alpha_3 X_{c,s,t} + \epsilon_{b,i,c,s,t} \tag{2.2}$$

Here $X_{b,i,c,s,t}$ represents branch specific control variables including local deposit market competition and bank branch networks, $X_{i,t}$ represents commercial bank control variables and $X_{c,s,t}$ represents county social economic control variables. We then re-scale $\epsilon_{b,i,c,s,t}$ to match the mean and standard deviation of raw markups in our full sample and use it as our alternative specification for the markup.

2.3 Identification

At monthly frequency we estimate by OLS regressions of the dispersion of markups ($Dispersion_t$) on their monthly average ($\overline{Markup_t}$):

$$Dispersion_t = \alpha_0 + \beta_0 \overline{Markup}_t + \epsilon_t \tag{2.3}$$

Here β_0 is the coefficient of interest and standard errors are clustered by month. We consider two measures of markup dispersion: the monthly standard deviation of markups (SD_t) and monthly coefficient of variation (CV_t).

2.4 Results

Figure 1 shows the correlations between national monthly markup dispersion and average markups. The panels of the top row of the figure show that standard deviations of markups are positively correlated with their averages for both raw and orthogonalised markups. The correlation between standard deviations and averages of raw markup is 0.752. The second row of Figure 1 shows that the coefficients of variation are negatively correlated with their averages for both raw and orthogonalised markups. The correlation for the case of the raw markup is -0.857.

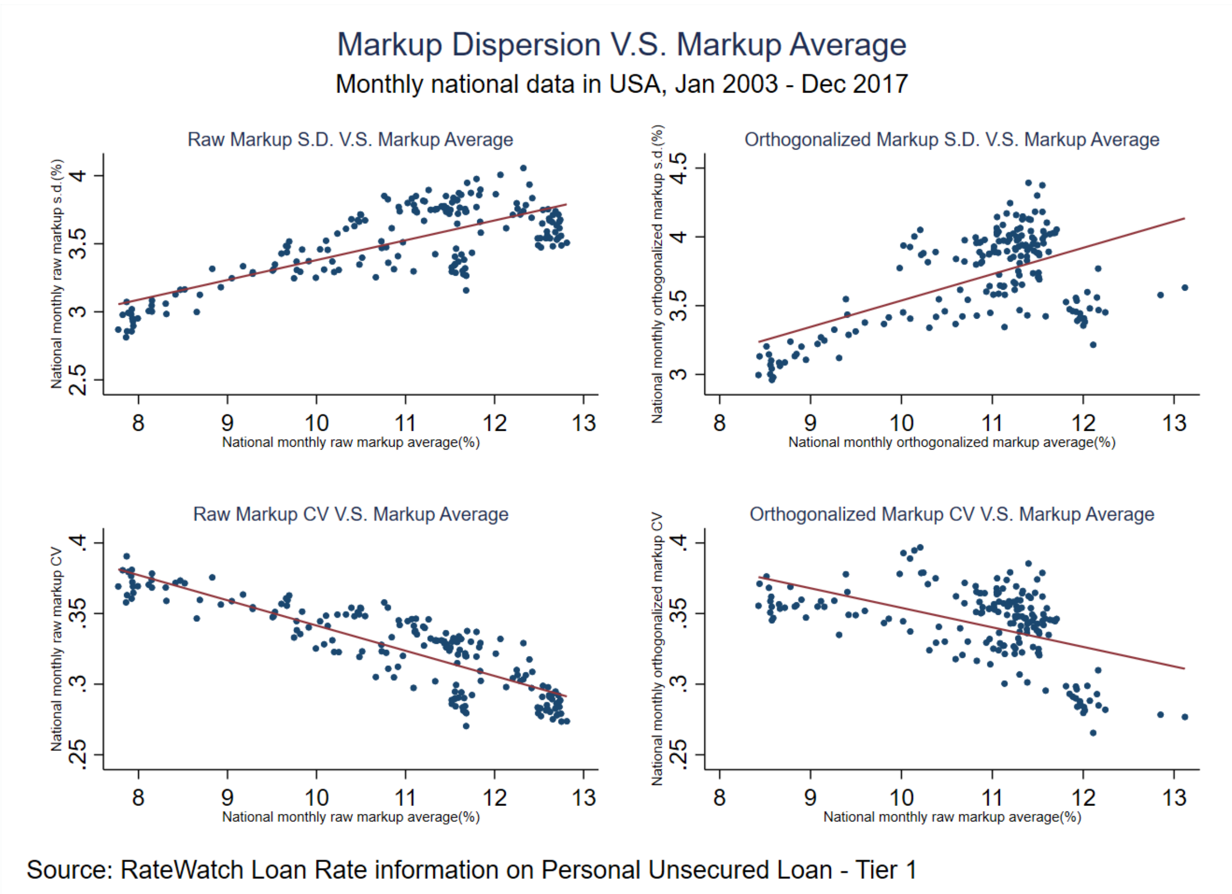
Table 1 illustrates the regression results of national monthly markup dispersions on markup averages. Columns (1) and (3) show a positive and statistically significant relationship between standard deviations and averages for the two markup measures. Using the raw markup, the coefficient in column (1) indicates one percentage point increase in markup average can lead to 0.146 percentage point increase in the standard deviation. Using the orthogonalised markup, the coefficient in column (3) indicates one percentage point increase in markup average can lead to 0.192 percentage point increase in the standard deviation. Similarly, columns (2) and (4) show negative and statistically significant relationships between the coefficients of variation and average markups.

Table 1: Regression for markup dispersion at national level

	Markup dispersion: $Dispersion_t$			
	Raw Markup		Orthogonalized markup	
	(1)	(2)	(3)	(4)
	SD_t	CV_t	SD_t	CV_t
$\overline{Markup_t}$	0.146***	−0.018***	0.192***	−0.014***
	(41.56)	(-56.61)	(18.70)	(-17.31)
Constant	1.924***	0.520***	1.621***	0.492***
	(49.09)	(136.80)	(14.57)	(55.55)
N	180	180	180	180
adj. R^2	0.554	0.733	0.333	0.259

Note: $^{*} p < 0.1$; $^{**} p < 0.05$; $^{***} p < 0.01$

Figure 1: Relationship between markup dispersion and average



2.5 State-level analysis

In the appendix, consider markup dispersion at the state level by taking the standard deviation of branch markups from state s in month t. Consistent with the analysis at the national level, the standard deviations of markups are positively related to their average levels in the state-month panel data after controlling for state and time fixed effects.

3 Model

We build upon the model of Berentsen, Camera and Waller (2007) (BCW), making one departure with regard to the banking environment with is discussed in detail below. Following BCW we abstract from means of consumption insurance and smoothing other than bank loans, and focus on money as an essential medium of exchange, unit of account and store of value.[6] For simplicity we focus on credit where loan contracts are perfectly enforceable.[7]

Overview. Time is discrete and infinite. Following Lagos and Wright (2005) each period is divided into two sub-periods. In sequence, first comes the DM (Decentralized Market) and second the CM (Centralized Market). There are two consumption goods in each period, one for each of the DM and CM, and none of these are storable across either periods or sub-periods. When useful, we will refer to these as the *special* (DM) and *general* (CM) goods, respectively. There is one productive resource, labor, which again is specific to a period and sub-period.

There are four types of agents in the economy: Firms, households, banks and the government. There are unit measures each of DM *firms*, CM *firms* and *households*. DM firms are endowed with labor in the DM, they can also work and produce in the CM and consume only the CM general good. Households are endowed with labor only in the

[6] In general, we could allow for agents to own other assets (*e.g.*, claims to private equity or bonds). In order to rationalize equilibrium coexistence of fiat money alongside other asset claims, we could introduce costly asset liquidation in frictional secondary asset market through, for example, over-the-counter, random-matching trades as in Rocheteau and Rodriguez-Lopez (2014) and Duffie, Gârleanu and Pedersen (2005). This would render demand for multiple assets that have different liquidity premia in equilibrium. For the purposes of this paper, however, these details are unnecessary and would serve only to complicate the analysis without altering the main insights.

[7] This will be comparable to the first part of BCW. We can also consider the case with endogenous borrowing limits but, unlike BCW, in our setting bank-specific lending limits would have to be determined simultaneously with the equilibrium distribution of loan rates.

CM, consume both goods and can produce the general good (in the CM) [8] Preferences and production technologies are specified below. These agents are anonymous in the DM but can recognize each other in the CM. They are unable to commit to any action across time periods, and has limited ability in the DM to commit to actions in the upcoming CM.

In addition to these agents there is a *government* which engages in both fiscal and monetary policy. To begin with, it suffices to think of this entity as acting solely as a monetary authority, issuing a stock of perfectly divisible fiat currency and increasing it over time via transfers, either lump-sum or proportional, in both the DMand CM. Later, we will allow the government also to collect taxes subject to certain restrictions.

Note that the physical environment and agents of the three types above are completely analogous to those BCW. The principal novelty here is the nature of the *banking system*, which works in the following way. At the beginning of each period, prior to the opening of the DM, two events occur. First, a fraction $1-n$ of households discover that they are uninterested in consuming the special good. Then, all households and firms (*i.e.* DM-sellers) have the opportunity to access the banking system. This process takes place in two stages. First, both households and firms have the opportunity to deposit money with any of a large number of *depository institutions* that behave competitively. Second, all of these agents have the opportunity to search for a contact with any of a unit measure of *lending agents*. For the sake of brevity, we will also refer to these lending agents as *banks*. These interactions will be described in further detail below.

Timing, events and actions. We will use the following notation for date-dependent variables: $X \equiv X_t$ and $X_{+1} \equiv X_{t+1}$. Let ϕ be the date-t value of a unit of money in units of CM good x, and, let M denote the aggregate stock of nominal money supply at the beginning of date t.

In each period, events take place in the following sequence: First, given initial total stock of money M, the DM opens. Households are indexed by initial money balance m. They are active buyers (*i.e.* early consumers) in the DM with probability n or inactive buyers in the DM (*i.e.* late consumers) with probability $1-n$. Then, the government injects new money to the economy via transfers, $\tau_1 M$ where

$$\tau_1 := n\tau_b + (1-n)\tau_s \leq \tau, \tag{3.1}$$

[8] This represents a minor variation on BCW and is akin to the assumption in Rocheteau and Wright (2005).

and $\tau_b M$ and $\tau_s M$ go to active and inactive buyers, respectively. At this point, agents have the opportunity to make deposits (previewing equilibrium, only inactive buyers will do this). Next, active buyers search for lending agents (banks) following the *noisy search* process of Burdett and Judd (1983). If an active buyer makes at least one contact, then they make take out a *loan*. These loans may be thought of as unsecured lines of credit at a posted interest rate. Active buyers may then purchase goods q_b from firms using their money balance m, and/or money borrowed from banks (loan l) via their line of credit. Buyers and firms exchange in a competitive market where q_f denotes the production of a typical DM firm, of which there is a unit measure.

At this point the DM closes and the CM opens: An agent's initial state here is given by (m, l, d), *i.e.* their remaining money balance, outstanding loan and deposit balance. Those with deposit balances (inactive buyers) interest $1 + i_d$ on deposit d, and those with outstanding loans (active buyers) repay them with interest $1 + i$ on loan l. All agents may work h, consume x, and accumulate money to carrying into the next period, m_{+1}. At the end of the period, the government transfers new money, $\tau_2 M$ uniformly.

3.1 Households

Households' period utility is given by

$$\mathcal{U}(q, x, h) = u(q) + U(x) - h, \tag{3.2}$$

where $u(q)$ is utility from consumption of the special good q in the DM, $U(x)$ is utility of consumption good x in the CM, and h is the disutility of labor. We assume that $u' > 0$, $u'' < 0$ and that u satisfies the usual Inada conditions. Likewise for U. For concreteness now, and anticipating the quantitative analyses later, we restrict our attention to the constant-relative-risk-aversion (CRRA) family of functions:

$$u(q) = \lim_{\hat{\sigma} \to \sigma} \frac{q^{1-\hat{\sigma}} - 1}{1 - \hat{\sigma}}, \tag{3.3}$$

and, we will assume that $\sigma < 1$.[9]

[9] This restriction is empirically motivated, as it is required to enable the model to fit long-run money-demand data well. We also consider the case of $\sigma > 1$ but we do not discuss it here for brevity. The knife-edge case of $\sigma = 1$ is not well-defined in terms of equilibrium characterization. This is also the case studied by Head et al. (2012).

3.1.1 The Centralized Market

A buyer with individual state (m, l, d) may have been either active or inactive in the preceding DM. Given her state, her lifetime utility is given by

$$W(m, l, d) = \max_{x,h,m_{+1}} [U(x) - h + \beta V(m_{+1})] \tag{3.4}$$

subject to

$$x + \phi m_{+1} = h + \phi m + \phi(1 + i_d) d - \phi(1 + i) l + \pi + T, \tag{3.5}$$

where i_d is the market interest rate on deposits, i is the interest rate on the buyer's outstanding loan, π is aggregate of profits from bank ownership, and T is any lump-sum tax or transfer from the government in the CM.

Using (3.5) (3.4), the problem may be rewritten:

$$\begin{aligned} W(m, l, d) = \phi[m + \tau_2 M - (1 + i) l + (1 + i_d) d] + \pi + T \\ + \max_{x,m_{+1}} \{U(x) - x - \phi m_{+1} + \beta V(m_{+1})\}. \end{aligned} \tag{3.6}$$

The first-order conditions with respect to the choices of x and m_{+1}, respectively, are

$$U_x(x) = 1 \tag{3.7}$$

$$\beta V_m(m_{+1}) = \phi \tag{3.8}$$

where $V_m(m_{+1})$ is the marginal value of an additional unit of money taken into period $t + 1$. The envelope conditions are

$$W_m(m, l, d) = \phi, \quad W_l(m, l, d) = -\phi(1 + i), \quad \text{and,} \quad W_d(m, l, d) = \phi(1 + i_d). \tag{3.9}$$

Note that W is linear in (m, l, d) and optimal decisions characterized by (3.7) and (3.8) are independent of the agent's state (*i.e.* their *wealth*).

3.1.2 The Decentralized Market

Consider a buyer at beginning of the current period DM. Given money holdings, m, this agent has expected lifetime utility

$$V(m) = n\Bigg\{\alpha_0 B^0(m) + \alpha_1 \int_{[\underline{i},\bar{i}]} B(m;i)\,\mathrm{d}F(i) + \alpha_2 \int_{[\underline{i},\bar{i}]} B(m;i)\,\mathrm{d}\left[1-(1-F(i))^2\right]\Bigg\} + (1-n)\,W(m+\tau_s M - d, 0, d)\,. \quad (3.10)$$

Conditional on being an active DM buyer with probability n, a household searches for a bank to obtain a line of credit, taking as given the distribution $F(i) \equiv F(i;m,M,\tau_b)$ of banks' posted loan rates i.[10]. With probability $\alpha_0 \in (0,1)$, the buyer fails to find a bank. Her value is then $B^0(m)$. With probability $\alpha_1 \in (0, 1-\alpha_0)$, the buyer makes contact with one bank. Her ex-post value is then $B(m;i)$, where i is drawn from $F(i)$. With probability $\alpha_2 = 1-\alpha_0-\alpha_1$, the buyer has two independent, randomly drawn meetings with two banks. Her ex-post value is then $B(m;i)$ where i is the lower of the two prices drawn from $1-(1-F(i))^2$. (Without loss of generality, we assume buyers can only sample up to two banks at a time.)

Conditional on being inactive in the DM (with probability $1-n$), the buyer's value *ex-post* equals the value of continuing to the subsequent CM, $W(m-d,0,d)$, with the option to deposit her idle money with the depository institutions. Next we define the post-matching valuation functions B^0, B, and S and their respective supporting optimal demands for special goods and loans.

First, consider an active household who has not succeeded in meeting a lending agent. In such events, there is no possibility of taking out a loan from banks. *Ex post*, such a buyer has valuation:

$$B^0(m) = \max_{q_b \leq \frac{m+\tau_b}{p}} \left\{u(q_b) + W(m+\tau_b M - pq_b, 0, 0)\right\}. \quad (3.11)$$

Using (3.3), the agent's optimal demand for goods can be derived as:

$$q_b^{0,\star}(m;p,\phi,M,\tau_b) = \begin{cases} \frac{m+\tau_b M}{p} & \text{if } p < \hat{p} \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \end{cases}. \quad (3.12)$$

[10] We assume for now a compact support for F as $[\underline{i},\bar{i}]$.

Next, consider the post-match value of a buyer who has contacted at least one lending agent:

$$B(m) = \max_{q_b \leq \frac{m+l+\tau_b M}{p}, l \in [0,\bar{l}]} \{u(q_b) + W(m + \tau_b M + l - pq_b, l, 0)\}. \tag{3.13}$$

In our baseline environment, $\bar{l} = \infty$. This implies that loan contracts are perfectly enforceable as in the baseline case of BCW.

Using the Karush-Kuhn-Tucker conditions, from (3.13) we can derive the demands for special goods and loans. The former is given by:

$$q_b^\star(m; i, p, \phi, M, \tau_b) = \begin{cases} [p\phi(1+i)]^{-1/\sigma} & \text{if } 0 < p \leq \tilde{p}_i \text{ and } 0 \leq i \leq \hat{i} \\ \frac{m+\tau_b M}{p} & \text{if } \tilde{p}_i < p < \hat{p} \text{ and } i > \hat{i} \\ (p\phi)^{-1/\sigma} & \text{if } p \geq \hat{p} \text{ and } i > \hat{i} \end{cases}, \tag{3.14}$$

where

$$\hat{p} \equiv \hat{p}(m; \phi, M, \tau_b) = \phi^{\frac{1}{\sigma-1}}(m + \tau_b M)^{\frac{\sigma}{\sigma-1}} \qquad \text{and} \qquad \tilde{p}_i = \hat{p}(1+i)^{\frac{1}{\sigma-1}}, \tag{3.15}$$

respectively, correspond to a maximal DM price at which the agent will use both his own liquidity and credit line from the bank, and, a maximal price at which the agent's purchase results in her being liquidity constrained. As $\sigma < 1$, $0 < \tilde{p}_i < \hat{p} < +\infty$.

The maximial interest rate at which a buyer is willing to borrow is given by

$$\hat{i} \equiv \hat{i}(m; \phi, M, \tau_b) = (p\phi)^{\sigma-1}[\phi(m + \tau_b M)]^{-\sigma} - 1. \tag{3.16}$$

For any interest rate $i \in [0, \hat{i}]$, the buyer's loan demand is:

$$l^\star(m; i, p, \phi, M, \tau_b) = \begin{cases} p^{\frac{\sigma-1}{\sigma}}[\phi(1+i)]^{-\frac{1}{\sigma}} - (m + \tau_b M) & p \in (0, \tilde{p}_i];\ i \in [0, \hat{i}] \\ 0 & p \in (\tilde{p}_i, \hat{p});\ i > \hat{i} \\ 0 & p \geq \hat{p};\ i > \hat{i}. \end{cases} \tag{3.17}$$

From the first case of (3.14) and (3.17), we can see that if the special good's relative price ($p\phi$) and interest on bank loans (i) are sufficiently low, the agent borrows to top up her own money balance and her goods and loan demands are decreasing in both i and $p\phi$. If, however, the special good's relative price and interest on borrowing are higher (*i.e.*, the intermediate case), the agent prefers not to borrow, but rather to spend all her money on the special good and be liquidity constrained. In this case the loan rate does not matter for demand. Finally, if $p\phi$ and i are sufficiently high, the buyer prefers not only not to borrow but also not to spend all her money balance on the special good. The cutoff pricing functions $(\hat{p}, \tilde{p}_i, \hat{i})$ clearly depend on both the state of the economy and public policy.

3.2 Firms.

The Centralized Market A unit measure of firms converts total labor supplied into the general good, x in the CM. Total CM labor supply is describe in Section 4 below.

The Decentralized Market There is a unit measure of firms operating in DM, each with valuation:

$$S(m) = \max_{q_s} \{-c(q_s) + W(m + \tau_s M + pq_s, 0, 0)\}. \tag{3.18}$$

Here $c(q)$ represents the cost of producing quantity q of special goods, where $c(0) = 0$, $c_q(q) > 0$ and $c_{qq}(q) \geq 0$. The firms' optimal production plan satisfies

$$c_q(q_s) = p\phi. \tag{3.19}$$

That is, in the DM firms produce to the point where the marginal cost of producing good q_s equals its relative price. DM firms here are analogous to sellers in Rocheteau and Wright (2005). It is straightforward to show that in equilibrium their valuation will be $S(0)$ at the start of each DM—*i.e.*, they optimally carry no money into the DM.

3.3 Banking

We split the discussion of the banking sector into two parts: depository institutions and lending agents. The focus in this paper is on the nature of competition in lending, and so we will use the term *lending agents* and *banks* interchangeably. We assume there is a financial services record-keeping technology available in the banking system which banks operate at zero cost. This assumption is in the same spirit as BCW.

3.3.1 Depository institutions

First, consider the interaction between households and firms on the one hand and depository institutions on the other. Depository institutions have the ability to take deposits, lend them to lending agents, enforce repayment of these loans in the CM and to commit to return both deposits and contracted interest to individual agents, also in the CM. In addition, these institutions have the ability to invest deposits within the period with an essentially unmodeled group of "foreign" investors outside the model. Such investments pay an exogeneously specified nominal return, $r = \frac{\gamma}{\beta}$, where γ is the growth rate of the money stock and β the subjective discount factor of households.

Previewing the equilibrium we will consider, because these institutions behave competitively, they will be willing both to promise all depositors a gross return on deposits of $1 + i_d = r$, and to make advances to lending agents in exchange for repayment in the CM at gross rate r as well. Normally, only those households who are uninterested in consuming the special good this period will deposit. Other households will want to keep their money to make purchases in the DM (and possibly to borrow more, see below), and firms will not carry money into the period at all. Note that the choice of r is exogenous, with this being enabled by our "small open economy" assumption. The value of r chosen corresponds to that arising in the baseline version of BCW, which features a perfectly competitive banking system. As such, households will be insured against holding idle balances here to the same extent that they are in that environment.

3.3.2 Lending agents (banks)

The market for loans opens following the interaction of households and firms with depository institutions at the beginning of the period prior to the opening of the DM.

Lending agents (viz. banks) are able to contract with prospective borrowers before the start of the DM and can enforce repayment of loans in the CM. The banks behave in a manner similar to that of sellers in the basic model of Burdett and Judd (1983). That is, they post lending rate i, and commit to fill the demand for loans at that rate.

Households (and in principle, DM sellers) randomly a sample of posted loan rates and are able to borrow the amount they desire at the lowest rate they observe. We restrict attention to cases in which with probability α_k a prospective lender observes $k \in \{0, 1, 2\}$ quoted lending rate(s). On successfully contacting a borrower, the lending agent is able to obtain funds from depository institutions at gross marginal cost r. The details of the interest rate (price)-posting problem are described below. Again previewing equilibrium, lending agents will, on average, earn positive profits as all posted lending rates in equilibrium will exceed i_d. These profits can either be retained by the lending agents or returned lump-sum to households, firms or both in the CM.

Consider now the problem of a lending agent that takes the distribution of posted rates, $F(i)$ as given and has marginal cost of funds i_d. If the bank posts a loan rate i, its expected profit is

$$\Pi(i) \equiv \Pi(i; m, p, \phi, M, \tau_b) = n\left[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2 \zeta(i)\right] R(i), \tag{3.20}$$

where

$$\zeta(i) = \lim_{\varepsilon \searrow 0} \{F(p) - F(p - \varepsilon)\}, \tag{3.21}$$

$$R(i) \equiv R(i; m, p, \phi, M, \tau_b) = l^\star(m; i, p, \phi, M, \tau_b)\left[(1 + i) - (1 + i_d)\right], \tag{3.22}$$

$R(i)$ is profit per customer served, $l^\star(m; p, \phi, M, \tau_b)$ is the demand for loans, and $n\alpha_2\zeta(i)$ is the measure of consumers that contact both this bank and another which has posted the same rate, i.[11]

With regard to banks' optimal choice of i, consider first a hypothetical bank serving borrowers who have contacted *only* this one bank. This bank's *realized* profit is

$$\Pi^m(i) = n\alpha_1 R(i), \tag{3.23}$$

where the superscript, m, denotes that a bank serving only customers who observe a single rate acts effectively as a monopolist.

Second, consider a bank faced with customers who potentially observe more than one rate due to noisy search. The bank's realized profit is given by

$$\Pi^\star \equiv \Pi^\star(m, M, \tau_b) = \max_{i \in \text{supp}(F)} \Pi(i) \tag{3.24}$$

subject to (3.20), (3.21), (3.22) and (3.17).

As we restrict attention to linear pricing rules, tt can be proved that for any state (m, M) and government policy (γ, τ_b), $\Pi^m(\cdot)$ is twice continuously differentiable, strictly concave and always positive valued. Moreover, it can be shown that any bank facing more than one customer will also earn strictly positive profit. There is a maximal loan interest that is the smaller of either the monopolist's optimal loan rate i^m, or, the consumer's maximum willingness to pay $\hat{i}$, where the latter depends on both the state and policy. The natural lower bound on loan rates is i_d, and so the support of the distribution of posted loan rates, $F(i)$, is bounded. The support is also connected, as

[11] We assume that in such cases prospective borrowers randomize between the two lenders. In equilibrium, the probability of a borrower observing two identical lending rates goes to zero.

in Burdett and Judd (1983).[12]

From (3.20), it can be seen that each bank faces the following trade-off: It can raise its profit *per loan* by raising its loan rate (*i.e* by increasing its *markup*). A bank can, however, raise the *measure* of borrowers it serves by lowering its posted rate. Since banks are *ex ante* identical, we may think of the distribution $F(i)$ as representing different pure strategy choices or we may think of banks as mixing symmetrically over a range of interest rates that yield the same expected profit. In either case, borrowers face random loan rates, distributed via F:[13]

Lemma 1. *Suppose that government policy increases the money stock at rate $\gamma > \beta$.*

1. *if $\alpha_1 \in (0,1)$, there is a unique non-degenerate, posted-loan-rate distribution F. This distribution is continuous with connected support:*

$$F(i) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R\left(\bar{i}\right)}{R\left(i\right)} - 1\right], \tag{3.25}$$

where $\text{supp}\left(F\right) = \left[\underline{i}, \bar{i}\right]$, $R\left(\underline{i}\right) = \frac{\alpha_1}{\alpha_1+2\alpha_2}R\left(\bar{i}\right)$ *and* $\bar{i} = \min\{i^m, \hat{i}\}$.

2. *If $\alpha_2 = 1$, then F is degenerate at i_d:*

$$F(i) = \begin{cases} 0 & \textit{if } i < i_d \\ 1 & \textit{if } i \geq i_d \end{cases}. \tag{3.26}$$

3. *If $\alpha_1 = 1$, F is degenerate at the largest possible loan rate $\bar{i} := \min\{\hat{i}, i^m\}$ such that*

$$F(i) = \begin{cases} 0 & \textit{if } i < \bar{i} \\ 1 & \textit{if } i \geq \bar{i} \end{cases}. \tag{3.27}$$

This result is akin to the original notion of "firm equilibrium" in Burdett and Judd (1983, Lemma 2) and in the monetary version of Head and Kumar (2005, Proposition 3). For empirical relevance, we restrict attention to the first part of Lemma 1. That is, to equilibria in which the distribution of loan rates is non-degenerate. With regard to the extent of market power, this case is sandwiched between the two familiar extremes:

[12] These results, A.3.1 to A.3.6, are derived formally in the Online Appendix.

[13] We relegate the proof to the Online Appendix A.3.7.

A Bertrand equilibrium at the competitive price and a monopoly price equilibrium, described respectively in the second and third parts of Lemma 1.

3.4 Government

We adapt our notation for government policies from BCW. The government and monetary authority can make a lump-sum monetary injection or extraction in the CM, and can make targeted transfers, positive or negative, to active and inactive households in the DM. These policy instruments are denoted respectively τ_2, τ_b, and τ_s).

The total change to the money supply, $(\gamma - 1)M$, is split between DM and CM. That is,

$$M_{t+1} - M_t = (\gamma - 1)M = \tau_1 M + \tau_2 M, \tag{3.28}$$

where $\tau_1 = \tau_b + \tau_s$ as specified earlier in (3.1).

4 Stationary Monetary Equilibrium

We focus on a *stationary monetary equilibrium* (SME), in which the price level and money supply grow at the same constant rate: $\phi/\phi_{+1} = M_{+1}/M = \gamma$.

As the price level, $1/\phi$, grows over time in the SME, we divide all nominal variables by the CM price level. From here on, we work with these stationary *real* variables. Let $z = \phi m$ (and $Z = \phi M$) denote individual and aggregate real balances respectively. Also, $\rho = \phi p$ denotes the relative price between DM and CM goods, and, $\xi = \phi l$ refer to the real value of a loan. In an SME, DM sellers never accumulate money in the CM and never borrow, and inactive DM households deposit all their money balances with the depository institutions. Thus, we focus on the loan demand of active buyers only.

4.1 The distribution of posted lending rates

Consider the case of $\alpha_1 \in (0, 1)$ as stated in Lemma 1. Rewriting the distribution of loan rates, F, in terms of stationary variables, we have:

$$F(i; z, \rho, Z, \tau_b) = 1 - \frac{\alpha_1}{2\alpha_2} \left[\frac{R(\bar{i})}{R(i)} - 1 \right], \tag{4.1}$$

where $\mathrm{supp}\,(F) = \left[\underline{i}, \bar{i}\right]$, $\underline{i}$ solves

$$R\left(\underline{i}\right) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R\left(\bar{i}\right), \qquad \bar{i} = \min\{i^m, \hat{i}\}, \tag{4.2}$$

and,

$$R\left(i\right) \equiv R\left(i; z, \rho, Z, \gamma\right) = \left[\rho^{\frac{\sigma-1}{\sigma}} \left(1+i\right)^{-\frac{1}{\sigma}} - \left(z + \tau_b Z\right)\right] \left(i - i_d\right) \tag{4.3}$$

is (real) bank profit per customer served.

We now have the following useful comparative-static result regarding the relationship between household-level real balances and the distribution of posted lending rates:

Lemma 2. *Fix a long-run inflation target $\gamma > \beta$, and let $\alpha_0, \alpha_1 \in (0,1)$. Consider any two real money balances $z < z'$. The induced loan-price distribution $F(\cdot, z)$ first-order stochastically dominates $F(\cdot, z')$.*

The proof can be found in our Online Appendix A.6.1. From Lemma 2 we have that in a SME in which households carry higher (lower) real balances into the DM, they are more (less) likely to draw lower loan-rate quotes, *ceteris paribus*.

4.2 Demand for money and bank credit

We now derive an equation describing CM agents' optimal money demand. A general expression for this is shown in Online Appendix A.4.[14] For clarity, we restrict attention to a stationary monetary equilibrium (SME) in which both *ex ante* demand for money balances and *ex-post* demand for loans in the DM are positive. This in fact will be the equilibrium configuration that emerges under our calibration when we consider a range of computational experiments later.[15]

Lemma 3. *Fix a long-run inflation target $\gamma > \beta$ and let $\alpha_0, \alpha_1 \in (0,1)$. Assume that there is an SME in which real balances, $z^\star \in \left(0, \left(\frac{1}{1+\bar{i}(z^\star)}\right)^{\frac{1}{\sigma}}\right)$. Then,*

1. *The relative price of DM goods satisfies*

$$\rho = 1 < \tilde{\rho}_i(z^\star) \equiv (z^\star)^{\frac{\sigma}{\sigma-1}} (1+i)^{\frac{1}{\sigma-1}}, \tag{4.4}$$

[14] This is done by taking the partial derivative of (3.10) (i.e., marginal valuation of money) one period ahead, combining this with the first-order condition with respect to next-period money balance (3.8) and the optimal DM-good and loan demand functions in (3.12) and (3.14).

[15] These equilibrium properties rely on a sufficient condition that is not dependent *per se*on model primitives, but rather is imposed in our computational experiments.

for any $i \in \text{supp}(F(\cdot; z^{\star}))$.

2. *Loan demand is always positive.*

3. *Money demand is given by:*

$$\frac{\gamma - \beta}{\beta} = \underbrace{(1-n)i_d}_{[A]} \left(\frac{1}{1+\bar{i}(z^{\star})} \right)^{\frac{1}{\sigma}} + \underbrace{n\alpha_0 \left(u^{'}[q_b^0(z)] - 1 \right)}_{[B]}$$
$$+ \underbrace{n \int_{\underline{i}(z)}^{\bar{i}(z)} \mathbb{I}_{\{0<\rho<\tilde{\rho}_i\}} i \left[\alpha_1 + 2\alpha_2 (1 - F(i;z)) \right] dF(i;z)}_{[C]}. \tag{4.5}$$

The terms on the right-hand side of equation (4.5) reflect the marginal benefit (deposit interest) to *ex post* inactive buyers who deposit their idle money (A), the liquidity premium on own money holding in delivering consumption value (B), and the expected cost savings from incurring less loan liability at the margin (C).

Because active DM buyers fail to make contact with a lender with positive probability (α_0), and because it is costly to carry too much money into the next period ($\gamma > \beta$) agents may find it welfare improving *ex ante* to count on using bank credit to "top up" liquidity. This is in contrast to BCW where the only potential gains from banking arise from the payment of interest to *inactive* buyers. The market power of banks, however, (manifest in loan rate dispersion) tends to reduce the *ex ante* value of real balances and be welfare reducing. As such, whether or not the presence of banking improves welfare is ambiguous. To see this, we can rewrite condition (4.5) as an asset pricing relation from the perspective of a household at the end of each period's CM:

$$1 = \underbrace{\frac{\alpha_0 \left(u^{'}[q_b^0(z)] - 1 \right)}{i_d}}_{\text{Benefit of self-insurance}}$$
$$+ \underbrace{\int_{\underline{i}(z)}^{\bar{i}(z)} \mathbb{I}_{\{0\leq\rho<\tilde{\rho}_i\}} \underbrace{\left[\alpha_1 + 2\alpha_2 \left(1 - F(i;z,\gamma) \right) \right]}_{\text{Extensive margin}} \underbrace{\left(\frac{i}{i_d} \right)}_{\text{Intensive margin}} \mathrm{d}F(i;z,\gamma)}_{\text{Benefit, proportional to banks' markup, of reduced borrowing}}. \tag{4.6}$$

The left-hand-side of this no-arbitrage condition (4.6) is the normalized, relative price of giving up CM consumption today. On the right, we have the discounted expected real return from carrying money into the next-period DM. This return has two components inherited from the equivalent expression (4.5). These components—now measured relative to the opportunity cost of holding money (i_d are associated with the ability to consume without credit, and with a reduced loan-interest burden, respectively. The latter term reflects the fact that households must consider both the cost of a particular loan (intensive margin) and the likelihood of being able to choose among multiple lenders (extensive margin). It is useful at this point to consider two instructuve special cases. First, take (4.5) in the special case of BCW with competitive banks. With $\alpha_2 = 1$ and $\alpha_0 = 0$, by Proposition 1, F is degenerate on the singleton set $\{i = i_d\}$. In this case, money demand is

$$\frac{\gamma - \beta}{\beta} = [u^{'}(q_b) - 1] \equiv i. \tag{4.7}$$

This is the same as (22) in BCW with linear production in the DM, *i.e.* $c'(q_s) = 1$.

Consider now a pure currency economy without banks ($\alpha_0 = 1$). Money demand is

$$\frac{\gamma - \beta}{\beta} = n[u'(\tilde{q}_b) - 1]. \tag{4.8}$$

Comparing (4.7) and (4.8), it is clear that $q_b > \tilde{q}_b$, ans do perfectly competitive banks are always welfare-enhancing (*i.e.* essential) if $\gamma > \beta$. In contrast, market power can offset the welfare-enhancing role of intermediation, as the right-hand side of (4.5) with imperfectly competitive banks may be smaller than that of (4.8). Lending banks extract surplus in monetary trades, and this can outweigh their benefits as providers of insurance for idle liquidity. As such, in contrast to BCW, here financial intermediation may not always be essential.

4.3 Goods market equilibrium in the DM

DM sellers optimize and the Walrasian price-taking DM market clears:

$$\begin{aligned} q_s(z, Z, \gamma) \equiv c'^{-1}(\rho) = n\alpha_0 q_b^{0,\star}(z; \rho, Z, \gamma) \\ + n\left[\int_{\underline{i}}^{\bar{i}} [\alpha_1 + 2\alpha_2 - 2\alpha_2 F(i)]\, q_b^{\star}(z; \rho, Z, \gamma)\, \mathrm{d}F(i)\right]. \end{aligned} \tag{4.9}$$

Given $x^{\star} = 1$, and SME solutions $\{z^{\star}, q_b^{\star}(z^{\star}, \cdot), q_b^{0,\star} q_b^{\star}(z^{\star}, \cdot)\}$, we can also verify that the CM labor and goods markets clear.

4.4 Equilibrium lending

In equilibrium lenders must earn non-negative profits. In aggregate, this requires that total interest collected on real loans weakly exceeds that paid on total real deposits:

$$
\begin{aligned}
(1-n)i_d\delta^\star(z,Z,\gamma) &\equiv (1-n)i_d\left(\frac{z+\tau_b Z}{\rho}\right) \\
&\leq n\left\{\int_{\underline{i}}^{\bar{i}}\left[\alpha_1+2\alpha_2-2\alpha_2 F(i)\right] i\,\xi^\star(z;i,\rho,Z,\gamma)\,\mathrm{d}F(i)\right\}.
\end{aligned}
\tag{4.10}
$$

Definition 4. A *stationary monetary equilibrium with money and credit* is a steady-state allocation point $(x^\star, z^\star, Z)$, allocation functions $\{q_b^{0,\star}(z^\star,\cdot), q_b^\star(z^\star,\cdot), \xi^\star(z^\star,\cdot)\}$, and (relative) pricing functions $(\rho, F(\cdot; z^\star, \rho, Z, \tau_b))$ such that given government policy (γ, τ_b, τ_s).[16]

1. $x^\star = 1$;
2. $z^\star$ solves (4.5);
3. $Z = z^\star$;
4. given $z^\star$, $q_b^{0,\star}(z^\star,\cdot)$ and $q_b^\star(z^\star,\cdot)$, respectively, satisfy

$$
q_b^{0,\star}(z;\rho,Z,\tau_b) = \frac{z+\tau_b Z}{\rho} \qquad \text{for } \rho < \hat{\rho} \tag{4.11}
$$

and,

$$
q_b^\star(z;\rho,Z,\gamma) = \left[\rho\,(1+i)\right]^{-\frac{1}{\sigma}} \qquad \text{for } 0<\rho\leq\tilde{\rho}_i \text{ and } 0\leq i<\hat{i}; \tag{4.12}
$$

5. $\xi^\star(z^\star,\cdot)$ satisfies:

$$
\xi^\star(z;i,\rho,Z,\gamma) = \rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}} - (z+\tau_b Z) \quad \text{for } \rho\in(0,\tilde{\rho}_i], \quad i\in[0,\hat{i}); \tag{4.13}
$$

6. ρ solves (4.9);
7. $F(\cdot; z^\star, \rho, Z, \tau_b)$ is determined by (4.1); and,
8. aggregate loans supplied is feasible according to (4.10),

[16]Note that τ_s does not materially affect equilibrium determination, and so we can set $\tau_s = 0$ without changing our basic results. For the baseline calibration of the model, we will let $\tau_b = 0$ also, so that there is no redistributive tax/transfer policy in place. Later we will consider counterfactual analyses involving differential tax policies.

5 SME with money and credit

Under sufficient conditions, there exists a unique SME with money and credit:

Proposition 5. *Assume loan contracts are perfectly enforceable. If $\gamma > \beta$, $z^\star \in \left(0, (1+\bar{i}(z^\star)^{-\frac{1}{\sigma}}\right)$, and there exists an endogenous lower bound $N(z^\star) \in [0,1]$ such that $n \geq N(z^\star)$, then there exists a unique SME with both money and credit.*

For a proof, see Appendix A.6.4. Formal proofs of intermediate results can be found in Appendices A.6.1, A.6.2, and A.6.3. Here we sketch the basic idea. Fix $\gamma > \beta$, first we show that lending banks' posted loan-price distribution F is decreasing (in the sense of first-order stochastic dominance) with respect to households' real balance, z. The intuition is that as households carry more money into the DM, the marginal benefit of bank credit falls. See Lemma 2 for details. As such, households with higher z are more likely to observe a lower best interest rate.

Second, with probability α_0, a household contacts no lending agent and so its marginal benefit from holding an extra dollar falls as real balances rise. Togetherm, these factors establish that the right-hand side of (4.5) is a continuous and monotone decreasing function of z. Since the left-hand side of (4.5) is constant in z, there exists a unique real money balance $z^\star$ for a given $\gamma > \beta$.

The second condition ensures that $z^\star$ is bounded and that the maximal loan interest is not too high. This guarantees positive loan demand. The third condition requires that the measure of active DM buyers not be too small. While neither of these conditions are determined solely by model primitives, but rather depend on equilibrium objects, they both can be easily verified by numerical calculations.

While the results obtain for $\gamma > \beta$, it is also of interest to consider the case of the Friedman rule ($\gamma = \beta$):

Proposition 6. *If $\gamma = \beta$, then there is no SME with loan interest rate dispersion. Moreover, if $\alpha_0 > 0$, the Friedman rule attains the first-best allocation $q^{\star,FB}$.*

The banking system is redundant at the Friedman rule, for the simple reason that it is costless to carry money across periods. As such households can insure themselves perfectly against the risk of not having trading opportunities and so there is no gain to redistributing liquidity across agents in an SME. From this point onward, we restrict attention to cases in which $\gamma > \beta$.

6 Quantitative Analyses

In this section, we first calibrate the baseline model to macro-level data and then use it to investigate the effects of various parameters and alternative policies. We also compare the model's predictions to micro-level empirical observations for external validity.

6.1 Baseline calibration

Our approach is to match the empirical money demand and the average gross lending markup in the macro data.[17] In our model, we measure the average gross lending markup by

$$S = \int_{i_{\min}(z)}^{i_{\max}(z)} \frac{i}{i_d(\gamma)} \mathrm{d}F^{\star}(i; z, \gamma). \tag{6.1}$$

The search probabilities (α_0, α_1) affect directily the lending rate distribution, $F^{\star}(\cdot)$, and thus banks' average markup.

The CM goods, $U(x)$, is also assumed to be CRRA(σ_{CM}). With quasi-linear preferences, real CM consumption is then given by $x^{\star} = \bar{U}_{\mathrm{CM}}(U')^{-1}(A)$, where the scaling parameter, $\bar{U}_{\mathrm{CM}}$, determines the relative importance of CM and DM consumption. We $\sigma_{CM} = 1.01$ to be very close to log utility.

We interpret a model period as a year and calibrate to annual data. The model has eight parameters: $(\tau, \beta, \sigma_{DM}, \sigma_{CM}, \bar{U}_{\mathrm{CM}}, n, \alpha_0, \alpha_1)$. Some parameters can be determined directly by observable statistics. We use the Fisher relation to pin down money growth rate (inflation), τ, and discount factor, β. The share of inactive buyers (depositors) $\tilde{n} \equiv 1 - n$ is set to match the average share of household depositors with commercial banks per thousand adults in the United States.[18] We then choose jointly $\left(\sigma_{DM}, \bar{U}_{\mathrm{CM}}, \alpha_0, \alpha_1\right)$ to match the money demand and average gross lending markup using spline functions fitted to annual data.

Our parameter values and targets are summarized in Table 2. We use annual data 1948 to 2007, so as to avoid both the Second World War and the Great Recession. In the latter, the nominal interest is at its zero lower bound.[19] Figure 2 provides a scatter-plot

[17] We use bank prime loan rate / three-month u.s. Treasury Bill rate (i) as a proxy for the average gross lending markup. We use three-month T-bill rate to be consistent with the model and money demand data used in Lucas and Nicolini (2015). Alternatively, we could use the federal funds rate and this would not alter the general shape of the function.

[18] Source: St. Louis Fed. Series (USAFCDODCHANUM), use of financial services, key indicators.

[19] Note: data for the bank loan prime rate is only available from 1931 onward.

of the data, the spline-fitted model using the data, and the calibrated model's implied money demand and average gross lending markup.

Table 2: Calibration and targets

Parameter	**Value**	**Empirical Targets**	**Description**
$1+\tau$	$(1+0.0382)$	Inflation rate[a]	Inflation rate
$1+i$	$(1+0.0481)$	3-month T-bill rate[a]	Nominal interest rate
β	0.9906	-	Discount factor, $\frac{(1+i)}{(1+\tau)}$
σ_{DM}	0.525	Aux reg. $(i,M/PY)$[b]	CRRA(q)
$\bar{U}_{CM}$	0.8	Aux reg. $(i,M/PY)$[b]	CM preference scale
σ_{CM}	1.01	Normalized	CRRA(x)
$\tilde{n}$	0.35	household depositors[c]	Proportion of inactive DM buyers
α_0, α_1	$0.04, 0.08$	Aux reg. (i,markup)[d]	Prob. $k = 0, 1$ bank contacts

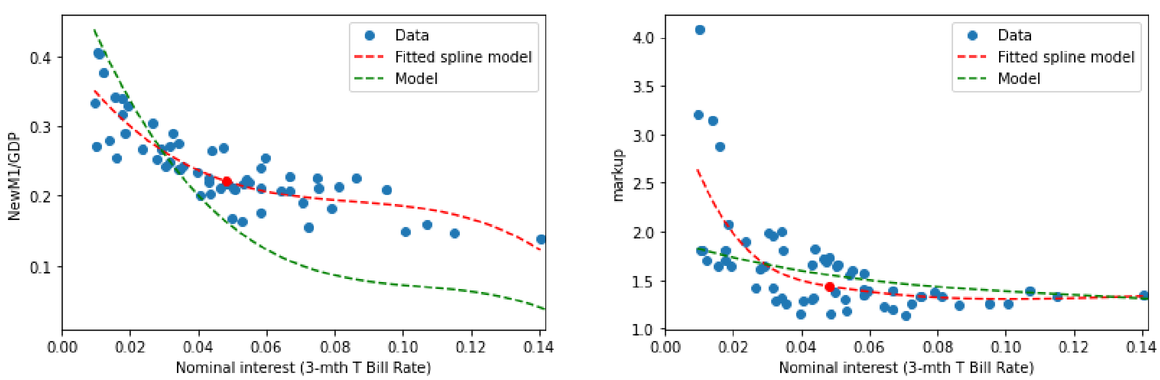
[a] Annual nominal interest and inflation rates.

[b] Auxiliary statistics (data) via spline function fitted to the annual data: 3-month T-bill (i); Lucas-Nicolini New-M1-to-GDP ratio (M/PY). Elasticity of M/PY with respect to i.

[c] Household depositors with commercial banks per 1000 adults for the United States.

[d] Auxiliary statistics (data) via spline function fitted to the annual data: 3-month T-bill (i); Average banking (gross) markup ratio (bank loan prime rate / i). Elasticity of average (gross) markup with respect to i.

Figure 2: Money demand and Average banking markup: Model and Data



(a) Calibration and money demand data

(b) Calibration and average (gross) markup data

In Figure 2, it can be seen that model's fit to aggregate money demand is not perfect, especially at higher nominal interest rates. This is due to a tension between matching both real money demand and the average banking markup simultaneously. In the model, a higher nominal interest leads to both a reduction in real money demand and an increase in the cost of funds for banks (i_d). The latter effect reduces the average markup. In order to match the high average markup in data, DM buyers in the model would have to hold even lower real balances, given that real balances are inversely related to lending rates. Nonetheless, we view the model's fit under our benchmark calibration to be reasonable.

6.2 Comparative steady-states

We now consider SME's indexed by different steady-state rates of inflation, $\gamma - 1 \equiv \tau$. We ask the following questions: First, under what circumstances are banks essential (*i.e.* welfare improving)? Second, what mechanisms are at work and what are their testable empirical predictions? And third, how do those mechanisms affect markups and the pass-through of monetary-policy pass to loan rates?

As noted above, in contrast to BCW, financial intermediation of the type studied here need not always be welfare improving. Moreover, in our theory the design of optimal monetary and (redistributive) tax policy depends on the policy and state dependent loan rate distribution, F, in equilibrium.[20]

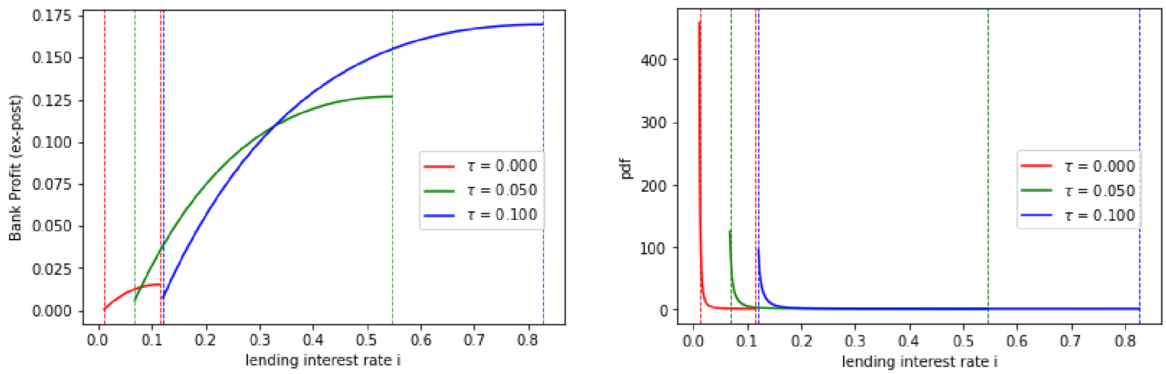
We begin with a discussion of the tradeoff faced by a lending bank in its rate setting decision. Figure 3 depicts realized profit per customer and posted loan rate densities for steady-state inflation rates at zero, five and ten percent. Lenders trade off between profit per customer (the *intensive margin*), which is increasing in the posted loan rate and the number of customers that it successfully serves (the *extensive margin*) which is *decreasing* in the posted rate. As inflation τ, rises, not only does the equilibrium support of F shift to the right, but the mass of the density also shifts rightward relative to the lower bound. We identify this latter effect with the extensive margin, as it implies that banks lose fewer customers as they raise their lending rates.[21]

We now consider the effect inflation on banks' market power. Specifically, we vary inflation from the Friedman rule, $\tau = \beta - 1$ to 300% ($\bar{\tau} = 3$). As shown in Proposition 2, the posted loan interest rate distribution $F(i)$ increases with inflation in the sense of first-order stochastic dominance. As such, the average loan interest rate rises with inflation. As, however, banks' cost of funds also increases with inflation, it is necessary to look at markups in order to determine how inflation affects banks' market power. To this end, we measure market power by both the average *level* and the dispersion of

[20]We will discuss this redistributive policy in Section 7.

[21]In the figure red, green and blue indicate annual inflation of 0%, 5%, and 10%, respectively. The dashed vertical lines indicate the bounds of the support.

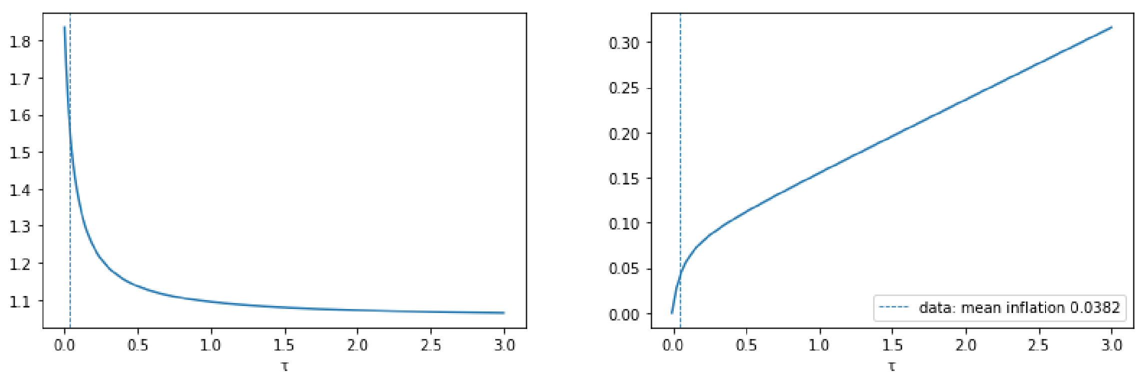
Figure 3: Lending banks' extensive versus intensive margin trade off for various $\gamma = 1+\tau$



(a) intensive margin: profit per customer

(b) extensive margin: queue-length

Figure 4: The effects of inflation on lending banks' market power for $\tau \in [\beta - 1, \bar{\tau}]$



(a) average loan (gross) markups

(b) dispersion loan (gross) markups

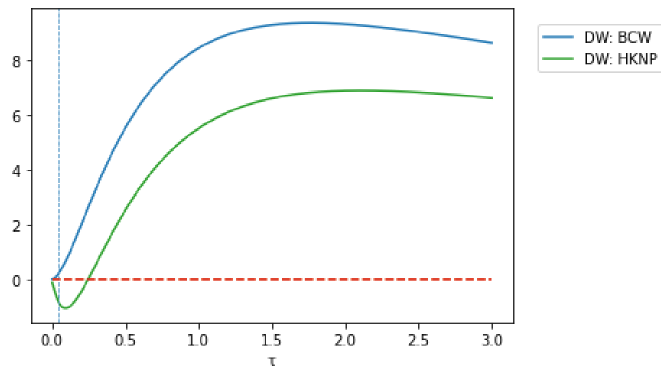
banks' markups. The average markup, S, is given by:

$$S = \int_{\underline{i}}^{\bar{i}} \frac{i}{i_d(\gamma)} \mathrm{d}F(i; \gamma, z) \tag{6.2}$$

and we use the coefficient of variation of markups across banks as our measure of dispersion. These two measure are depicted in two panels of Figure 4.

As trend inflation rises, market power measured by the average markup declines, especially sharply at low inflation. At the same time, the support of F shifts right and becomes wider, reflecting an increase in dispersion. Those banks posting the lowest rates post closer to their marginal cost in an attempt to serve a large number of borrowers, many of whom have made contact with more than one bank. On the other hand, those posting high rates (which for the most part serve only customers with no alternative)

Figure 5: Welfare gain from financial intermediation: with banks vs. without banks



raise their rates by a large amount to take advantage of their borrowers' high marginal utility of consumption driven by their lower real balances. At higher inflation, the mass of F spreads out from it's lower bound so that posted interest rates become more dispersed. At the same time, that lower bound falls toward banks' marginal cost of funds. The average gross is highest at low inflation (although it collapses to one at the Friedman Rule) then converges back to one as τ (and inflation) rises sufficiently.

With regard to welfare, we identify two opposing channels through which anticipated inflation works through financial intermediation. On one hand, as in BCW improve welfare by providing insurance against holding idle money in the DM as an inactive buyers. On the other, there is a negative, welfare-reducing effect emanating from banks' market power in the loan market. By raising the cost of additional funds, banks effectively extract surplus from buyers in DM trades, putting downward pressure on the value of real balances. Overall, whether banks raise of lower welfare in equilibrium depends on the *net* effect of these two opposing channels.[22] As such, it depends on the extent of imperfect competition (measured by markups) in equilibrium.

Figure 5 shows the welfare effects of inflation in both our imperfectly competitive benchmark (green) and the BCW economy (blue) which our model nests by setting $\alpha_2 = 1$. In both the BCW economy and our baseline the relationship between trend inflation and welfare is non-monotonic. In the former case, welfare is increasing at low inflation and only begins to fall at very high inflation as the inflation tax eventually outweighs the gains from insurance. In our imperfectly competitive benchmark, however, the additional effect of imperfect competition leads to a *negative* welfare effect of banking

[22]Note: Our welfare criterion is measured in terms of households (ex-ante) lifetime utility. To understand whether there is welfare gain from financial intermediation, we compare the households lifetime utility in an economy with banks relative to an economy without banks.

overall at low inflation. When trend inflation is low, the gains from insurance are also low and are easily outweighed by the high markups (recall Figure 4) which arise in equilibrium. As inflation rises, markups fall and the gains to insurance rise so that net effect of the banking system to be positive. Welfare gains are, however, always lower in the imperfectly competitive benchmark than in the BCW case.[23]

Overall, imperfect competition in banking has the potential to offset the welfare-enhancing effects of financial intermediation. This finding suggests that policymakers, especially in low-inflation countries may rightly be concerned with market power in the banking sector.

6.3 Imperfect pass-though: a testable empirical prediction

The previous analysis illustrates two implications of the theory: First, there is positive markup of lending rates on average over the cost of bank funds in equilibrium, implying the potential for imperfect pass-through of monetary policy to lending rates. Second, there is a positive correlation between equilibrium average markup and dispersion markup in the loan market. That is, as inflation rises, banks pass-through the increase in costs *differentially* to their lending rates in a manner analogous to that described by Head, Kumar and Lapham (2010).

Using the annualized version of the monthly data from Section **??**, we find that the correlation between the dispersion and average level of residual or orthogonalized markup is 0.64. In our baseline calibration, the implied correlation is 0.69. This provides an external validity check on the model's empirical relevance.

7 Optimal stabilization policy

In this section, we construct optimal Ramsey stabilization policies in response to fluctuations in aggregate demand. Our policy exercise is in the same spirit as Berentsen and Waller (2011). The government commits to long-run money supply growth path, determined by $\tau \equiv \gamma - 1$. Our analysis in novel in that the distribution of lending rates, F, is both state and policy dependent. As such, banks' market power in the loan market is endogenous and responds to policy changes. We consider two regimes for government policy in response to aggregate demand fluctuations:[24]

[23]Note that the welfare gain from financial intermediation in both economies approaches to zero as $\tau \to \infty$. This is because the value of liquidity is of very little value at very high inflation.

[24]Details of the problem setup can be found in Appendix A.7.

1. **An active central bank**: The policymaker commits to an *ex ante* optimal policy that maximizes social welfare in a steady-state equilibrium (SME). Specifically, let $\omega = (n, \epsilon) \in \Omega$ denote the aggregate state (vector) in DM, where $\Omega = [\underline{n}, \bar{n}] \times [\underline{\epsilon}, \bar{\epsilon}]$ and $\psi(\omega)$ is the density of ω. The central bank solves[25]

$$
\begin{aligned}
\max_{\{q_b^0(\omega),\ q_b^1(\omega), \tau_b(\omega)\}_{\omega\in\Omega}} \quad & U(x) - x - c(q_s) \\
& + \int_{\omega\in\Omega} n\alpha_0 \epsilon u \left[q_b^0(z;\gamma,\tau_b,\omega)\right] \psi(\omega)\,\mathrm{d}\omega \\
& + \int_{\omega\in\Omega} n \int_{\underline{i}}^{\bar{i}} \left[\alpha_1 + 2\alpha_2\left(1 - F(i;z,\gamma,\tau_b,\omega)\right)\right] \\
& \times \epsilon u\left[q_b^1(z;i,\gamma,\tau_b,\omega)\right] \mathrm{d}F(i;z,\gamma,\tau_b,\omega)\,\psi(\omega)\,\mathrm{d}\omega
\end{aligned}
\tag{7.1}
$$

The policy plan prescribes ω-contingent liquidity injections. That is, $\tau_1(\omega) = \tau_b(\omega) \geq 0$.

2. **A passive central bank**: In this regime, the policymaker is constrained by $\tau_1(\omega) = \tau_2(\omega) = 0$ for all $\omega \in \Omega$. In this case, equilibrium outcomes are similar to those of the deterministic baseline SME.

Assume n is distributed uniformly on $\{n_1, \ldots n_K\}$ where $n_i < n_{i+1}$. We interpret shocks to the number of active DM buyers (n) as a proxy for aggregate demand fluctuations. We fixing the long-run inflation target at $\tau > \beta - 1$ (away from the Friedman rule), and thus banks' marginal cost of funds is fixed.[26]

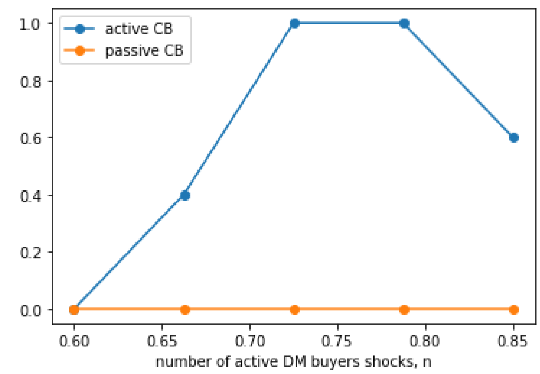
We identify two opposing forces of state-contingent liquidity injections, $\tau_b(n)$, on both allocations and welfare.[27] First, higher $\tau_b(n)$ shifts the support of the loan interest rate distribution F to the left, and reduces its dispersion. This is welfare-improving, as it reduces banks' market power directly. Second, state-contingent liquidity injections in the DM crowd out money demand, lowering z on average, and reducing welfare. This effect also tends to increase banks' market power, shifting the distribution, F to the right in the sense of first-order stochastic dominance. The net welfare consequences of stabilization policy thus depend on the combination of these two opposing effects.

[25] Recall that n is the measure of active buyers in the DM. Here ϵ is a multiplicative shock affected utility from DM consumption. When considering shocks to n, ϵ is held constant. For computational details, please see the pseudocode summarized in Appendix **??**.

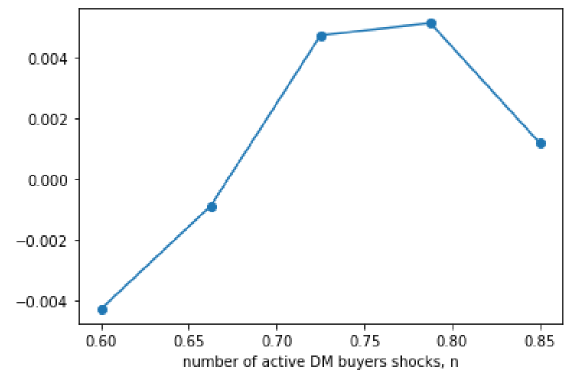
[26] Note: If $\tau = \beta - 1$, holding money is costless, there is no need for either the insurance banks provide or for stabilization policy. Our focus here is not about the optimality of the Friedman rule, but rather on optimal stabilization policy when inflation is constrained to be away from it exogenously. We set τ equals to the average inflation rate in Lucas and Nicolini (2015) data (from 1948-2007).

[27] These can be deduced from (A.7.9) and (A.7.8) in Appendix A.7.

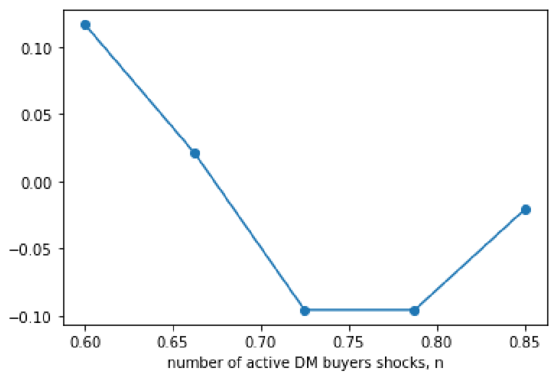
Figure 6: The effects of state-contingent monetary transfers on allocations, market power and welfare



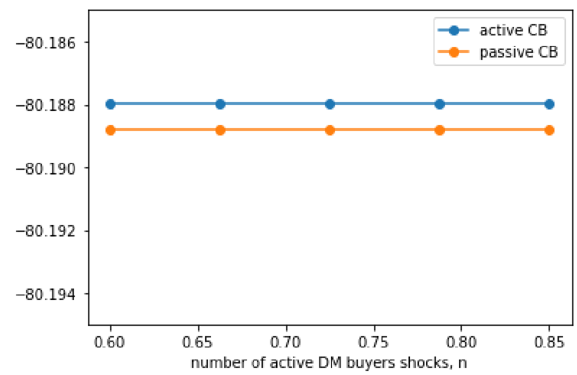
(a) state-contingent liquidity injection



(b) difference in DM allocation



(c) difference in gross loan markup



(d) ex-ante welfare; active vs. passive policy

The key insight of this section is that there is room for demand stabilization via liquidity-management policy, given a long-run inflation target.[28] When aggregate demand "heats up", the optimal stabilization prescribes injecting relatively more liquidity to ex-post high-aggregate demand state.[29] Overall, we find there is inefficiency arising from a passive policy regime as shown in Figure 6.The reason is as follows.

There is more demand for liquidity when more people shop in the DM as n increases. The active central bank commits to inject more liquidity into the market in high demand

[28]Note: we have also consider idiosyncratic taste shock ϵ for the DM goods to be a proxy for demand fluctuation. The results are similar but we do not discuss it here due to limited space. The key difference of ϵ shock relative to n shock is that there is an extra moving part in the equilibrium loan-price distribution F. In particular, lending banks' trade-offs are changing with respect to both ϵ-state-contingent policy and ϵ shock simultaneously. Whereas the n shock case, there is one less moving part in this aspect.

[29]Note: this policy prescription (somewhat counter-intuitive to Keynesian stabilization policy) makes sense when the policymaker needs to take into account endogenous market power and markup response by banks. Moreover, the policy prescription is consistent with the elastic currency mandate of the FED.

state relative to a passive policy regime. This liquidity injection induces relatively more consumption (less) in the state with ex-post more (less) active buyers than the passive policy regime. Similarly, the active policy induces a relatively increase (reduction) in ex-post markups in low n states (high n states). This happens via two opposing forces. On one hand, higher n prescribes higher $\tau_b(n)$, which leads to a reduction in ex-ante z (relative to the passive policy). The lower ex-ante z induces higher (ex-post) average markup in low n state (implied by first-order stochastic dominance and cost of funds is state-independent). On the other, higher $\tau_b(n)$ directly lowers the (ex-post) average markup through reducing monopoly price (support of F) in high demand states.

In summary, the efficiency gains from active policy come from the ability of the central bank to reduce some of the (ex-post) market power in the banking system. As a result, the active demand-side stabilization policy through liquidity provision results in higher ex-ante welfare for households.

8 Conclusion

We construct and study a microfounded monetary economy where market power of lenders (banks) is endogenous to policy. We show that imperfect competition may render an otherwise useful banking system (i.e., one that provides an insurance role for holders of idle funds) by re-allocating liquidity) detrimental if inflation is sufficiently low.

We showed that the model can rationalizd incomplete pass-through of monetary policy, via banks' borrowing cost, to lending interest rates. We also demonstrate a testable positive relationship between the average markup and the dispersion markup. This is corroborated by evidence from micro-level data on consumer loans in the U.S.

Our welfare analysis speaks to why policymakers in many low-inflation countries may, rightly, be concerned with market power in the banking sector. We close by studying optimal interest-rate and tax policies designed to alleviate banking demand instabilities. We find there are efficiency gains from demand stabilization via liquidity-management policy, given a long-run inflation target.

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Online Appendix

A.1 Empirical Analysis of Markups at the State Level

In this section, we calculate the markup standard deviations and means at the state-month level and construct a panel data. Figure A.7 demonstrates markup standard deviation and average are positively correlated at state-month level.

Then, we run OLS regressions of markup standard deviation on markup average after controlling for state fixed effects and time fixed effects. Specifically, we estimate β_0 in the following sepecification,

$$Dispersion_{s,t} = \alpha_0 + \beta_0 \overline{Markup}_{s,t} + \alpha_1 Z_s + \alpha_2 Z_t + \epsilon_{s,t} \tag{A.1.1}$$

Whereas s stands for the state and t stands for the month. We cluster the standard errors by state and month.

Table A.1 illustrates how state-month markup standard deviations are associated with markup average. Column (1) to (3) examine how markup standard deviation is related with markup average using raw markups. Column (4) to (6) examine how markup standard deviation is related with markup average using orthogonalized markups. All columns show a positive and statistically significant relationship between markup standard deviation and markup average. The magnitude of the coefficient is also economically significant. From column (6), the coefficient indicates one percentage point increase in orthogonalized markup average can lead to 0.286 percentage point increase in the standard deviation, after controlling for state fixed effects and time fixed effects.

Figure A.7: Relationship between markup dispersion and average

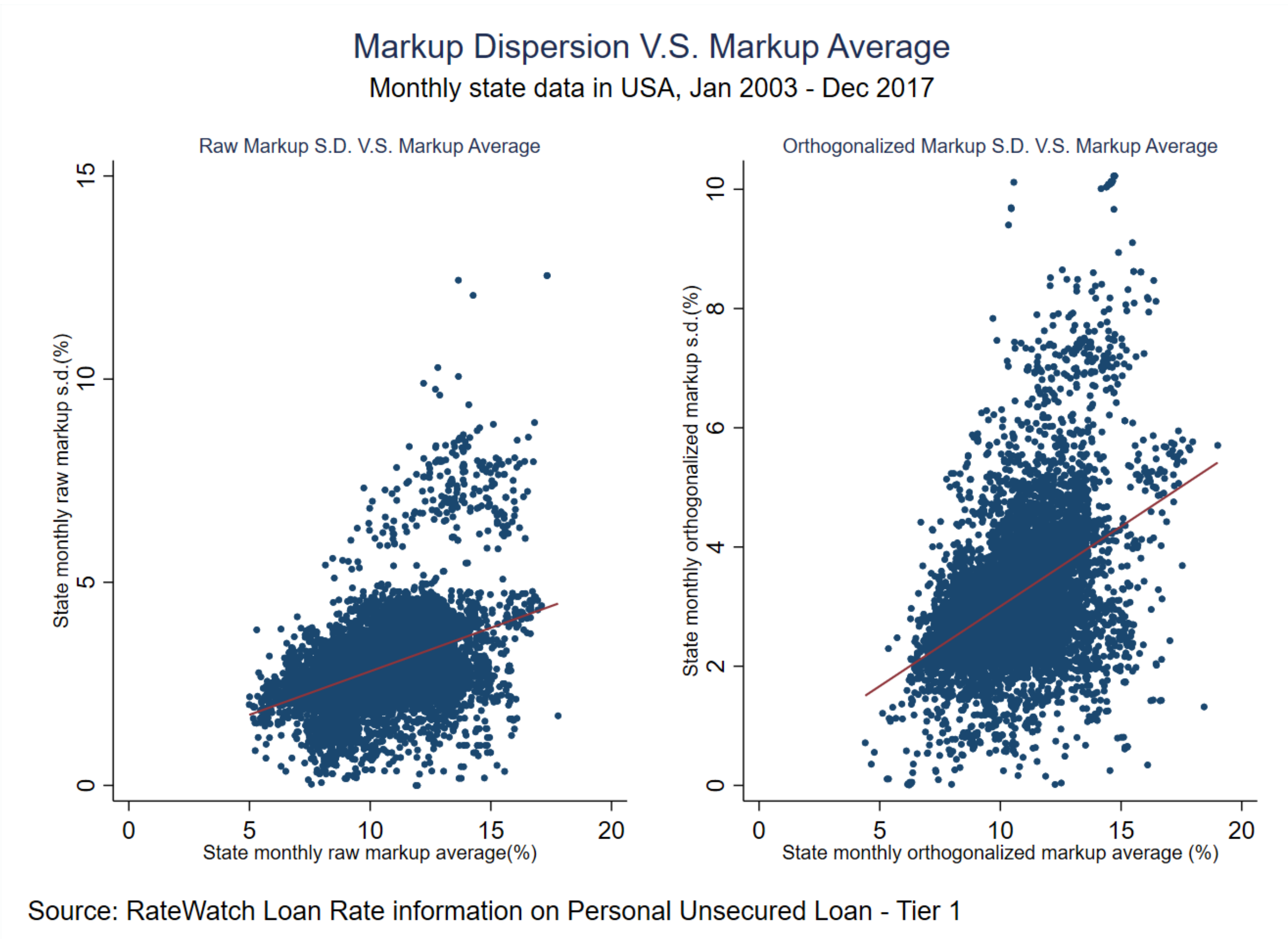

Table A.1: OLS regressions on state markup standard deviation and state markup mean from January 2003 to December 2017.

	Markup dispersion: $Dispersion_{s,t}$					
	Raw markup			Orthogonalized markup		
	(1)	(2)	(3)	(4)	(5)	(6)
	State FE	Time FE	Both FE	State FE	Time FE	Both FE
$\overline{Markup}_{s,t}$	0.179***	0.290***	0.353***	0.220***	0.304***	0.286***
	(6.06)	(3.08)	(4.56)	(3.99)	(3.86)	(3.42)
State fixed effects	X		X	X		X
Time fixed effects		X	X		X	X
N	8237	8237	8237	7463	7463	7463
adj. R^2	0.618	0.178	0.646	0.538	0.203	0.577

Nota: $^{*}\, p < 0.1$; $^{**}\, p < 0.05$; $^{***}\, p < 0.01$

A.2 Control variables list

Table A.2: This table shows the list of control variables to obtain the orthogonalised markup.

(a) Panel A: County variables

Variable	Data source	Frequency	Details
Real GDP	BEA	Annual	Annual county real GDP
GDP growth	BEA	Annual	Real GDP growth
Establishments	BLS	Annual	Number of establishments within county
Unemployment	BLS	Annual	County unemployment rate
House price	U.S. Census	Annual	Average housing pricing in the county
Median income	U.S. Census	Annual	Median Household Income
Population	U.S. Census	Annual	ln(Total population)
Poverty	U.S. Census	Annual	Proportion of county population under poverty-line

(b) Panel B: Local competition

Variable	Data source	Frequency	Details
Within county share	SOD	Annual	Total branch deposits / Total county deposits
County deposit HHI	SOD	Annual	HHI of county's deposit holdings
County branch count	SOD	Annual	Number of branch counts in the county

(c) Panel C: Bank branch network

Variable	Data source	Frequency	Details
Within bank share	SOD	Annual	Total branch deposits / Total bank deposits
Bank deposit HHI	SOD	Annual	HHI of commercial bank's deposit holdings across its branches
Bank branch count	SOD	Annual	Number of branch counts in the commercial bank

(d) Panel D:Commercial bank controls

Variable	Data source	Frequency	Details
Deposit reliance	Call reports	Quarter	Total deposits / Total liabilities
Leverage	Call reports	Quarter	Total equity / Total assets
Credit risk	Call reports	Quarter	Allowance for Loan and Lease Losses /Total Loans
Bank size	Call reports	Quarter	ln(Total assets)

A.3 Omitted Proofs

This appendix supplies the intermediate results and proofs that lead to the characterization of an equilibrium distribution of loan rates in the noisy-search model for loans. We first begin by proving for case 1 in Lemma 1 (since the case where $\alpha_1 \in (0,1)$ is our main focus of the model). Then we lay out the proof for the remaining cases (pure monopoly bank on one limit, and competitive banks on the other).

The characterization is arrived at in a few intermediate steps. First, in section A.3.1, we show any bank faced with just one customer ex post will earn strictly positive profit. Second, in section A.3.2 we show that banks that ex post face more than one customer will also earn strictly positive profit. Third, in section A.3.3 we show that there is a unique upper bound on loan prices. Fourth, if the upper bound loan rate is the monopoly rate, we show (in section A.3.4) that this rate is uniquely determined as a function of the state of the economy. There is a natural lower bound on loan rates, which is i_d. These results help establish that the equilibrium support on the distribution of loan rate F is bounded.

In a noisy search equilibrium, the banks will be indifferent between a continuum of pure-strategy price posting outcomes. For example, a bank can choose some lower rate in return for attracting a larger measure of borrowers. Or it can post some higher rate to increase its intensive-margin markup but attract a smaller measure of borrowers. Or it can charge a monopolist price. The intermediate results in Lemmata 11 to 13 (in section A.3.5 to A.3.6) show that there is a continuum of pure-strategy price posting outcomes that deliver the same maximal monopoly profit. Thus, banks can ex-ante mix over these pure strategies, and in equilibrium, borrowers face a lottery over loan rates, given by a distribution function F. Finally, we can summarize F as an analytical expression in Lemma 1. The proof of this is in section A.3.7.

A.3.1 Positive monopoly bank profit

Lemma 7. $\Pi^m(i) > 0$ *for* $i > i_d$.

Proof. For any positive markup $i - i_d$,

$$\begin{aligned}\Pi^m(i) &= n\alpha_1 R(i)\\ &= n\alpha_1 l^\star(m; i, p, \phi, M, \tau_b)\left[(1+i) - (1+i_d)\right].\end{aligned}$$

Since $l^\star(m; i, p, \phi, M, \tau_b) > 0$ and $i - i_d > 0$, then $\Pi^m(i) > 0$. □

A.3.2 All banks earn positive expected profit

Now, we prove that banks will earn strictly positive expected profits:

Lemma 8. $\Pi^{\star} > 0$.

Proof. Since we are restricting to a class of linear pricing rules, then, for any markup over marginal cost $\mu > 1$, the profit from positing $i = \mu i_d$ is

$$\begin{aligned}\Pi(\mu i_d) &= n\left[\alpha_1 + 2\alpha_2\left(1 - F(\mu i_d)\right) + \alpha_2 \xi(\mu i_d)\right] R(\mu i_d) \\ &> n\alpha_1 R(\mu i_d) = \Pi^m(\mu i_d) > 0,\end{aligned}$$

where $R(i) = l^{\star}(m; i, p, \phi, M, \tau_b)\left[(1+i) - (1+i_d)\right]$. The last inequality is from Lemma 7. From the definition of the max operator in (3.24),

$$\begin{aligned}\Pi^{\star} &= \max_{i \in \text{supp}(F)} \Pi(i) \\ &\geq \Pi(\mu i_d) > \Pi^m(\mu i_d) > 0.\end{aligned}$$

□

A.3.3 Maximal loan pricing

Third, we can also show that:

Lemma 9. *The largest possible price in the support of F is the smaller of the monopoly price and ex-post borrower's maximum willingness to pay: $\bar{i} := \min\{i^m, \hat{i}\}$.*

Although the monopoly rate i^m is the maximal possible price in defining an arbitrary support of F, it may be possible in some equilibrium that this exceeds the maximum willingness to pay by households, $\hat{i}$. We condition on this possibility when characterizing an *equilibrium* support of F later.

Proof. First assume the case that $\hat{i} \geq i^m$. Suppose there is a $\bar{i} \neq i^m$ which is the largest element in $\text{supp}(F)$. Then $\Pi^m(\bar{i}) = n\alpha_1 R(\bar{i})$. Since $F(i^m) \geq 0$ amd $\zeta(i^m) \geq 0$, then

$$\begin{aligned}\Pi(i^m) &= n\left[\alpha_1 + 2\alpha_2\left(1 - F(i^m)\right) + \alpha_2 \zeta(i^m)\right] R(i^m) \\ &\geq n\alpha_1 R(i^m) = \Pi^m(i^m) \\ &> \Pi^m(\bar{i}).\end{aligned}$$

The last inequality is true by the definition of a monopoly price i^m. Therefore $\Pi(i^m) > \Pi^m(\bar{i})$. The equal profit condition would require that, $\Pi^m\left(\bar{i}\right) = \Pi^\star \geq \Pi^m(i^m)$. Therefore $\bar{i} = i^m$ if $\hat{i} \geq i^m$.

Now assume $\hat{i} < i^m$. In this case, the most that a bank can charge for loans is $\hat{i}$, since at any higher rate, no ex-post buyer will execute his line of credit (i.e., he will not borrow). Thus trivially, $\bar{i} = \hat{i}$ if $\hat{i} < i^m$. □

A.3.4 Unique monopoly loan rate

Fourth, under a mild parametric regularity condition on preferences, we show that there is a unique monopoly loan rate.

Lemma 10. *Assume $\sigma < 1$. For an arbitrarily small constant $\epsilon > 0$, if $\sigma \geq \epsilon/(2+\epsilon)$, then there is a unique monopoly-profit-maximizing price i^m that satisfies the first-order condition*

$$\frac{\partial \Pi^m(i)}{\partial i} = n\alpha_1 \left[\frac{\partial l^\star(m; i, p, \phi, M, \tau_b)}{\partial i}(1+i) + l^\star(m; i, p, \phi, M, \tau_b) - \frac{\partial l^\star(m; i, p, \phi, M, \tau_b)}{\partial i}(1+i_d) \right] = 0.$$

Proof. Assume $\hat{i} > i^m$. Using the demand for loans from (3.17) the FOC at $i = i^m$ is explicitly

$$\underbrace{-\frac{m+\tau_b M}{p^{\frac{\sigma-1}{\sigma}}\phi^{-\frac{1}{\sigma}}}}_{f(i)} + \underbrace{\frac{1}{\sigma}(1+i)^{-\frac{1}{\sigma}}\left[(\sigma-1) + \frac{1+i_d}{1+i}\right]}_{g(i)} = 0. \tag{A.3.1}$$

Note that: Given individual state m, aggregate state M, and policy/prices (τ_b, p, ϕ), $f(i)$ is a constant w.r.t. i. Given i_d, $g(i)$ has these properties:

1. $g(i)$ is continuous in i;
2. $\lim_{i\searrow 0} g(i) = +\infty$;
3. $\lim_{i\nearrow +\infty} g(i) = 0$, and,
4. the RHS is monotone decreasing, $g'(i) < 0$.

The first three properties are immediate from (A.3.1). Since $\Pi^m(i)$ is twice-continuously differentiable, the last property can be shown by checking for a second-order condition:

For a maximum profit at $i = i^m$, we must have $\left.\frac{\partial^2 \Pi^m(i)}{\partial i^2}\right|_{i=i^m} \leq 0$. Observe that the second-derivative function is

$$\frac{\partial^2 \Pi^m(i)}{\partial i^2} = g'(i) = -\underbrace{\frac{1}{\sigma^2}(1+i)^{-\frac{1}{\sigma}-1}}_{>0}\left[(\sigma-1) + \frac{(1+\sigma)(1+i_d)}{(1+i)}\right] \tag{A.3.2}$$

For (A.3.2) to hold with ≤ 0, Case A ($\sigma < 1$) would require

$$\frac{(1+\sigma)(1+i_d)}{(1+i)} \geq 1 - \sigma$$

for all $i \geq i_d$.

Let $1+i \equiv (1+\epsilon)(1+i_d)$ since $i^m \geq i > i_d$. The above inequality can be re-written as

$$\frac{1}{1+\epsilon} \geq \frac{1-\sigma}{1+\sigma},$$

which implies

$$1 > \sigma \geq \frac{\epsilon}{2+\epsilon}. \tag{A.3.3}$$

Condition (A.3.3) is a sufficient condition on parameter σ to ensure that a well-defined and unique monopoly profit point exists with monopoly price $i^m \geq \underline{i} > i_d$ if $\frac{\epsilon}{2+\epsilon} \leq \sigma < 1$. □

A.3.5 Distribution is continuous

In the next two results, we show that the loan pricing distribution is continuous with connected support.

Lemma 11. *F is a continuous distribution function.*

We will prove Lemma 11 in two parts. First, we document a technical observation that the per-customer profit difference is always bounded above:

Lemma 12. *Assume there is an $i' < i$ and an $i'' < i'$, with*

$$\zeta(i) = \lim_{i' \nearrow i} \{F(i) - F(i')\} > 0,$$

and $\zeta(i') = \lim_{i'' \nearrow i'} \{F(i') - F(i'')\} > 0$, and that $R(i') > 0$. The per-customer profit difference is always bounded above: $\Delta := R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$.

Proof. The expected profit from posting i is

$$\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2 \zeta(i)] R(i).$$

The expected profit from posting i' is

$$\Pi(i') = n[\alpha_1 + 2\alpha_2(1 - F(i')) + \alpha_2 \zeta(i')] R(i').$$

A firm would be indifferent to posting either prices if $\Pi(i) - \Pi(i') = 0$. This implies that

$$(\alpha_1 + 2\alpha_2)[R(i) - R(i')] + \alpha_2 \zeta(i) R(i) - \alpha_2 \zeta(i') R(i') \\ - 2\alpha_2 [F(i) R(i) - F(i') R(i')] = 0.$$

Rearranging and using the definition of $\zeta(i) = \lim_{i' \nearrow i} \{F(i) - F(i')\} > 0$:

$$\begin{aligned} (\alpha_1 + 2\alpha_2)[R(i) - R(i')] &= \alpha_2 [F(i) R(i) - F(i') R(i')] - \alpha_2 \zeta(i') R(i') \\ &< \alpha_2 [F(i) R(i) - F(i') R(i')] \leq \alpha_2 \lim_{i' \nearrow i} \{F(i) - F(i')\} R(i). \end{aligned}$$

The strict inequality is because $R(i') > 0$ and $\zeta(i') > 0$. The subsequent weak inequality comes from the fact that $R(i)$ is continuous, so that we can write

$$\lim_{i' \nearrow i} \{F(i) R(i) - F(i') R(i')\} = \lim_{i' \nearrow i} \{F(i) - F(i')\} R(i).$$

Since $\zeta(i) = \lim_{i' \nearrow i} \{F(i) - F(i')\}$, the last inequality implies that $R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$. □

The follow provides the proof to Lemma 11.

Proof. Suppose there is a $i \in \text{supp}(F)$ such that $\zeta(i) > 0$ and

$$\Pi(i) = n[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2 \zeta(i)] R(i).$$

R is clearly continuous in i. Hence there is a $i' < i$ such that $R(i') > 0$ and from

Lemma 12, $\Delta := R(i) - R(i') < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$. Then

$$\begin{aligned}\Pi(i') &= n\left[\alpha_1 + 2\alpha_2(1 - F(i')) + \alpha_2 \zeta(i')\right] R(i') \\ &\geq n\left[\alpha_1 + 2\alpha_2(1 - F(i)) + \alpha_2 \zeta(i)\right]\left[R(i) - \Delta\right] \\ &\geq \Pi(i) + n\left\{\alpha_2 \zeta(i)\left[R(i) - \Delta\right] - (\alpha_1 + 2\alpha_2)\Delta\right\}.\end{aligned}$$

The first weak inequality is a consequence of $F(i) - F(i') \geq \zeta(i)$. Since $R(i) > \Delta$ and $\Delta < \frac{\alpha_2 \zeta(i) R(i)}{\alpha_1 + 2\alpha_2}$, then the last line implies $\Pi(i') > \Pi(i)$. This contradicts $i \in \text{supp}(F)$. □

A.3.6 Support of distribution is connected

Lemma 13. *The support of F, $\text{supp}(F)$, is a connected set.*

Proof. Pick two prices i and i' belonging to the set $\text{supp}(F)$, and suppose that $i < i'$ and $F(i) = F(i')$. The expected profit under these two prices are, respectively,

$$\Pi(i) = n\left[\alpha_1 + 2\alpha_2(1 - F(i))\right] R(i),$$

and,

$$\Pi(i') = n\left[\alpha_1 + 2\alpha_2(1 - F(i'))\right] R(i').$$

Since $F(i) = F(i')$, then the first terms in the profit evaluations above are identical:

$$n\left[\alpha_1 + 2\alpha_2(1 - F(i))\right] = n\left[\alpha_1 + 2\alpha_2(1 - F(i'))\right].$$

However, since i and i' belonging to the set $\text{supp}(F)$, then clearly, $i_d < i < i' \leq i^m$. From Lemma 10, we know that $R(i)$ is strictly increasing for all $i \in [i_d, i^m]$, so then, $R(i) < R(i')$. From these two observations, we have $\Pi(i) < \Pi(i')$. This contradicts the condition that if firms are choosing i and i' from $\text{supp}(F)$ then F must be consistent with maximal profit $\Pi(i) = \Pi(i') = \Pi^\star$ (viz. the equal profit condition must hold). □

A.3.7 Proof of Proposition 1

Proof. Consider the case where $\alpha_1 \in (0, 1)$. Since F has no mass points by Lemma 13, and is continuous by Lemma 11, then expected profit from any $i \in \text{supp}(F)$ is a continuous function over $\operatorname{supp}(F)$,

$$\Pi(i) = n\left[\alpha_1 + 2\alpha_2\left(1 - F(i)\right)\right] R(i),$$

where the image $\Pi\left[\operatorname{supp}(F)\right]$ is also a connected set. From Lemma 9, profit is maximized at $\Pi^m(i^m) = n\alpha_1 R(i^m)$. For any $i \in \operatorname{supp}(F)$, the induced expected profit must also be maximal, i.e.,

$$\Pi(i) = n\left[\alpha_1 + 2\alpha_2\left(1 - F(i)\right)\right] R(i) = n\alpha_1 R(i^m).$$

Solving for F yields the analytical expression (3.25).

Proof for the remaining case 2 and case 3 in Lemma 1 follows directly from Lemma 1 and Lemma 2 in Burdett and Judd (1983). The pricing outcomes, $\bar{i}$ and i_d are, respectively, the upper bound (the monopoly price) and the lower bound (Bertrand price) on the support of F. □

A.4 General money demand Euler equation

Evaluating the partial derivation of (3.10) one period ahead, combining this with the first-order condition (3.8) and the optimal goods demand functions in (3.12) and (3.14), we can derive an Euler functional describing the optimal money demand function. Re-writing this in terms of stationary variables, we have the steady-state Euler equation on z as:

$$\begin{aligned}
\underbrace{\frac{\gamma-\beta}{\beta}}_{\text{MC of extra dollar}} &= \underbrace{\Theta(z,Z;\tau_b)-1}_{\text{Net MB of extra dollar}} \\
&+ \underbrace{\mathbb{I}_{\{0\leq\rho<\hat{\rho}\}}\times n\alpha_0\left[\frac{1}{\rho}\left(\frac{z+\tau_b Z}{\rho}\right)^{-\sigma}-1\right]}_{\text{Liquidity cons, no bank contact (self-insure): Net MB of consumption from extra dollar}} \\
&+ \underbrace{n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{0\leq\rho<\tilde{\rho}_i\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i\right)\right)\right]i\mathrm{d}F(i)}_{\text{Borrow, MB of a dollar's less of borrowing is } i} \\
&+ \underbrace{n\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i\right)\right)\right]\left[\frac{1}{\rho}\left(\frac{z+\tau_b Z}{\rho}\right)^{-\sigma}-1\right]\mathrm{d}F(i)}_{\text{No Borrow, MB of consumption from extra dollar}}
\end{aligned}, \tag{A.4.1}$$

where the net marginal benefit of an extra dollar is decomposable as:

$$\begin{aligned}
\Theta(z,Z;\tau_b)-1 := &\underbrace{(1-n)(1+i_d)}_{\text{Expected gross value of additional deposit}} \\
&+ n\left[\alpha_0+\int_{\underline{i}}^{\bar{i}}\mathbb{I}_{\{0\leq\rho<\tilde{\rho}_i\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i\right)\right)\right]\mathrm{d}F(i)\right. \\
&\quad+\int_{\bar{i}}^{i^m}\mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i\right)\right)\right]\mathrm{d}F(i) \\
&\quad\left.+\int_{\bar{i}}^{i^m}\mathbb{I}_{\{\rho>\hat{\rho}\}}\left[\alpha_1+2\alpha_2\left(1-F\left(i\right)\right)\right]\mathrm{d}F(i)\right]-1
\end{aligned} \tag{A.4.2}$$

A.5 Omitted Proofs - First best allocation

A.5.1 Friedman rule

Proof. Suppose that $\gamma = \beta$ but that there is an SME with a non-degenerate distribution of loan interest rates, F.

Since we focus on $\alpha_1 \in (0, 1)$, from Lemma 1 (part 1), we know that if there is an SME, then the posted loan-rate distribution F is non-degenerate and continuous with connected support, $\text{supp}(F) = [\underline{i}, \bar{i}]$.

If there is an SME, then the general Euler condition for money demand (A.4.1) holds. However the marginal cost of holding money—i.e., LHS of (A.4.1)—is zero at the Friedman rule ($\gamma = \beta$). Also, the liquidity premium of carrying more real money balance at the margin into next period is always non-negative $u'(q)/c'(q) - 1 \geq 0$. What remains on the RHS of (A.4.1) are all the (net) marginal benefit of borrowing less at the margin when one has additional real balance, i.e., the integral terms. These terms are also non-negative measures. Thus, for an SME to hold, it must be that F is degenerate on a singleton set.

Since (A.4.1) holds in any SME, then our previous reasoning must further imply that the integral terms reduce to the condition $u'(q^f) = c'(q^f)$. We can compare this with the first best allocation. Given our CRRA preference representation assumption, the first-best allocation solving $u'(q^*) = c'(q^*)$ will yield $q^* = 1$.

Thus if there is an SME at the Friedman rule, then F must be degenerate. Moreover, at the Friedman rule, the allocation is Pareto efficient: $q^f = q^* = 1$.

□

A.6 Omitted Proofs - SME

This appendix supplies the intermediate results and proofs for establishing existence and uniqueness of a stationary monetary equilibrium with co-existing money and credit.

The conclusion is arrived at in a few intermediate steps. First, in section A.6.1 we show that a posted loan-price distribution with lower real money balance first-order stochastic dominance a distribution with higher real money balance, given a monetary policy rule $\gamma > \beta$. Second, in section A.6.2 we show that the general money demand Euler equation (A.4.1) can be simplified to (4.5), and the candidate real money balance solution to the money demand Euler equation is bounded. Third, we use results from section A.6.1 and section A.6.2 together in section A.6.3 to show there exists a unique real money balance that solves the money demand Euler equation in (4.5). This establishes existence. Finally, we prove for the uniqueness of a SME with co-existing money and credit in section A.6.4.

A.6.1 First-order stochastic dominance: Proof of Lemma 2.

Proof. The analytical formula for the loan-price distribution $F(i;\gamma,z)$ is characterized in (4.1). Suppose we fix $\bar{i}(z)=\bar{i}(z^{'})$, and denote it as $\bar{i}$. In general, the lower and upper support of the distribution F is changing with respect to z and policy γ. By fixing the upper support at both z and $z^{'}$ here, we are checking whether the curve of the cumulative distribution function, $F(\cdot)$, is lying on top or below for z relative to $z^{'}$. Next, differentiate $F(i;\gamma,z)$ with respect to z, we

$$\frac{\partial F(i;\gamma,z)}{\partial z}=\underbrace{\frac{\alpha_1}{2\alpha_2}}_{>0}\left[\frac{(\bar{i}-i_d)R(i;\gamma,z)-(i-i_d)R(\bar{i};\gamma,z)}{\underbrace{(R(i;\gamma,z))^2}_{>0}}\right]$$

For $\partial F(i;\gamma,z)/\partial z>0$ to hold, one needs to show the numerator is positive. Suppose this were not the case. Then we have

$$\begin{aligned}
&(\bar{i}-i_d)R(i;\gamma,z)-(i-i_d)R(\bar{i};\gamma,z)\leq 0\\
\implies\ &(\bar{i}-i_d)\underbrace{\left[(1+i)^{\frac{-1}{\sigma}}-z\right](i-i_d)}_{=R(i;\gamma,z)}\leq(i-i_d)\underbrace{\left[(1+\bar{i})^{\frac{-1}{\sigma}}-z\right](\bar{i}-i_d)}_{=R(\bar{i};\gamma,z)}\\
&\implies\left[(1+i)^{\frac{-1}{\sigma}}-z\right]\leq\left[(1+\bar{i})^{\frac{-1}{\sigma}}-z\right]
\end{aligned}$$

The last inequality contradicts to the fact that the loan demand curve is downward sloping in i, and $\bar{i}$ is the highest possible loan-price posted by banks (lending agents). Thus, the numerator must be >0, and $\partial F(i;\gamma,z)/\partial z>0$. This shows that a loan-price distribution $F(z)$ first-order stochastically dominates $F(z^{'})$, for $z<z^{'}$. □

A.6.2 Money and credit: Proof of Lemma 3

Proof. We want to show equivalence in the three claims in Lemma 3. The proof relies on a CRRA(σ) preference representation and linear cost of producing the DM good $c(q)=q$.

1. We say that the DM relative price ρ is sufficiently low if real money balance z is such that

$$\rho = 1 < \tilde{\rho}_i(z) \equiv (z)^{\frac{\sigma}{\sigma-1}} (1+i)^{\frac{1}{\sigma-1}}, \qquad 0 < \sigma < 1. \tag{A.6.1}$$

The following is a sufficient requirement: If $z < \left(\frac{1}{1+i}\right)^{\frac{1}{\sigma}}$, then inequality (A.6.1) holds. From Lemma 1, if $\alpha_1 \in (0,1)$, the distribution F is non-degenerate and $\text{supp}(F) = [\underline{i}(z), \bar{i}(z)]$ exists. This implies that for all $i \in \text{supp}(F)$, the inequality $z < \left(\frac{1}{1+\bar{i}(z)}\right)^{\frac{1}{\sigma}}$ is also true. Since SME $z = z^\star$ exists and $z^\star < \left(\frac{1}{1+\bar{i}(z^\star)}\right)^{\frac{1}{\sigma}}$, then ρ is sufficiently low and satisfies inequality (A.6.1).

2. From Claim 1 above, the DM relative price ρ satisfies inequality (A.6.1). From (4.13), there is ex-post positive loan demand by the active DM buyers who meet at least one bank. In the opposite direction: If there is ex-post positive loan demand, then condition (A.6.1) must hold, thus implying Claim 1.

3. Combining Claim 2 with agents' first-order condition for optimal money demand, we can reduce their Euler equation (A.4.1) to (4.5). In reverse, (4.5) implies that there is positive demand for loans and money (Claim 2).

□

A.6.3 Unique real money balance

Lemma 14. *Fix a long-run inflation target $\gamma > \beta$. Assume $\alpha_0, \alpha_1 \in (0,1)$. In any SME, there is a unique real money demand, $z^\star \equiv z^\star(\gamma)$.*

Proof. Consider the case where the long-run inflation target is set away from the Friedman rule, i.e., $\gamma > \beta$. From lemma 3, the money demand Euler equation is characterized by

$$i_d = \frac{\gamma - \beta}{\beta} = \underbrace{\alpha_0 \left(u'[q_b^0(z^\star)] - 1 \right)}_{=:A} + \underbrace{\int_{\underline{i}(z^\star)}^{\bar{i}(z^\star)} \mathbb{I}_{\{0<\rho<\tilde{\rho}_i\}} i \mathrm{d}J\left(i; z^\star\right)}_{=:B} \tag{A.6.2}$$

where

$$\mathrm{d}J(i;z^\star) = \underbrace{\left\{\alpha_1 + 2\alpha_2(1 - F(i;z^\star))\right\} f(i;z^\star)}_{=:j(i;z^\star)}\,\mathrm{d}i$$
$$\equiv \alpha_1 + 2\alpha_2(1 - F(i;z^\star))\mathrm{d}F(i;z^\star)$$

Recall that $1 \equiv \rho < \tilde{\rho}_i(z^\star)$ from lemma 3, the ex-post DM goods demand function for the event where the active DM buyer failed to meet with a lending bank is given by $q_b^0 = \frac{z}{\rho}$, i.e., she is liquidity constrained with own money balance. Thus, $\partial q_b^0/\partial z > 0$. Since $u^{''} < 0$, then $u^{'} \circ q_b^0(z)$ is continuous and decreasing in z. Thus, term A is continuous and decreasing in z.

Next, let

$$H(z) := \int_{\underline{i}(z)}^{\bar{i}(z)} i\mathrm{d}J(i;z)$$

and apply integration by parts, it yields

$$H(z) = \bar{i}(z) - \underbrace{\int_{\underline{i}(z)}^{\bar{i}(z)} J(i;z)\mathrm{d}i}_{\tilde{H}(z)}$$

Then apply Leibniz rule to $\tilde{H}(z)$, we have

$$\tilde{H}^{'}(z) = \bar{i}^{'}(z) + \int_{\underline{i}(z)}^{\bar{i}(z)} \frac{\partial J(i;z)}{\partial z}\mathrm{d}i$$

Overall, we have

$$H^{'}(z) = \bar{i}^{'}(z) - \tilde{H}^{'}(z) \;\; = -\int_{\underline{i}(z)}^{\bar{i}(z)} \frac{\partial J(i;z)}{\partial z}\mathrm{d}i$$

From lemma 2, we show $J(i;z) \underset{\text{FOSD}}{>} J(i;z^{'})$ for all $z < z^{'}$. Thus, $\partial J(i;z)/\partial z > 0$, which implies $H^{'}(z) < 0$. Thus, both terms A and B on the RHS of the money demand equation (A.6.2) are continuous and monotone decreasing in z. Moreover, the LHS of money demand equation (A.6.2) is constant with respect to z. Therefore, there exists a unique real money demand $z^\star(\gamma)$ that solves (A.6.2). Moreover, $z^\star(\gamma)$ is bounded from lemma 3. ∎

A.6.4 SME with money and credit: Proof of Proposition 5

Proof. From lemma: 2, 3, and 14, we show there exists a unique money demand such that $0 < z^\star \equiv z^\star(\gamma) < 1$ for a given $\gamma > \beta$. To complete for uniqueness, we need to check where equilibrium requirements (market clearing, loans feasibility) are simultaneously satisfied at $z^\star$.

First, recall the loans feasibility constraint evaluating at $z = z^\star$ requires to be:

$$\underbrace{n \int_{\underline{i}(z)}^{\bar{i}(z)} \{\alpha_1 + 2\alpha_2(1 - F(i;,\gamma,z))\} \xi(i;\gamma,z) i \mathrm{d}F(i;\gamma,z)}_{\text{total loans interest}} \geq \underbrace{(1-n) i_d \delta(z;\gamma)}_{\text{total deposits interest}} \tag{A.6.3}$$

Let $\mathrm{d}J(i;\gamma,z) := \{\alpha_1 + 2\alpha_2(1 - F(i;,\gamma,z))\} dF(i;\gamma,z)$.

Recall: depositors deposit all of their money balance in the bank (depository institution), $\delta(z;\gamma) = z > 0$, if and only if $i_d > 0$ (i.e., $\gamma > \beta$).

Rewrite (A.6.3) as

$$n \int_{\underline{i}(z)}^{\bar{i}(z)} \xi(i;\gamma,z) i \mathrm{d}J(i;\gamma,z) \geq (1-n) i_d z \tag{A.6.4}$$

Observe that both sides on (A.6.4) must be non-negative value. Then rearrange, we can get

$$n \geq \frac{z i_d}{z i_d + \int_{\underline{i}(z)}^{\bar{i}(z)} \xi(i;\gamma,z) i \mathrm{d}J(i;\gamma,z)} \tag{A.6.5}$$

Consider both limiting cases in (A.6.5):

- $\int_{\underline{i}(z)}^{\bar{i}(z)} \xi(i;\gamma,z) i \mathrm{d}J(i;\gamma,z) \to \infty$. In this case, if total loans interest is sufficiently high, the RHS of (A.6.5) approaches to zero. This says n need to be at least as great as a very small number (approaching to zero). Intuitively, if total revenue from loans is sufficiently high, banks do not need to have a high n (share of active DM buyers) to be able to (weakly) cover deposit interest.

- $\int_{\underline{i}(z)}^{\bar{i}(z)} \xi(i;\gamma,z) i \mathrm{d}J(i;\gamma,z) \to 0$. Similarly as above, this case would mean n need to be at least as great as a very larger number (approaching to one) for banks to cover deposit interest.

Clearly, n can't be the case that $n < 0$ nor $n > 1$ since it's a probability of consuming in the DM. Thus, a sufficient condition for deposit interest feasible at equilibrium given $z^\star(\gamma > \beta)$, n needs to satisfy an endogenous bound pinned down by (A.6.5), such that $n \geq N[z^\star(\gamma)] \subset [0, 1]$ require to hold.

Next, the competitive price-taking DM goods market clearing condition is

$$q_s(z;\gamma) = n\left\{\alpha_0 q_b^0(z;\gamma) + \int_{\underline{i}(z)}^{\bar{i}(z)} q_b(i;z,\gamma)\mathrm{d}J(i;\gamma,z)\right\} \tag{A.6.6}$$

DM firms' optimal production rule is pinned down by a constant marginal cost of production since they face with linear production technology. Thus, aggregate supply (by DM firms' labor endowment) has to equal to the aggregate demand in the DM goods market. Finally, given a constant optimal CM consumption (due to quasi-linear preference) $x^\star$, real money balance $z^\star$ and DM allocations $(q_b^{0,\star}(\cdot), q_b^\star(\cdot))$, we can verify that the CM goods and labor market also clear. Overall, these results and lemma 14 together establish that there exists a unique SME with co-existing money and credit. □

A.7 A stochastic version of the baseline model

This appendix supplies a stochastic version of the baseline model setup that lead to the characterization of a SME with shocks (demand fluctuation), and the objective of a Ramsey policy problem for demand stabilization.

In contrast to the perfectly-competitive banking environment of Berentsen, Camera and Waller (2007), our model now has non-trivial consequences for the design of optimal monetary and (redistributive) tax policy. Mathematically, the latter can be gleaned from the fact that the money-demand Euler equation in (A.4.1) now also depends on τ_b the tax/transfer to active buyers, which can be treated differentially from the tax/transfers from/to other agents. The counterpart of τ_b in Berentsen, Camera and Waller (2007) disappears from the equilibrium characterization, since perfect competition in banks eliminate the need for redistributive taxation. With noisy search for loans, this is no longer the case.

In this section, we construct what would be optimal Ramsey policies for taxation, taking monetary supply growth as a given constraint on the policy problem. In particular, we consider a similar long-run Ramsey policy design to that in Berentsen and

Waller (2011). We will show how differential or redistributive tax policies provide an additional tool to over risk or instability in the overall demand for loans.

A.7.1 Shocks

We now consider n and ϵ are being random variables to capture demand fluctuation in the DM. The random variable ϵ is interpreted as taste shock of the DM special goods, and the random variable n is interpreted as shock that affects the number of active DM buyers. In particular, n has support $[\underline{n},\bar{n}] \in (0,1)$ and ϵ has support $[\underline{\epsilon},\bar{\epsilon}], 0 < \underline{\epsilon} < \bar{\epsilon} < \infty$. Let $\omega = (n, \epsilon) \in \Omega$ denote as the aggregate state (vector) in DM, where $\Omega = [\underline{n},\bar{n}] \times [\underline{\epsilon},\bar{\epsilon}]$. Let $\psi(\omega)$ denote the density function of ω.

A.7.2 Monetary policy

Monetary policy here is similar to Berentsen and Waller (2011). In particular, the central bank implements it's long-term inflation goal by providing lump-sum injections of liquidity, τM, to the households at the beginning of the period. Let $\tau_1(\omega)$ and $\tau_2(\omega)$ denote the total state-contingent policy in DM and CM respectively given the realization of state ω. We also assume any state-contingent injection of liquidity received by the DM agents will be undone in CM, i.e., $\tau_2(\omega) = -\tau_1(\omega)$.[30]

Notice that the central bank could potentially treat DM active buyers, DM inactive buyers and (measure of one) sellers differently with their state-contingent policy in response to ω shocks by $\tau_1(\omega) = n(\omega)\tau_b(\omega) + [1 - n(\omega)]\hat{\tau}_b(\omega) + \tau_s(\omega) \geq 0$.[31]

Given the assumption of DM state-contingent policy will be undone in the CM, the total new money transferred within each period is deterministic since τM is state independent,

$$(\gamma - 1) M = (\tau + \tau_1(\omega) + \tau_2(\omega)) M = \tau M \tag{A.7.1}$$

or equivalently, the gross growth rate of money stock is depending only on the

[30]The state-contingent policy plan can be thought as repo agreement make by the central bank where they sell money in DM and promised to buy that back in CM.

[31]What really matters in our current setup is $\tau_b(\omega)$ and $\hat{\tau}_b(\omega)$ since $\tau_s(\omega)$ do not show up in the Euler equation for real money demand (A.4.1), and we can show the sellers do not carry money in each DM. Also, the state-contingent tax/transfer to inactive buyers $\hat{\tau}_b(\omega)$ affect the quantity of deposit which will affect the feasibility constraint for loans (how relaxed or binding of that is) without affecting the real money demand equation. For this reason, we let $\hat{\tau}_b(\omega) = 0$, $\tau_s(\omega) = 0$, and focus on $\tau_b(\omega)$ for now. However, there could be a redistribution tax policy (between active and inactive buyers) to be considered.

(deterministic) inflation rate τ

$$\gamma = 1 + \tau = \frac{M_{+1}}{M} \tag{A.7.2}$$

A.7.3 Characterization of SME with shocks

The markets structure of the model is the same as in baseline except that ϵ and n are random variables now.[32] We also work with stationary variables and restrict attention to stationary monetary equilibrium where end-of-period real money balances are both time and state invariant

$$\phi M = \phi_{+1} M_{+1} = z, \qquad \text{for all } \omega \in \Omega$$

and stationary money supply growth is

$$\frac{\phi}{\phi_{+1}} = \frac{M_{+1}}{M} = \frac{p_{+1}}{p} = \gamma = 1 + \tau$$

meaning that the central bank engages in price-level targeting by choosing a path for the money stock in CM as in Berentsen and Waller (2011).

Before we go into the description of stationary monetary equilibrium with shocks, let's first discuss the components of it.

Note: For ease of notation, the explicit state-dependency sometimes might be dropped.[33]

Ex-post households with positive bank contact. In events with probability measure α_1 and α_2, the buyer's optimal demand for DM consumption and loan is respectively characterized by

$$q_b^{1,\star}(z; i, \rho, Z, \gamma, \tau_b, \omega) = \begin{cases} \epsilon^{\frac{1}{\sigma}} \left[\rho(1+i)\right]^{-\frac{1}{\sigma}} & \text{if } 0 < \rho \leq \tilde{\rho}_i \text{ and } 0 \leq i \leq \hat{i} \\ \frac{z + \tau_b Z}{\rho} & \text{if } \tilde{\rho}_i < \rho < \hat{\rho} \text{ and } i > \hat{i} \\ \epsilon^{\frac{1}{\sigma}} \rho^{-\frac{1}{\sigma}} & \text{if } \rho \geq \hat{\rho} \text{ and } i > \hat{i} \end{cases}, \tag{A.7.3}$$

[32]If we treat ϵ and n as parameter, and set $\epsilon = 1$, then we are back to the baseline case.

[33]Note: there's time and state dependency of variable, e.g., $X_t(s)$ and $X_{t+1}(s)$. Since we now work with stationary variable, it becomes $X(s)$, and we just write X when we drop the state dependency notation.

and,

$$\xi^{\star}(z;i,\rho,Z,\gamma,\tau_b,\omega)=\begin{cases}\epsilon^{\frac{1}{\sigma}}\rho^{\frac{\sigma-1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}-(z+\tau_b Z) & \text{if } 0<\rho\leq\tilde{\rho}_i \text{ and } 0\leq i\leq\hat{i}\\ 0 & \text{if } \tilde{\rho}_i<\rho<\hat{\rho} \text{ and } i>\hat{i}\\ 0 & \text{if } \rho\geq\hat{\rho} \text{ and } i>\hat{i}\end{cases}, \tag{A.7.4}$$

where

$$\hat{\rho}:=\hat{\rho}(z;Z,\gamma,\omega)=\epsilon^{-\left(\frac{1}{\sigma-1}\right)}(z+\tau_b Z)^{\frac{\sigma}{\sigma-1}},$$
$$\tilde{\rho}_i:=\hat{\rho}\,(1+i)^{\frac{1}{\sigma-1}},$$
$$\text{and } \hat{i}=\epsilon\,(z+\tau_b Z)^{-\sigma}\rho^{\sigma-1}-1>0$$

where both ex-post demand functions require to hold for each realization of state $\omega\in\Omega$.

Ex-post households with zero bank contact. The buyer's optimal demand for DM consumption (for events with probability measure α_0) is

$$q_b^{0,\star}(z;\rho,Z,\gamma,\tau_b,\omega)=\begin{cases}\frac{z+\tau_b Z}{\rho} & \text{if } \rho\leq\hat{\rho}\\ \epsilon^{\frac{1}{\sigma}}\rho^{-\frac{1}{\sigma}} & \text{if } \rho\geq\hat{\rho}\end{cases} \tag{A.7.5}$$

where

$$\hat{\rho}:=\hat{\rho}(z;Z,\gamma,\omega)=\epsilon^{-\left(\frac{1}{\sigma-1}\right)}(z+\tau_b Z)^{\frac{\sigma}{\sigma-1}}$$

which requires to hold for each realization of state $\omega\in\Omega$.

Firms. The firm's optimal production plan satisfies

$$c_q(q_s)=p\phi, \qquad \omega\in\Omega \tag{A.7.6}$$

where the marginal cost of producing is equal to the real relative price of DM goods for each realization of state ω.

Hypothetical monopolist lending bank. We can derive the closed-form loan-price posting distribution similar to the baseline, except that the distribution is both state and policy dependent now. Given a realization of shock ω, this bank's "monopoly" profit function is

$$\Pi^m(i) = n\alpha_1 R(i)$$

To pin down an monopoly loan price, differentiate the bank's "monopoly" profit function wrt. i, the (stationary variable version) FOC is

$$-\underbrace{z + \tau_b Z}_{f(i)} + \underbrace{\frac{1}{\sigma}\epsilon^{\frac{1}{\sigma}}(1+i)^{-\frac{1}{\sigma}}\left[(\sigma-1) + \frac{1+i_d}{1+i}\right]}_{g(i)} = 0 \tag{A.7.7}$$

which needs to hold for each realization of state ω.

Observe that in (A.7.7), for a given individual state z, aggregate state Z, trend inflation rate τ, state ω, and $\omega \mapsto \tau_b(\omega)$, $f(i)$ is a constant w.r.t. i, and $g(i)$ is decreasing in i. Thus, there exists a unique monopoly-profit-maximizing price i^m that satisfies the above FOC for each realization of state ω.

Once we pin down this $i^m(\omega)$, then we use the equal profit condition combining with the upper support of the distribution $\bar{i}(\omega) := \min\{i^m(\omega), \hat{i}(\omega)\}$ to derive the lower support of the distribution $\underline{i}$, which together pin down the closed-form loan-price posting distribution for each realization of state ω.

Equilibrium real money demand. Similar to the baseline case, we differentiate the DM value function wrt. m, update one period and plug that into CM first-order condition (wrt. m_{+1}). Convert the result using stationary variables and combining that with ex-post optimal goods demand functions (A.7.3) and (A.7.5) in DM, and then we get the Euler equation for real money demand as

$$
\begin{aligned}
\frac{\gamma-\beta}{\beta} &= \theta\left(z, Z; \tau_b, \omega\right) - 1 \\
&+ \int_{\omega\in\Omega} n\mathbb{I}_{\{\rho<\hat{\rho}\}}\alpha_0 \left[\frac{1}{\rho}\epsilon\left(\frac{z+\tau_b(\omega)z}{\rho}\right)^{-\sigma} - 1\right]\psi\left(\omega\right)\mathrm{d}\omega \\
&+ \int_{\omega\in\Omega} n \int_{\underline{i}}^{\bar{i}} \mathbb{I}_{\{\rho<\tilde{\rho}_i\}}\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i;\omega\right)\right)\right] i\mathrm{d}F\left(i;\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+ \int_{\omega\in\Omega} n \int_{\underline{i}}^{\bar{i}} \mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i;\omega\right)\right)\right] \\
&\times \left[\frac{1}{\rho}\epsilon\left(\frac{z+\tau_b(\omega)z}{\rho}\right)^{-\sigma} - 1\right]\mathrm{d}F\left(i;\omega\right)\psi\left(\omega\right)\mathrm{d}\omega
\end{aligned}
\tag{A.7.8}
$$

where $F(i;\omega) \equiv F(i;\omega,\tau_b(\omega))$, and,

$$
\begin{aligned}
\theta\left(z, Z; \tau_b, \omega\right) - 1 := &\int_{\omega\in\Omega} (1-n)\left(1+i_d\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+ \int_{\omega\in\Omega} n\alpha_0\psi\left(\omega\right)\mathrm{d}\omega \\
&+ \int_{\omega\in\Omega} n \int_{\underline{i}}^{\bar{i}} \mathbb{I}_{\{\rho<\tilde{\rho}_i\}}\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i;\omega\right)\right)\right]\mathrm{d}F\left(i;\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+ \int_{\omega\in\Omega} n \int_{\bar{i}}^{i^{\mathrm{m}}} \mathbb{I}_{\{\tilde{\rho}_i\leq\rho<\hat{\rho}\}}\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i;\omega\right)\right)\right]\mathrm{d}F\left(i;\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&+ \int_{\omega\in\Omega} n \int_{\bar{i}}^{i^{\mathrm{m}}} \mathbb{I}_{\{\hat{\rho}\leq\rho\}}\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i;\omega\right)\right)\right]\mathrm{d}F\left(i;\omega\right)\psi\left(\omega\right)\mathrm{d}\omega \\
&- 1
\end{aligned}
$$

Interpretation of money demand equation (A.7.8): LHS captures the marginal cost of accumulating an extra unit of real money balance at the end of each CM, and RHS captures the expected marginal utility value of that extra unit of money balance (evaluated at the beginning of next DM before shock is realized and before buyer types, matching and trading occurs).

Equilibrium loan price-posting distribution. We restrict to the case $\alpha_1 \in (0,1)$ for the stochastic version here. Distribution of loan (interest-rate) price posts $F(i; z, \gamma, \tau_b, \omega)$

at each state ω is given by:

$$F(i;\omega) := F\left(i;z,\gamma,\tau_b,\omega\right) = 1 - \frac{\alpha_1}{2\alpha_2}\left[\frac{R\left(\bar{i};\omega\right)}{R\left(i;\omega\right)} - 1\right], \tag{A.7.9}$$

and, $\text{supp}\left(F\right) = \left[\underline{i}(\omega), \bar{i}(\omega)\right]$, and, given largest possible price $\bar{i}(\omega) = \min\{i^m(\omega), \hat{i}(\omega)\}$, $\underline{i}(\omega)$ solves:

$$R\left(\underline{i};\omega\right) = \frac{\alpha_1}{\alpha_1 + 2\alpha_2} R\left(\bar{i};\omega\right) \tag{A.7.10}$$

where the (real) bank profit per customer served is

$$R\left(i;\omega\right) := R\left(i;z,\rho,Z,\gamma,\omega\right) = \left[\epsilon^{\frac{1}{\sigma}}\rho^{\frac{\sigma-1}{\sigma}}\left(1+i\right)^{-\frac{1}{\sigma}} - \left(z+\tau_b Z\right)\right]\left(i - i^d\right) \tag{A.7.11}$$

Observe that in (A.7.3), (A.7.4), all those cut-off functions (in terms of relative price of DM goods or lending interest rate) are all now depending on a given $\omega \mapsto \tau_b(\omega)$ function, and also on $\omega := (\epsilon, n)$ states of the economy.

Similarly, for the loan pricing mechanism, the upper support and lower support of the loan interest rate distribution distribution (A.7.9) is now also depending on a given $\omega \mapsto \tau_b(\omega)$ function, and also on $\omega := (\epsilon, n)$ states of the economy. This can be seen from the optimal monopoly rate (solved by the hypothetical monopolist bank's FOC (A.7.7)), households' reservation interest rate $\hat{i}(\omega)$, and the associate lowest possible loan rate of the distribution $\underline{i}(\omega)$. The key difference between ϵ shocks and n shocks is that the former have one extra moving part in affecting (A.7.9) (via banks' trade-offs), and the latter have one less moving part.

Equilibrium competitive price taking and goods market clearing. DM goods market clears for all ω:

$$\begin{aligned} q_s\left(z;Z,\gamma\right) &\equiv c^{'-1}\left(\rho\right) \\ &= \int_\Omega n\alpha_0 q_b^{0,\star}\left(z;\rho,Z,\gamma,\tau_b,\omega\right)\psi(\omega)\mathrm{d}\omega \\ &+ \int_\Omega n\int_{\underline{i}}^{\bar{i}}\left[\alpha_1 + 2\alpha_2\left(1 - F\left(i;\omega\right)\right)\right] q_b^{1,\star}\left(z;i,\rho,Z,\gamma,\tau_b(\omega),\omega\right)\mathrm{d}F\left(i;\omega\right)\psi(\omega)\mathrm{d}\omega \end{aligned} \tag{A.7.12}$$

We can also verify that the CM labor and goods market clear given the SME solutions $\{z^\star, q_b^{0,\star}, q_b^{1,\star}\}$

Aggregate feasibility of loanable funds in banking market. Interests on total loans weakly exceed that on total deposits

$$\int_\Omega n \int_{\underline{i}}^{\bar{i}} [\alpha_1 + 2\alpha_2 (1 - F(i;\omega))] \xi^\star (z; i, \rho, Z, \gamma, \tau_b, \omega) i \mathrm{d}F(i;\omega) \psi(\omega) \mathrm{d}\omega$$

$$\geq \int_\Omega (1-n) i_d \delta^\star(z, Z, \gamma, \omega) \psi(\omega) \mathrm{d}\omega \qquad \text{(A.7.13)}$$

$$\equiv \int_\Omega (1-n) i_d \left(\frac{z + \hat{\tau}_b(\omega) Z}{\rho} \right) \psi(\omega) \mathrm{d}\omega$$

for each realization of state $\omega \in \Omega$.

We now summarize the description of a SME below.

Definition 15. Assume $\sigma < 1$. Given money supply growth γ, and redistributive policy plan $\{\tau_1(\omega), \tau_2(\omega)\}_{\omega\in\Omega}$ a *Stationary Monetary Equilibrium (SME)* is a list of time- and state-invariant CM consumption allocation and residual real money balance outcomes $\{x^\star \equiv 1, z^\star\}$, and time-independent allocation functions for DM goods and loans, $\{q^\star(z^\star, \omega), \xi^\star(z^\star, \omega)\}$, and loan pricing (distribution) function, $F(\cdot; z^\star, \omega, \gamma, \tau_b)$ such that:

1. household optimization satisfies the Euler equation for money demand (A.7.8)
2. distribution of posted loan (interest-rate) price satisfies (A.7.9)
3. DM goods market clearing satisfies (A.7.12)
4. loans feasibility satisfies (A.7.13)
5. government budget constraint holds for each ω, i.e.,

$$\frac{\gamma - \beta}{\beta} = \tau + \tau_1(\omega) + \tau_2(\omega), \quad \tau_1(\omega) = -\tau_2(\omega) \qquad \text{(A.7.14)}$$

A.7.4 Optimal stabilization policy over SME with shocks

To understand how the stabilization policy in response to demand fluctuation may work, we compare two types of government policy:

1. **Active central bank**. The policymaker commits to an ex-ante, optimal policy plan that maximizes social welfare over a steady-state equilibrium (i.e., a SME). In particular, the active central bank solves

$$\begin{aligned} \max_{\{q_b^0(\omega),\ q_b^1(\omega),\tau_b(\omega)\}_{\omega\in\Omega}} \quad & U(x) - x - c(q_s) \\ & + \int_{\omega\in\Omega} n\alpha_0 \epsilon u\left[q_b^0(z;\gamma,\tau_b,\omega)\right]\psi(\omega)\,\mathrm{d}\omega \\ & + \int_{\omega\in\Omega} n \int_{\underline{i}}^{\bar{i}} \left[\alpha_1 + 2\alpha_2\left(1 - F(i;z,\gamma,\tau_b,\omega)\right)\right] \\ & \times \epsilon u\left[q_b^1(z;i,\gamma,\tau_b,\omega)\right] \mathrm{d}F(i;z,\gamma,\tau_b,\omega)\,\psi(\omega)\,\mathrm{d}\omega \end{aligned} \tag{A.7.15}$$

subject to optimal money demand (A.7.8) distribution of loan interest rates (A.7.9), DM goods market clearing (A.7.12), loans feasibility (A.7.13) and government budget feasibility (A.7.14), where q_s is given by (A.7.12).

Note: The policy plan prescribes ω-contingent liquidity injections. That is, $\tau_1(\omega) = \tau_b(\omega) \geq 0$.

2. **Passive central bank**. In this regime, the policymaker is constrained by $\tau_1(\omega) = \tau_2(\omega) = 0$ for all $\omega \in \Omega$. The outcomes will be very similar to our deterministic, baseline SME.

Objective of the active central bank is similar to Berentsen and Waller (2011). New insights arises from the equilibrium varying dispersion of loan-price markups since $F(i;\omega)$ is now both state and policy dependent.