General equilibrium analysis of the Eaton-Kortum model of international trade
Alvarez, F. and Lucas, R (JME, 2007)

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Summary

- Competitive, CRS, multicountry Ricardian model of trade
- Determinants of cross-country distribution of trade volumes (size, distance, tariffs)
- Estimate welfare gains from hypothetical trade liberalizations

Closed Economy Environment

- L consumers. Utility: $u(c) = c$
- Continuum of intermediate goods. Aggregated

$$q = \left[\int_0^\infty q(x)^{1-1/\eta} \phi(x) dx \right]^{\eta/(\eta-1)} \tag{1}$$

- Labor and aggregate are allocated in the two sectors:

$$s_f + \int_0^\infty s(x)\phi(x)dx \leq 1 \tag{2}$$

$$q_f + \int_0^\infty q_m(x)\phi(x)dx \leq q \tag{3}$$

- Intermediate goods production ($x \sim exp(\lambda)$, $x^{-\theta} \sim$ Frechet and $\uparrow \theta \rightarrow \uparrow var$)

$$q(x) = x^{-\theta} s(x)^\beta q_m(x)^{1-\beta} \tag{4}$$

- One final good

$$c = s_f^\alpha q_f^{1-\alpha} \tag{5}$$

Closed Economy Solution

All prices set at marginal cost: w (wage), p (final good), $p(x)$ (tradables) and p_m (intermediate aggregate)

$$p = \alpha^{\alpha}(1-\alpha)^{-(1-\alpha)} w^{\alpha} p_m^{1-\alpha} \tag{6}$$

$$p(x) = Bx^{\theta} w^{\beta} p_m^{1-\beta} \tag{7}$$

$$p_m = \left(\lambda \int_0^{\infty} e^{-\lambda x} p(x)^{1-\eta} dx\right)^{1/(1-\eta)} = ABw^{\beta} p_m^{1-\beta} \lambda^{-\theta} \tag{8}$$

where $B = \beta^{\beta}(1-\beta)^{-(1-\beta)}$ and $A = \left(\int_0^{\infty} e^{-z} z^{\theta(1-\eta)} dz\right)^{1/(1-\eta)}$

For $A < \infty$ we need $1 + \theta(1-\eta) > 0$. So elasticity of substitution is bounded above. η changes units of measurement of $p(x)$ but not allocations.

Quantities are recovered from Cobb-Douglas share formulas

n Countries Model

- $L = (L_1, ..., L_n)$, $\lambda = (\lambda_1, ..., \lambda_n)$, $x = (x_1, ..., x_n)$. θ, β, α and η constant
- "Iceberg" costs: 1 unit shipped from j to i results in κ_{ij} in i
- Tariffs: 1 dollar spent in i on goods from j becomes ω_{ij} dollars paid to seller in j. The remainder $(1 - \omega_{ij})$ is lump sum transfer to people in i
- Costs are independent across countries

$$\phi(x) = \left(\prod_{i=1}^{n} \lambda_i \right) exp \left\{ - \sum_{i=1}^{n} \lambda_i x_i \right\} \tag{9}$$

n Countries Model

- Let $q_i(x)$, q_i consumption of tradeable x and the aggregate in i. With prices $p_i(x)$ and p_{mi} respectively. Adjusting 7:

$$p_i(x) = B \min_j \left[\frac{w_j^{\beta} p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} x_j^{\theta} \right] \Rightarrow p_i(x)^{1/\theta} = B^{1/\theta} \min_j \left[\frac{w_j^{\beta/\theta} p_{mj}^{(1-\beta)/\theta}}{(\kappa_{ij}\omega_{ij})^{1/\theta}} x_j \right] \tag{10}$$

- Use two properties of exponential distribution:
  - $x \sim exp(\lambda) \Rightarrow kx \sim exp(\lambda/k)$
  - $x \sim exp(\lambda_x)$,$y \sim exp(\lambda_y)$ independent$\Rightarrow z = min(x, y) \sim exp(\lambda_x + \lambda_y)$
  - So $p_i(x)^{1/\theta}$ is distributed exponentially with parameter $\mu = B^{-1/\theta} \sum_{j=1}^{n} \psi_{ij}$ where $\psi_{ij} = \left(\frac{w_j^{\beta} p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} \right)^{-1/\theta} \lambda_j$
  - Price of the aggregate

$$p_{mi}^{1-\eta} = \int p_i(x)^{1-\eta} \phi(x) dx \Rightarrow p_{mi}(w) = AB \left(\sum_{j=1}^{n} \psi_{ij} \right)^{-\theta} \tag{11}$$

n Countries Model

- D_{ij} fraction of country i's spending on tradeables spent on goods from country j: probability that for a good x the lowest price is in j

$$D_{ij} = Pr\left\{ \frac{w_j^{\beta} p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} x_j^{\theta} \leq min_{k\neq j}\left[\frac{w_k^{\beta} p_{mk}^{1-\beta}}{\kappa_{ik}\omega_{ik}} x_k^{\theta} \right] \right\} \tag{12}$$

- Using the property: $x \sim exp(\lambda_x), y \sim exp(\lambda_y)$ independent $\rightarrow Pr(x \leq y) = \lambda_x/(\lambda_x + \lambda_y)$ then

$$D_{ij} = \frac{\psi_{ij}}{\sum_{k=1}^{n} \psi_{ik}} \tag{13}$$

- Trade balance:

$$L_i p_{mi} q_i \sum_{j=1}^{n} D_{ij}\omega_{ij} = \sum_{j=1}^{n} L_j p_{mj} q_j D_{ji}\omega_{ji} \tag{14}$$

n Countries Model

Accounts for country i as a 3 sector economy

Sector	Value added	Labor income / Taxes
Services	$L_i p_i c_i - L_i p_{mi} q_{fi}$	$L_i w_i s_{fi}$
Tradeables	$\sum_j L_j p_{mj} q_j D_{ji} \omega_{ji} - L_i p_{mi} q_{mi}$	$L_i w_i (1 - s_{fi})$
Importing	$L_i p_{mi} q_i - L_i p_{mi} q_i \sum_j D_{ij} \omega_{ij}$	$L_i p_{mi} q_i \sum_{j \neq i} D_{ij}(1 - \omega_{ij})$
Total	$L_i p_i c_i$	$L_i w_i + L_i p_{mi} q_i \sum_{j \neq i} D_{ij}(1 - \omega_{ij})$

Using trade balance (14) and the above identities:

$$L_i w_i (1 - s_{fi}) = \sum_{j=1}^{n} L_j \frac{w_j (1 - s_{fj})}{F_j} D_{ji} \omega_{ji} \tag{15}$$

where $s_{fi} = \frac{\alpha[1-(1-\beta)F_i]}{(1-\alpha)\beta F_i + \alpha[1-(1-\beta)F_i]}$ and $F_i = \sum_{j=1}^{n} D_{ij} \omega_{ij}$

Excess demand function

$$Z_i(w) = \frac{1}{w_i} \left[\sum_{j=1}^{n} L_j \frac{w_j (1 - s_{fj})}{F_j} D_{ji} \omega_{ji} - L_i w_i (1 - s_{fi}) \right] \tag{16}$$

$\omega = 1$ vs $\omega < 1$

Assume $L_i = L$ and $\lambda_i = \lambda$. Uniform tariff $\omega_{ij} = \omega$

- Gains by reducing prices but lost of revenue from tariffs
- $Welfare|_{\omega=0.7} - Welfare|_{\omega=0.8} < Welfare|_{\omega=0.6} - Welfare|_{\omega=0.7}$
- Effects of changes in ω on Import/GDP are large for all sizes

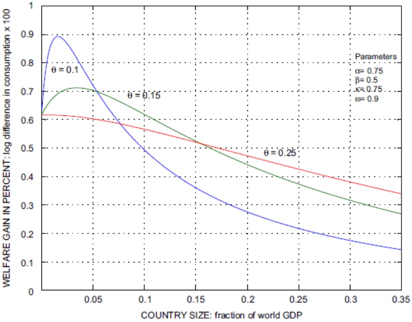
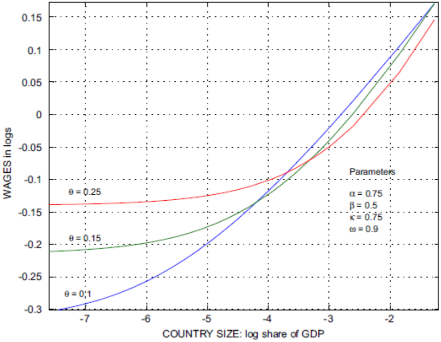
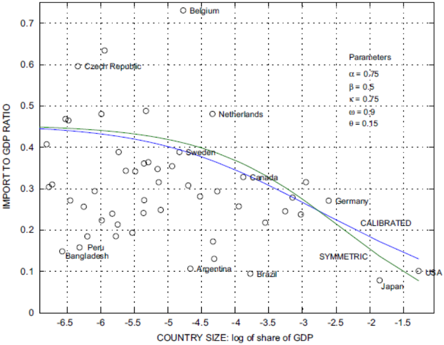


Figure: Welfare gains from eliminating tariffs

$\lambda/L = 1$

With $\lambda_i = \lambda$ small countries would produce as many goods as big countries, so wages are driven by size.
Two sets of equations: $Z(w, L) = 0$, $L\hat{w}(w, L) = Y$
Size matters for trade volume, but the configuration of the rest of the world matters very little.





$\lambda/L = 1$

Specification	(1)	(2)	(3)	(4)	(5)
Correlation	0.49	0.59	0.61	0.62	0.69
$\lambda_i = L_i$	X	X	X	X	X
$\omega_i \neq \omega_j$		X	X		X
FTA			X		X
$\kappa_{ij} < 1$				X	X

Table: Correlation between log trade in data and the model

Small Open Economy

$\omega_{1j} = \omega$, $\omega_{i,1} = \hat{\omega}$. Other economies behave as if there were $n-1$ countries.

Higher variability in productivity allows country to impose higher tariffs.

Optimal is around $\theta/(1+\theta)$, optimal markup charged by monopolist with same elasticity.

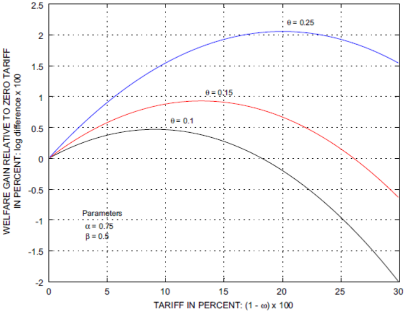


Figure: Welfare effect from eliminating tariffs - Small Open Economy

Relative Productivities / Relative Prices

Matching large variations of price of tradeable to non-tradeable goods requires goods measures of λ/L (productivities), which imply large variations in wages.

Use data from GDP and relative prices (p/p_m)

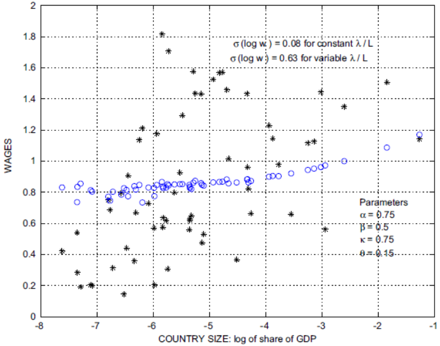


Figure: Wages for λ/L constant and variable

Conclusions

- Tractable model solved with simple algorithm
- Analysis of changes in tariffs. Effect on trade and welfare
- Does not include capital or technology difussion