ECON526: Problem Set 0

Viktoriya Hnatkovska Jesse Perla

Instructions

- This problem set covers linear algebra and optimization material from the MA Math Camp.
- Submit as PDF on Canvas

Question 1

Minimize the function $x^2 + y^2 + z^2$ subject to

$$\begin{aligned} 3x + y + z &= 7 \\ x + y + z &= 1 \end{aligned}$$

Question 2.1

For the following optimization problem:

1. Show the Lagrangian
2. Provide all of the first-order conditions for the optimization problem
3. Solve the optimization problem for the x value and any Lagrange multipliers, being formal in the use of equations and inequalities where possible.

$$\begin{aligned} &\max_x \{2x\} \\ &\text{s.t. } x^2 \leq 5 \end{aligned}$$

Question 2.2

For the following optimization problem:

1. Show the Lagrangian
2. Provide all of the first-order conditions for the optimization problem
3. Solve the optimization problem for the x value and any Lagrange multipliers, being formal in the use of equations and inequalities where possible.

$$\max_x \{2x\}$$
$$\text{s.t. } x^2 \geq 5$$

Question 3

Consider a household whose utility is defined over consumption of two goods, x and y, and is given by the following function:

$$u(x, y) = (ax^\eta + by^\eta)^{\frac{1}{\eta}},$$

where $\eta \leq 1$ is a parameter governing the elasticity of substitution between goods x and y, and $0 < a < 1$ and $0 < b < 1$.

Prices of the goods are determined in competitive markets and are given by p_x and p_y. The household has a total income of m. (i) Write down the budget constraint facing the household. (ii) Find the utility-maximizing demand for the two goods.

Question 4

Being formal and explicit about the rules of matrix algebra (e.g., when operations are commutative, distributive, and when invertibility is required), solve for $x \in \mathbb{R}^N$. Constants: vectors $b, c, d \in \mathbb{R}^N$, matrices $A, B, Q, R \in \mathbb{R}^{N \times N}$, and scalar $m \in \mathbb{R}$.

1. $(I - A)\, x = b + c$
2. $Ax + (x^\top B)^\top = d$

Question 5

1. Find a unit vector $x \in \mathbb{R}^2$ such that $x \cdot \begin{bmatrix} 3 & 4 \end{bmatrix} = 0$. Hint: perpendicular vectors to any $\begin{bmatrix} a & b \end{bmatrix}$ is $\begin{bmatrix} -b & a \end{bmatrix}$ or $\begin{bmatrix} b & -a \end{bmatrix}$ and a unit vector is one where $||x||_2 = 1$.
2. Given any nonzero vectors $u, v \in \mathbb{R}^N$, explain how to test using the norm and inner product whether they are (i) orthogonal and (ii) collinear, and if collinear, whether they point in the same or opposite direction.

Question 6

1. What is the span of the vectors $\begin{bmatrix} 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 2 & 4 \end{bmatrix}$ in $\mathbb{R}^2$?
2. For a set of vectors $u^1, \dots, u^k \in \mathbb{R}^n$, what are the possible dimensions of $\text{span}\{u^1, \dots, u^k\}$? State the minimum and maximum values and when they occur.